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Brief paper

Linear approximation and identification of MIMO Wiener–Hammerstein systems ☆

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ABSTRACT

This paper considers the linear approximation and identification of multi-input multi-output (MIMO) Wiener–Hammerstein systems, or LNL systems. Evaluating the input–output cross-covariance matrix of the MIMO LNL system for Gaussian inputs, we show that the best linear approximation of the MIMO LNL system in the mean square sense can be obtained by the orthogonal projection (ORT) subspace identification method. For each allocation of the poles of the best linear approximation between the two linear subsystems, the unknown parameters in the numerators of the linear subsystems and the coefficients of a basis function expansion of the nonlinearity are estimated by applying the separable least-squares. The best LNL system is the one that gives the minimum mean square output error. A numerical example is included to show the feasibility of the present approach.

1. Introduction

The classical result due to Bussgang (1952) that exactly computes the input–output correlation function of a nonlinear system has been applied to the identification of single-input single-output (SISO) Wiener–Hammerstein systems, or LNL systems (Billings & Fakhouri, 1982; Hunter & Korenberg, 1986), where a static nonlinearity (N) is sandwiched by two linear (L) subsystems. For recent developments in the identification of LNL systems; see Greblicki (2012), Mu and Chen (2014), Schoukens, Pintelon, and Enqvist (2008). Also, the result of Bussgang (1952) has been employed to develop the multivariable output-error state space (MOESP)-based methods for identifying multi-input multi-output (MIMO) Wiener and Hammerstein systems (Verhaegen & Westwick, 1996; Westwick & Verhaegen, 1996). The linear approximation problems for SISO nonlinear finite impulse response (NFIR) systems, that include LNL systems, have extensively been studied using the classical Wiener theory (Enqvist & Ljung, 2005), extending the results of Bussgang (1952). Moreover, a partitioning approach for identifying an SISO Wiener–Hammerstein system has been developed by using the best linear approximation of it (Sjöberg, Lauwers, & Schoukens, 2012; Sjöberg & Schoukens, 2012), therein the consistency of the initialization algorithm is shown.

In Ase and Katayama (2015), motivated by the subspace-based approaches (Verhaegen & Westwick, 1996; Westwick & Verhaegen, 1996) and by the partitioning method (Sjöberg & Schoukens, 2012), we have presented a subspace-based method of identifying the Wiener–Hammerstein benchmark model by using the orthogonal projection (ORT) subspace method (Picci & Katayama, 1996) and the separable least-squares (Golub & Pereyra, 1973). In this paper, we deal with the linear approximation and identification of MIMO Wiener–Hammerstein systems, extending the SISO results of Ase and Katayama (2015) to MIMO systems.

We first derive the input–output cross-covariance matrix of an MIMO LNL system for Gaussian inputs, from which we show that the best linear approximation, or the best linear model, of the LNL system in the mean square sense is obtained by replacing the static nonlinearity with the equivalent gain matrix. It is also shown that the best linear model is consistently identified using the ORT subspace method.

To identify the LNL system, the poles of the best linear model are then allocated between the two linear subsystems by

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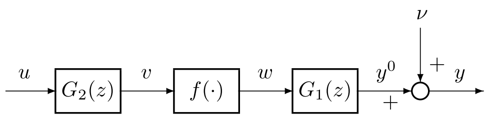

Fig. 1. LNL system.

using a state transformation (Ase & Katayama, 2015). For each realizable allocation of the poles, we estimate unknown system parameters and the coefficients of a basis function expansion of the nonlinearity by the separable least-squares, which is solved using a gradient-based optimization method (Wills & Ninness, 2008); the best LNL configuration is selected based on the mean square output error.

The paper is organized as follows. Section 2 computes the input–output cross-covariance matrix of MIMO LNL systems. In Section 3, we consider the best linear approximation of the LNL system from the point of view of the orthogonal projection. The identification procedure of the best linear model by the ORT method is discussed in Section 4, and Section 5 explains a method of allocating the poles of the best linear model between the two linear subsystems. Based on a basis function expansion of the nonlinearity, we outline a method of identifying MIMO LNL systems in Section 6, and a feasibility study is included in Section 7. Section 8 concludes this paper.

Notation: $E\{\cdot\}$ denotes the mathematical expectation, and $\hat{E}\{\cdot \mid \cdot\}$ orthogonal projection. Two random vectors a and b are said to be orthogonal, or uncorrelated, if $E\{ab^{\mathrm{T}}\} = 0$, which is expressed as $a \perp b$.

2. Input–output cross-covariance matrix

We consider the LNL system shown in Fig. 1, where $G_1(z)$ and $G_2(z)$ are linear subsystems, and $f : \mathbb{R}^r \to \mathbb{R}^q$ is a static nonlinearity. Also, $u(t) \in \mathbb{R}^m$ is the input, $v(t) \in \mathbb{R}^r$ is the output of $G_2(z)$, $w(t) \in \mathbb{R}^q$ is the output of the nonlinearity and the input to $G_1(z)$, $y^0(t) \in \mathbb{R}^p$ is the noise-free output, $y(t) \in \mathbb{R}^p$ is the output, and $\nu(t) \in \mathbb{R}^p$ is the output noise.

To ensure that all the variables are 2nd-order stationary random processes, we assume the following.

Assumption 1. (i) The input $u(t)$ is a zero mean stationary Gaussian process with a finite covariance matrix, and $\nu(t)$ is a zero mean white noise sequence.

(ii) The linear subsystems $G_1(z)$ and $G_2(z)$ are stable, and there is no pole-zero cancellation between them (Anderson & Gevers, 1981).

(iii) The nonlinearity is a measurable function, and the variance of output of the nonlinearity is bounded, i.e.

$$E\{|f_{ij}(v)|^2\} < \infty, \quad i = 1, \dots, q;\ j = 1, \dots, r$$

and each element of the nonlinearity is well approximated by a basis function expansion (Ljung, 1999). □

The input–output cross-covariance matrix is defined by $R_{yu}(\tau) = E\{y(t)u^{\mathrm{T}}(t-\tau)\}$, $\tau = 0, \pm 1, \dots$, and other covariance matrices are defined similarly. Let $G_i^{(\cdot)}$ be the impulse response matrices of $G_i(z)$, i.e.

$$G_i(z) = \sum_{k=0}^{\infty} G_i^{(k)} z^{-k}, \quad i = 1, 2.$$

Under Assumption 1, we show the following result, an MIMO extension of the Bussgang's result (Bussgang, 1952).

Proposition 1. *The input–output cross-covariance matrix of the LNL system of Fig. 1 is given by*

$$R_{yu}(\tau) = \sum_{k=0}^{\infty} G_1^{(k)} F^e \sum_{j=0}^{\infty} G_2^{(j)} R_{uu}(\tau - j - k) \tag{1}$$

where $R_{uu}(\cdot)$ is the covariance matrix of the input $u(t)$, and F^e is the equivalent gain matrix of the nonlinearity defined by $F^e := E\{f(v(t))v^{\mathrm{T}}(t)\}R_{vv}^{-1}(0) \in \mathbb{R}^{q\times r}$ (Roberts & Spanos, 1990), where $R_{vv}(0)$ is the covariance matrix of $v(t)$.

Proof. See Appendix. □

In the next section, we derive a useful orthogonal projection result from (1).

3. Linear approximation of LNL system

Let $\mathcal{U}_t^- := \overline{\operatorname{span}}\{u(t), u(t-1), \dots\}$ be the linear subspace spanned by the past inputs, where the over-bar denotes the closure in mean square. Let the orthogonal projection onto $\mathcal{U}_t^-$ be defined by $\hat{E}\{\cdot \mid \mathcal{U}_t^-\}$.

By the definition of the covariance matrix, we see that (1) is equivalent to the following condition

$$y(t) - \sum_{j=0}^{\infty} G_1^{(j)} \sum_{k=0}^{\infty} F^e G_2^{(k)} u(t-j-k) \perp u(t-\tau)$$

for $\tau = 0, \pm 1, \dots$. Define

$$y^d(t) = \sum_{j=0}^{\infty} G_1^{(j)} F^e \sum_{k=0}^{\infty} G_2^{(k)} u(t-j-k). \tag{2}$$

Then, $y^d(t)$ satisfies $y(t) - y^d(t) \perp \mathcal{U}_t^-$ and $y^d(t) \in \mathcal{U}_t^-$, implying that $y^d(t)$ is the orthogonal projection of $y(t)$ onto the subspace $\mathcal{U}_t^-$, i.e. $y^d(t) = \hat{E}\{y(t) \mid \mathcal{U}_t^-\}$.

It follows from (2) that $y^d(t) = G_1(z)F^eG_2(z)u(t)$. By the property of orthogonal projection, we see that $y^d(t)$ is the linear least-squares estimate of $y(t)$ given the past inputs (Enqvist & Ljung, 2005). Hence,

$$G_d(z) := G_1(z)F^eG_2(z) \tag{3}$$

is the best linear approximation, or the best linear model, of the LNL system. The best linear model is also called the deterministic system.

Remark 1. The best linear model $G_d(z)$ minimizes the effect of the nonlinearity in the mean square sense (Enqvist & Ljung, 2005; Schoukens, Pintelon, Dobrowiecki, & Rolain, 2005), so that it depends on the statistics of the input to the nonlinearity. In fact, $G_d(z)$ is the transfer matrix obtained by replacing the static nonlinearity $f(\cdot)$ in Fig. 1 with the equivalent gain matrix F^e. □

Remark 2. Since there exists no pole-zero cancellation in the right-hand side of (3) by Assumption 1(ii), the poles of $G_d(z)$ are the sum of poles of $G_1(z)$ and $G_2(z)$. This fact will be utilized for partitioning the poles of the best linear model into the two linear subsystems.[1] □

[1] We can partition the zeros of $G_d(z)$, if the inverse $G_d^{-1}(z)$ exists, i.e. if the matrix $D_1F^eD_2$ of (7) is nonsingular. In this paper, however, we do not consider the partitioning of zeros, since it seems that the case where $G_d(z)$ is invertible is very rare in practice.