Automatica 71 (2016) 143–150

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Exact predictor feedbacks for multi-input LTI systems with distinct input delays☆

Daisuke Tsubakino [a,1], Miroslav Krstic [b], Tiago Roux Oliveira [c]

[a] Department of Aerospace Engineering, Nagoya University, Nagoya 464-8603, Japan
[b] Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093-0411, USA
[c] Department of Electronics and Telecommunication Engineering, State University of Rio de Janeiro, Rio de Janeiro, RJ 20550-900, Brazil

ARTICLE INFO

Article history:
Received 18 May 2015
Received in revised form
29 December 2015
Accepted 15 April 2016
Available online 31 May 2016

Keywords:
Delay compensation
Predictor feedback
Multi-input systems
Backstepping
Lyapunov stability

ABSTRACT

This paper proposes a predictor-based state feedback controller for multi-input linear time-invariant (LTI) systems with different time delays in each individual input channel. The controller is derived based on the backstepping method. Since the conventional backstepping transformation is not applicable to the systems due to the differences among delays, a modified transformation is introduced. This transformation enables us to design an exponentially stabilizing controller under which the plant behaves as if the delays were absent after a finite time interval. As a dual of the controller design, we also present the observer design for multi-output LTI systems with distinct sensor delays. A numerical simulation confirms the performance of the proposed controller.

1. Introduction

This paper considers a stabilization problem of the following linear time-invariant (LTI) systems with distinct input delays:

$$\dot{X}(t) = AX(t) + \sum_{i=1}^{m} b_i U_i(t - D_i), \tag{1}$$

where $X(t) \in \mathbb{R}^n$ is the state and the ith control channel $U_i(t) \in \mathbb{R}$ is delayed by $D_i > 0$, $i \in \{1, \ldots, m\}$. Stabilization of dynamical systems in the presence of input delays has been widely studied in the field of control engineering (Gu & Niculescu, 2003; Richard, 2003). A typical approach is the predictor-based controller (Artstein, 1982; Kwon & Pearson, 1980; Lewis, 1979; Manitius & Olbrot, 1979; Watanabe & Ito, 1981). According to them, we can design the following control law:

$$U_i(t) = k_i^\top \left[e^{AD_i} X(t) + \sum_{j=1}^{m} \int_{t-D_j}^{t} e^{A(t+D_i-\theta-D_j)} b_j U_j(\theta) d\theta \right], \tag{2}$$

where $i \in \{1, 2, \ldots, m\}$ (see Example 5.2 in Artstein, 1982). This control law exponentially stabilizes the system (1), if the gains $k_i \in \mathbb{R}^n$, $i = 1, \ldots, m$ are chosen so that the matrix

$$A + \sum_{i=1}^{m} e^{-AD_i} b_i k_i^\top e^{AD_i} \tag{3}$$

is Hurwitz.

The idea of predictor-feedback is to realize $U_i(t) = k_i^\top X(t + D_i)$ (or, equivalently, $U_i(t - D_i) = k_i^\top X(t)$). The variation-of-constants formula shows that the solution $X(t)$ of (1) satisfies

$$X(t + D_i) = e^{AD_i} X(t) + \sum_{j=1}^{m} \int_{t-D_j}^{t-D_{ji}} e^{A(t+D_i-\theta-D_j)} b_j U_j(\theta) d\theta, \tag{4}$$

where $D_{ji} := D_j - D_i$ for each $i, j \in \{1, 2, \ldots, m\}$. Clearly, (2) does not imply $U_i(t) = k_i^\top X(t + D_i)$, unless $m = 1$ or $D_1 = D_2 = \cdots = D_m$. This fact seems to be of little importance, because the exponential stability of the closed-loop system is guaranteed as

☆ This work was supported in part by JSPS KAKENHI Grant Number 15K18085. The material in this paper was partially presented at the 2015 American Control Conference, July 1–3, 2015, Chicago, IL, USA. This paper was recommended for publication in revised form by Associate Editor Jamal Daafouz under the direction of Editor Richard Middleton.

E-mail addresses: tsubakino@nuae.nagoya-u.ac.jp (D. Tsubakino), krstic@ucsd.edu (M. Krstic), tiagoroux@uerj.br (T.R. Oliveira).

[1] Tel.: +81 52 789 4499.

http://dx.doi.org/10.1016/j.automatica.2016.04.047
0005-1098/© 2016 Elsevier Ltd. All rights reserved.

long as the matrix (3) is Hurwitz. However, the stability condition depends on delays except in the case of $m = 1$ or $D_1 = D_2 = \cdots = D_m$. Hence, even if the nominal feedback law $U_i(t) = k_i^\top X(t)$ stabilizes the undelayed system, the predictor feedback (2) does not always stabilize (1). We need to abandon the nominal design.

Recently, another interpretation of the predictor-feedback was made in Krstic (2009) for single-input systems. The predictor-based controller is naturally derived by applying the infinite-dimensional backstepping method (Krstic & Smyshlyaev, 2008; Meurer, 2013; Vazquez & Krstic, 2008). In this approach, we represent systems with an input delay by a cascade of an ordinary differential equation (ODE) and a transport partial differential equation (PDE). Then, a state transformation, called the backstepping transformation, is used to convert the original system into a stable target system. The feedback control law is obtained as a condition under which the transformation is accomplished. As the main feature of the backstepping approach, we can construct an explicit Lyapunov functional of the closed-loop system, which brings some benefit as pointed out in Krstic (2009, 2008).

Actually, an extension of the backstepping approach to the systems given by (1) is available by specializing the result in Bekiaris-Liberis and Krstic (2011), which deals with more general distributed delays. The resulting controller is the same as (2). However, this result does not seem to be a multi-input counterpart of the result in Krstic (2009). First of all, we need to use the backstepping-forwarding transformation. Since the system (1) does not contain distributed delays, the forwarding part should be unnecessary. In addition, the transformation involving the forwarding part is not always invertible (Bribiesca Argomedo & Krstic, 2015). The other reason is that the structure of the target system used there is completely different from the one in Krstic (2009).

The purpose of this paper is to obtain a predictor-based controller that is more compatible with the variation-of-constant formula (4) by extending the backstepping approach to multi-input LTI systems with distinct input delays. The goal is achieved by introducing a new backstepping-like transformation. This is the main contribution of this paper. The resulting controller has a structure that is naturally expected from the variation-of-constants formula. Furthermore, it is guaranteed that the closed-loop system behaves as if the nominal static feedback control were realized after a finite time interval. This fact is an advantage of the proposed approach, since we can exploit the nominal feedback gain. An explicit Lyapunov functional for the closed-loop system is available. In addition to the predictor-feedback controller, we also derive an observer for multi-output systems with distinct output delays by developing a dual method. This is one of the substantial differences from our earlier conference paper (Tsubakino, Oliveira, & Krstic, 2015).

The organization of the paper is as follows. In Section 2, we present controller design using the proposed transformation. Section 3 is devoted to the stability analysis of the closed-loop system. A Lyapunov functional is introduced. In Section 4, we develop an observer design method as a dual result of the foregoing two sections. The effectiveness of the proposed controller is demonstrated by a numerical simulation in Section 5.

2. Controller design

Without loss of generality, we can assume that the control inputs are ordered so that $0 \le D_1 \le D_2 \le \cdots \le D_m$. It is convenient to let $D_0 = 0$. Set $B = (b_1, b_2, \ldots, b_m) \in \mathbb{R}^{n\times m}$. We also suppose that the pair (A, B) is stabilizable. In other words, there exists a matrix $K = (k_1, k_2, \ldots, k_m)^\top \in \mathbb{R}^{m\times n}$ such that $A + BK$ is Hurwitz. Let us represent the system (1) as the ODE–PDE cascade

$$\dot{X}(t) = AX(t) + \sum_{i=1}^{m} b_i u_i(0, t), \tag{5}$$

$$\partial_t u_i(x, t) = \partial_x u_i(x, t), \quad x \in (0, D_i), \tag{6}$$

$$u_i(D_i, t) = U_i(t), \quad i \in \{1, 2, \ldots, m\}. \tag{7}$$

The equivalence between (1) and (5)–(7) can be seen by noticing that the solution of (6) under the condition (7) is given by $u_i(x, t) = U_i(x + t - D_i)$ for $x \in [0, D_i]$ and $t \ge D_i - x$.

The main procedure of backstepping is to find a state transformation and a state feedback control law that convert the system (5)–(7) into a stable target system. We employ the following target system:

$$\dot{X}(t) = (A + BK)X(t) + \sum_{i=1}^{m} b_i w_i(0, t), \tag{8}$$

$$\partial_t w_i(x, t) = \partial_x w_i(x, t), \quad x \in (0, D_i), \tag{9}$$

$$w_i(D_i, t) = 0, \quad i \in \{1, 2, \ldots, m\}. \tag{10}$$

The solution to (9) with (10) satisfies $w_i(x, t) = 0$ for any $x \in [0, D_i]$ after $t = D_i$. Hence, the state X satisfies

$$\dot{X}(t) = (A + BK)X(t), \quad t \ge D_m.$$

Thus, the plant obeys the nominal closed-loop equation after $t = D_m$. The stability with respect to an appropriate norm will be discussed later.

If $m = 1$, we can use the standard backstepping transformation proposed in Krstic (2009). Even if $m \ne 1$, we can easily obtain a multi-variable version of the backstepping transformation in the case of identical delays, that is, $D_1 = D_2 = \cdots = D_m$. The main difficulty in our case is that each u_i has a different spatial domain $[0, D_i]$ due to the discrepancy of delays. For this reason, we propose a new state transformation that is suitable to the system (5)–(7).

2.1. Backstepping-like transformation

For each $i \in \{1, 2, \ldots, m\}$, define a function $\phi_i : [0, D_m] \to [0, D_i]$ and the matrix $A_i \in \mathbb{R}^{n\times n}$ by

$$\phi_i(x) = \begin{cases} x, & 0 \le x \le D_i, \\ D_i, & D_i < x \le D_m, \end{cases} \tag{11}$$

$$A_i = A_{i-1} + b_i k_i^\top, \tag{12}$$

where $A_0 = A$. Obviously, we have $A_m = A + BK$. Let Φ be the state transition matrix generated by

$$F(t) = \begin{cases} A, & t \in [0, D_1), \\ A_i, & t \in [D_i, D_{i+1}),\ i = 1, 2, \ldots, m-1, \\ A_m, & t \ge D_m. \end{cases}$$

The explicit expression of $\Phi(x, y)$ is given by

$$\Phi(x, y) = e^{A_i(x-D_i)} e^{A_{i-1}(D_i - D_{i-1})} \cdots e^{A_{j+1}(D_{j+2}-D_{j+1})} e^{A_j(D_{j+1}-y)}, \quad D_i \le x \le D_{i+1}, \quad D_j \le y \le D_{j+1} \tag{13}$$

for $i, j \in \{0, 1, \ldots, m-1\}$ such that $i > j$, and

$$\Phi(x, y) = e^{A_i(x-y)}, \quad D_i \le y \le x \le D_{i+1} \tag{14}$$

for any $i \in \{0, 1, \ldots, m-1\}$. We must understand (13) as $\Phi(x, y) = e^{A_i(x-D_i)} e^{A_{i-1}(D_i-y)}$ if $j = i - 1$. See Fig. 1 for the case of $m = 3$. It should be noted that Φ is continuous, but not differentiable on the lines represented by $x = D_i$ or $y = D_i$ for some $i \in \{1, \ldots, m-1\}$.

Consider the following transformation:

$$w_i(x, t) = u_i(x, t) - k_i^\top \Phi(x, 0)X(t) - \sum_{j=1}^{m} \int_0^{\phi_j(x)} k_i^\top \Phi(x, y) b_j u_j(y, t) dy \tag{15}$$