Automatica 71 (2016) 151–158

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Robust H_∞ filter design with past output measurements for uncertain discrete-time systems[☆]

Luciano Frezzatto[a], Maurício C. de Oliveira[b,1], Ricardo C.L.F. Oliveira[a], Pedro L.D. Peres[a]

[a] School of Electrical and Computer Engineering, University of Campinas – UNICAMP, 13083-852, Campinas, SP, Brazil

[b] Department of Mechanical and Aerospace Engineering, University of California San Diego, La Jolla, CA, 92093-0411, USA

ARTICLE INFO

Article history:
Received 12 November 2015
Received in revised form
22 January 2016
Accepted 15 April 2016
Available online 31 May 2016

Keywords:
Robust H_∞ filtering
Memory filters
Linear time-invariant systems
Uncertain discrete-time systems
Linear Matrix Inequalities

ABSTRACT

This paper is concerned with the design of robust filters for uncertain discrete-time linear systems. The uncertainty is assumed to be time-invariant belonging to a polytope. An upper bound to the H_∞ norm of the transfer-function from the system input to the filtering error is used as performance criterion. The novelty is that the resulting robust filter has order greater than the order of the system being filtered by incorporating past output measurements. The design conditions are expressed in terms of convex problems constrained by Linear Matrix Inequalities. Performance is assessed through parameter-dependent Lyapunov functions. The results generalize existing robust filtering procedures without bringing in any additional conservatism. Simple numerical examples illustrate the procedure.

1. Introduction

Estimation and filtering of signals corrupted by noise have a vast range of applications, for instance in navigation and control of aerospace vehicles. In these applications, the Kalman filter (Kalman, 1960) is widely deployed. Kalman filters assume that all noise terms and measurements have Gaussian distributions and that a precise knowledge of the underlying linear system model is available (Anderson & Moore, 1979). H_∞ filters have become a popular way to overcome the first assumption (El Sayed & Grimble, 1989; Grimble, 1988). Optimal H_∞ filters are designed to minimize the ℓ_2-induced gain from the noise input to the estimation error. Other than having bounded energy, no assumption is made on the nature of the input noise distribution for H_∞ filters. More recently, H_∞ filters have been extended to take into account uncertainties in the dynamic models, as an attempt to overcome the second assumption on Kalman filters (Geromel, Bernussou, Garcia, & de Oliveira, 2000; Palhares, de Souza, & Peres, 2001).

Robust H_∞ filters have been designed based on Riccati equations (de Souza, Shaked, & Fu, 1995) or by means of Linear Matrix Inequalities (LMIs) (Geromel et al., 2000). LMIs provide the basic framework for the present paper. For uncertain systems, it is very difficult to design filters while exactly minimizing the H_∞ norm of the filtering error transfer-function. Virtually all conditions in the literature that can be computed efficiently have some degree of conservativeness and much effort has been dedicated to try to reduce such conservativeness. At first, LMIs for H_∞ filtering were based on the concept of quadratic stability, where a common Lyapunov matrix is used to ensure the stability of the entire uncertainty domain (Geromel, 1999; Geromel et al., 2000; Palhares et al., 2001). LMI conditions based on parameter-dependent Lyapunov functions were later introduced to overcome the conservativeness of quadratic stability (Chesi, 2007; Geromel, de Oliveira, & Bernussou, 2002). These conditions make use of Lyapunov functions that are affine in the uncertain parameter with the introduction of slack variables (de Oliveira & Skelton, 2001). Slack variables were later used to introduce Lyapunov functions with higher-order polynomial dependence on the uncertainty, with an associate certificate of stability and performance being obtained as the degree grows (Chesi, Garulli, Tesi, & Vicino, 2003; Scherer, 2005).

☆ The first, third, and fourth authors are supported by the Brazilian agencies CAPES, CNPq and grant 2014/23074-2 from São Paulo Research Foundation (FAPESP). The material in this paper was partially presented at the 2016 American Control Conference, July 6–8, 2016, Boston, MA, USA. This paper was recommended for publication in revised form by Associate Editor Erik Weyer under the direction of Editor Torsten Söderström.

E-mail addresses: luciano@dt.fee.unicamp.br (L. Frezzatto), mauricio@ucsd.edu (M.C. de Oliveira), ricfow@dt.fee.unicamp.br (R.C.L.F. Oliveira), peres@dt.fee.unicamp.br (P.L.D. Peres).

[1] Tel.: +1 858 822 3492.

http://dx.doi.org/10.1016/j.automatica.2016.04.050
0005-1098/

Some newer strategies have been recently introduced in an attempt to further reduce conservativeness and bring the H_∞ norm bounds closer to the worst-case norm of the uncertain filtered system. One of such strategies includes designing filters whose order is higher than the order of the system been filtered, as it is done in the present paper. Some examples are Geromel and Korogui (2008), Lee and Joo (2014) and Frezzatto, Lacerda, Oliveira, and Peres (2015). In the present paper, a design procedure for memory filters that employs past output measurements is proposed. As illustrated by simple numerical examples, the proposed procedure is able to design filters associated with less conservative H_∞ bounds when compared to some competing approaches (Frezzatto et al., 2015; Lee & Joo, 2014), usually with filters of smaller order. Moreover, we also show that the H_∞ guaranteed costs attained by the proposed method do not increase when an increasing number of past output measurements are considered. Given a desired number of past output measurements, the robust filter is computed by solving a convex problem with LMI constraints. A much shorter preliminary and limited version of some of the results in the present paper has appeared in (Frezzatto, de Oliveira, Oliveira, & Peres, 2016).

The paper is organized as follows. In Section 2, some preliminary results are introduced and the filtering problem is stated. The main results and discussions are addressed in Section 3. Section 4 proves the non-increasing behavior of the H_∞ guaranteed cost and proposes a procedure to impose constraints on the order of the designed filters. Finally, we close the paper in Section 5 with some conclusions and final remarks.

The notation used throughout this paper is standard. For a symmetric matrix, $X \succ 0$ ($X \prec 0$) means that X is positive (negative) definite. For matrices or vectors $(^T)$ indicates the transpose. The symbol $\star$ means a symmetric term in a block matrix.

2. Preliminaries and problem statement

Consider the linear time-invariant discrete-time uncertain system described by

$$\begin{aligned} x(k+1) &= A(\xi)\,x(k) + B(\xi)\,w(k) \\ y(k) &= C_y(\xi)\,x(k) + D_y(\xi)\,w(k) \\ z(k) &= C_z(\xi)\,x(k) + D_z(\xi)\,w(k) \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $w(k) \in \mathbb{R}^m$ is the disturbance input, $z(k) \in \mathbb{R}^p$ is the signal to be estimated and $y(k) \in \mathbb{R}^q$ is the measured output. The uncertain matrices have compatible dimensions and are assumed to belong to a polytopic domain described in terms of a vector of time-invariant parameters ξ. For instance,

$$A(\xi) = \sum_{i=1}^{N} \xi_i A_i, \quad \xi \in \Xi_N \tag{2}$$

where Ξ_N is the unit simplex given by

$$\Xi_N \triangleq \left\{ \xi \in \mathbb{R}^N : \sum_{i=1}^{N} \xi_i = 1,\ \xi_i \geq 0, i = 1, \dots, N \right\}. \tag{3}$$

The aim is to design a robust filter that at instant k employs the last h, $h \geq 1$, output measurements, $y(k-\ell)$, $\ell = 0, \dots, h-1$, in order to obtain filters with improved performance. The filter to be designed has a linear time-invariant discrete-time realization:

$$\begin{aligned} \hat{x}(k+1) &= \hat{A}\,\hat{x}(k) + \sum_{\ell=0}^{h-1} \hat{B}_\ell\, y(k-\ell) \\ \hat{z}(k) &= \hat{C}\,\hat{x}(k) + \sum_{\ell=0}^{h-1} \hat{D}_\ell\, y(k-\ell) \end{aligned} \tag{4}$$

where $\hat{x}(k) \in \mathbb{R}^{\hat{n}}$ and $\hat{z}(k) \in \mathbb{R}^p$ is the estimated output and all matrices have compatible dimensions. Note that filter (4) has the following state-space realization:

$$\begin{aligned} x_f(k+1) &= A_f\, x_f(k) + B_f\, y(k) \\ z_f(k) &= C_f\, x_f(k) + D_f\, y(k) \end{aligned} \tag{5}$$

where

$$\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right] = \left[\begin{array}{ccccc|c} 0 & I & & & 0 & 0 \\ & \ddots & \ddots & & & \vdots \\ & & & \ddots & I & 0 \\ 0 & & & 0 & 0 & I \\ \hat{B}_{h-1} & \hat{B}_{h-2} & \cdots & \hat{B}_1 & \hat{A} & \hat{B}_0 \\ \hline \hat{D}_{h-1} & \hat{D}_{h-2} & \cdots & \hat{D}_1 & \hat{C} & \hat{D}_0 \end{array}\right] \tag{6}$$

which reveals that the true order of the filter is $n_f = \hat{n} + (h-1)q$. Note also that the realization (5) reflects the fact that the filter needs to have as input only $y(k)$, with previous outputs being buffered internally.

After defining the vector

$$s(k) = \begin{bmatrix} y(k-h+1)^T & \cdots & y(k-1)^T & x(k)^T \end{bmatrix}^T \in \mathbb{R}^{n+(h-1)q}, \tag{7}$$

the augmented system is given by

$$\begin{aligned} s(k+1) &= \begin{bmatrix} \mathsf{A}_2 & \mathsf{A}_1 \\ 0 & \tilde{A}(\xi) \end{bmatrix} s(k) + \begin{bmatrix} 0 \\ \tilde{B}(\xi) \end{bmatrix} w(k) \\ y(k) &= \begin{bmatrix} 0 & \tilde{C}_y(\xi) \end{bmatrix} s(k) + \tilde{D}_y(\xi)\, w(k) \\ z(k) &= \begin{bmatrix} 0 & \tilde{C}_z(\xi) \end{bmatrix} s(k) + \tilde{D}_z(\xi)\, w(k) \end{aligned} \tag{8}$$

where

$$\begin{aligned} \tilde{A}(\xi) &= \begin{bmatrix} 0 & C_y(\xi) \\ 0 & A(\xi) \end{bmatrix}, & \tilde{B}(\xi) &= \begin{bmatrix} D_y(\xi) \\ B(\xi) \end{bmatrix}, \\ \tilde{C}_y(\xi) &= \begin{bmatrix} I & 0 \\ 0 & C_y(\xi) \end{bmatrix}, & \tilde{D}_y(\xi) &= \begin{bmatrix} 0 \\ D_y(\xi) \end{bmatrix}, \\ \tilde{C}_z(\xi) &= \begin{bmatrix} 0 & C_z(\xi) \end{bmatrix}, & \tilde{D}_z(\xi) &= D_z(\xi), \end{aligned} \tag{9}$$

and

$$\mathsf{A}_1 = \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ I & 0 \end{bmatrix}, \quad \mathsf{A}_2 = \begin{bmatrix} 0 & I & & 0 \\ & \ddots & \ddots & \\ & & \ddots & I \\ 0 & & & 0 \end{bmatrix}. \tag{10}$$

One can verify that defining

$$\bar{s}(k) = \begin{bmatrix} s(k)^T & \hat{x}(k)^T \end{bmatrix}^T \in \mathbb{R}^{n+\hat{n}+(h-1)q}, \tag{11}$$

the connection of filter (4) with system (8) produces a system of the form:

$$\begin{aligned} \bar{s}(k+1) &= \mathbb{A}(\xi)\,\bar{s}(k) + \mathbb{B}(\xi)\,w(k) \\ e(k) &= \mathbb{C}(\xi)\,\bar{s}(k) + \mathbb{D}(\xi)\,w(k) \end{aligned} \tag{12}$$

where $e(k) = z(k) - \hat{z}(k)$ is the *filtering error* and

$$\left[\begin{array}{c|c} \mathbb{A}(\xi) & \mathbb{B}(\xi) \\ \hline \mathbb{C}(\xi) & \mathbb{D}(\xi) \end{array}\right] = \left[\begin{array}{ccc|c} \mathsf{A}_2 & \mathsf{A}_1 & 0 & 0 \\ 0 & \tilde{A}(\xi) & 0 & \tilde{B}(\xi) \\ \hat{\mathsf{B}}_2 & \hat{\mathsf{B}}_1\tilde{C}_y(\xi) & \hat{A} & \hat{\mathsf{B}}_1\tilde{D}_y(\xi) \\ \hline -\hat{\mathsf{D}}_2 & \tilde{C}_z(\xi) - \hat{\mathsf{D}}_1\tilde{C}_y(\xi) & -\hat{C} & \tilde{D}_z(\xi) - \hat{\mathsf{D}}_1\tilde{D}_y(\xi) \end{array}\right], \tag{13}$$