Automatica 71 (2016) 159–168

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Interval observer for a class of uncertain nonlinear singular systems ☆

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ARTICLE INFO

Article history:
Received 14 October 2014
Received in revised form
7 August 2015
Accepted 23 March 2016
Available online 31 May 2016

Keywords:
Delayed singular systems
Commensurate delays
Observer

ABSTRACT

This paper investigates the problem of observer design for a general class of linear singular time-delay systems, in which the time delays are involved in the state, the output and the known input (if there exists). The involvement of the delay could be multiple which however is rarely studied in the literature. Sufficient conditions are proposed which guarantee the existence of a Luenberger-like observer for the general system.

1. Introduction

Singular system approach (known as well descriptor system/algebraic–differential system) was introduced to model a large class of systems in many different domains, such as physical, biological, and economic ones, for which the standard representation sometimes cannot be applied (Campbell, 1980, 1982). The structure of this type of systems contains both the dynamic equations and the algebraic ones, and due to this characteristic, many well-defined concepts dealing with the observability problem for regular (non-singular) systems have to be reconsidered. In Yip and Sincovec (1981), the authors have studied the solvability, controllability and observability concepts for singular systems with regular matrix pencil. The algebraic duality between controllability and observability for singular systems with regular matrix pencil is proven by using the Schwartz distribution framework in Cobb (1984). In Hou and Muller (1999a), the concept of causal observability was proposed for singular systems. The strong observability and strong detectability of a general class of singular linear systems with unknown inputs are recently tackled in Bejarano, Floquet, Perruquetti, and Zheng (2011, 2013) by converting the singular system into a regular one with unknown inputs and algebraic constraints. The observability problem for nonlinear singular system has been treated in Bejarano, Perruquetti, Floquet, and Zheng (2012) and Bejarano, Floquet, Perruquetti, and Zheng (2015).

Concerning the observer design, a Luenberger-like observer has been proposed in Paraskevopoulos and Koumboulis (1992) for linear singular systems. Darouach and Boutayeb (1995) gave necessary and sufficient condition for the existence of a reduced order observer for the linear singular systems with known inputs, and the result was extended to treat the linear singular systems with unknown inputs in Darouach, Zasadzinski, and Hayar (1996). In Hou and Muller (1999b), a generalized observer was studied by involving the derivative of input and output. For linear singular systems with unknown inputs, a proportional–integral observer was proposed in Koenig and Mammar (2002), and its extension by involving multiple integrations to design an unknown input observer was studied in Koenig (2005). For the nonlinear singular system, Kaprielian and Turi (1992) studied an observer for a class of nonlinear singular systems in which the system was linearized around the equilibrium point. The same technique was used in Boutayeb and Darouach (1995) to study a reduced order observer for a class of nonlinear singular systems. Other techniques, such as LMI (Darouach & Boutat-Baddas, 2008b; Lu & Ho, 2006) and convex optimization (Koenig, 2006), are proposed

☆ The material in this paper was partially presented at the 54th IEEE Conference on Decision and Control, December 15–18, 2015, Osaka, Japan. This paper was recommended for publication in revised form by Associate Editor Delin Chu under the direction of Editor Ian R. Petersen.

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http://dx.doi.org/10.1016/j.automatica.2016.04.002
0005-1098/© 2016 Published by Elsevier Ltd.

as well to design an observer for nonlinear singular systems with known (or unknown) inputs. And recently, the technique of regularization by applying the geometrical differential method to the nonlinear singular systems was introduced in Boutat, Zheng, Boutat-Baddas, and Darouach (2012).

Most of the cited references are for the asymptotic estimation of the state for singular systems without uncertainties. The observer design becomes complicated when considering the systems with uncertain terms in the state and in the measurement. In this situation, the exact estimation may be not possible, and one solution is to provide the upper and lower bound estimation of the admissible values for the state by applying the theory of set-membership or interval estimation (Efimov, Fridman, Raïssi, Zolghadri, & Seydou, 2012; Gouzé, Rapaport, & Hadj-Sadok, 2000; Mazenc & Bernard, 2010; Raïssi, Efimov, & Zolghadri, 2012).

Concerning the interval observer design for singular system, the only work existing in the literature is Efimov, Polyakov, and Richard (2014), where the problem of interval observer design is addressed for a class of linear singular systems with delays. In this work, the studied singular system was decoupled into two parts $x = (x_1, x_2)^T$: the dynamical one and the algebraic one. The authors then proposed an interval observer by restrictively assuming that x_2 can be fully described as the function of x_1. Thus the algebraic equation can be removed and the studied singular system is equivalent to a regular one. Without imposing this restrictive condition, this paper proposes a method to design an interval observer for a class of nonlinear singular systems with uncertainties. By imposing the observability rank condition for the linear part and assuming some boundedness properties of the studied system, this paper shows that an interval observer for this class of uncertain nonlinear singular systems always exists, which provides the upper and the lower estimations of the system's state.

In this paper, the following notation is used. $\mathbb{R}$ denotes the set of real numbers and $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$. $a\mathcal{R}b$ represents the element-wise relation $\mathcal{R}$ (a and b are vectors or matrices): for example $a < b$ (vectors) means $\forall i : a_i < b_i$. For a matrix $\mathcal{A} \in \mathbb{R}^{m\times n}$, define $\mathcal{A}^+ = \max\{0, \mathcal{A}\}$[1] and $\mathcal{A}^- = \mathcal{A}^+ - \mathcal{A}$. For a vector $x \in \mathbb{R}^n$, define $x^+ = \max\{0, x\}$ and $x^- = x^+ - x$. For a matrix (function) $\mathcal{A}$ the symbol $\mathcal{A}_i$ denotes its ith column, for a vector (function) b the symbol b_i denotes its corresponding element. A matrix $\mathcal{A} \in \mathbb{R}^{n\times n}$ is called Metzler if all its elements outside the main diagonal are nonnegative.

2. Problem statement and backgrounds

2.1. Problem statement

Consider the following uncertain nonlinear singular system:

$$\Sigma_\xi : \begin{cases} \bar{E}\dot{\xi} = \bar{A}\xi + \bar{f}(\xi, u) + v(t) \\ y = \bar{C}\xi + w(t) \end{cases} \tag{1}$$

where $\xi \in \mathbb{R}^n$ is the state whose initial value belongs to a compact set $I_0(\xi(t_0)) = [\underline{\xi}(t_0), \bar{\xi}(t_0)]$; $y \in \mathbb{R}^p$ and $u \in \mathbb{R}^m$ are respectively the output and the input. $\bar{E} \in \mathbb{R}^{n\times n}$, $\bar{A} \in \mathbb{R}^{n\times n}$, $\bar{C} \in \mathbb{R}^{p\times n}$, and the vector field $\bar{f}$ represents the nonlinear term with the appropriate dimension. $w(t)$ and $v(t)$ are the disturbance in the output and in the model, respectively.

When the matrix $\bar{E}$ is nonsingular, then (1) can be written as:

$$\begin{cases} \dot{\xi} = \bar{E}^{-1}\bar{A}\xi + \bar{E}^{-1}\bar{f}(\xi, u) + \bar{E}^{-1}v(t) \\ y = \bar{C}\xi + w(t) \end{cases} \tag{2}$$

[1] It means each element of $\mathcal{A}^+$ is the maximum value between 0 and the maximum value of $\mathcal{A}$ at the same position.

which becomes a classical regular system. We can cite lots of existing methods to design an observer for the system without the uncertainties, if it is observable. For the case with uncertainties in the state and in the output, an interval observer was proposed in Raïssi et al. (2012) for the observable system (2) to give the upper and lower estimation of the real state with the only requirement of the uncertainty boundedness. This method is extended in Zheng, Efimov, and Perruquetti (2013, 2016) to treat the case even if the system is not observable. When the matrix $\bar{E}$ is singular, (1) represents a large class of nonlinear singular systems with uncertainties in the state and in the output, covering those studied in Darouach and Boutat-Baddas (2008a) and Koenig (2006). This paper is devoted to designing an interval observer for this larger class of uncertain nonlinear singular systems. Before this, let us firstly recall the basic background on comparison systems which will be used for interval observer design.

2.2. Background on comparison systems

When dealing with a qualitative property involving solutions of a complex system, it is sometimes of interest to obtain a simpler system whose solutions overvalue the solutions of the initial system in some sense. For ODE, the contributions of Kamke (1932), Müller (1926) and Wazewski (1950) are probably the most important in this field: they give necessary and sufficient hypotheses ensuring that the solution of $\dot{x} = f(t, x)$, with initial state x_0 at time t_0 and function f satisfying the inequality $f(t, x) \leq g(t, x)$ is overvalued by the solution of the so-called "comparison system" $\dot{z} = g(t, z)$, with initial state $z_0 \geq x_0$ at time t_0, or, in other words, conditions on function g that ensure $x(t) \leq z(t)$ for $t \geq t_0$. These results were extended to many different classes of dynamical systems (Bitsoris, 1978; Borne, Dambrine, Perruquetti, & Richard, 2003; Dambrine, Goubet, & Richard, 1995; Grujič, Martynyuk, & Ribbens-Pavella, 1987; Perruquetti & Richard, 1996; Perruquetti, Richard, & Borne, 1995a; Perruquetti, Richard, Grujić, & Borne, 1995b). From these results one can deduce the following proposition:

Proposition 1 (*Smith, 1995*). *Assume that A is a Metzler matrix, $b(t) \in \mathbb{R}^n_+$, $\forall t \geq t_0$, where t_0 represents the initial time such that the system:*

$$\frac{dx(t)}{dt} = Ax + b(t), \tag{3}$$

possesses, for every $x(t_0) \in \mathbb{R}^n_+$, a unique solution $x(t)$ for all $t \geq t_0$. Then, for any $x(t_0) \in \mathbb{R}^n_+$, the inequality $x(t) \geq 0$ holds for every $t \geq t_0$.

In other words, under conditions of Proposition 1, $\mathbb{R}^n_+$ is positively invariant w.r.t. (3). Therefore, there are two important issues when designing an interval observer for the uncertain nonlinear singular system (1): find the Metzler matrix and bound the positive term, which will be discussed in the following sections.

3. Assumptions and preliminary results

Concerning the observability for singular systems, there exist several different definitions in the literature, including observability, R-observability and Impulse-observability (Dai, 1989; Yip & Sincovec, 1981). Generally speaking, they characterize the state reconstruction ability from different aspects: R-observability defines the ability to estimate the reachable set of the studied system; Impulse-observability corresponds to the ability to estimate the impulse term of the studied system and the observability covers both mentioned abilities to estimate all states of the studied system.