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Brief paper

Output feedback negative imaginary synthesis under structural constraints☆

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ABSTRACT

The negative imaginary property is a property that many practical systems exhibit. This paper is concerned with the negative imaginary synthesis problem for linear time-invariant systems by output feedback control. Sufficient conditions are developed for the design of static output feedback controllers, dynamic output feedback controllers and observer-based feedback controllers. Based on the design conditions, a numerical algorithm is suggested to find the desired controllers. Structural constraints can be imposed on the controllers to reflect the practical system constraints. Also, the separation principle is shown to be valid for the observer-based design. Finally, three numerical examples are presented to illustrate the efficiency of the developed theory.

1. Introduction

The study of negative imaginary systems has attracted much attention in recent years (Cai & Hagen, 2010; Ferrante & Ntogramatzidis, 2013; Lanzon & Petersen, 2008; Mabrok, Kallapur, Petersen, & Lanzon, 2015, 2014; Wang, Lanzon, & Petersen, 2015a,b; Xiong, Petersen, & Lanzon, 2012). By appropriately choosing the system input and output, many practical dynamic systems can be modelled as negative imaginary systems. Examples could be found in active vibration control systems (Das, Pota, & Petersen, 2015; Fanson & Caughey, 1990; Moheimani, Vautier, & Bhikkaji, 2006; Petersen & Lanzon, 2010) and circuit systems (Petersen & Lanzon, 2010). An important class of results for negative imaginary systems is the stability results developed in Lanzon and Petersen (2008), Xiong, Petersen, and Lanzon (2010), Mabrok et al. (2014), Liu and Xiong (2015). For positive-feedback interconnected negative imaginary systems, necessary and sufficient conditions are established to test the system stability. These results can be considered as a generalization of the positive position control results in Moheimani et al. (2006), Fanson and Caughey (1990), and depend only on the system gains at zero and infinite frequencies. In other words, the interconnected systems might have large control gains over other frequencies. In contrast, the small gain theorem requires that the control gains be small over all frequencies. An important application of the results is to robust control problems, where system uncertainty can be modelled by a negative imaginary system. Then, the closed-loop system will be stable as long as the controller is negative imaginary and satisfies the gain conditions. An illustrative example can be found in Xiong et al. (2010), where the uncertainty parameter is allowed to be arbitrarily large. Therefore, the results in Lanzon and Petersen (2008), Xiong et al. (2010), Mabrok et al. (2014), Liu and Xiong (2015) provide an attractive tool for robust control.

The underlying motivation of this study is to extend the application areas of negative imaginary systems theory. Consider the case that the uncertainty part in a system is negative imaginary while the remaining part of the system is not. The stability results in Lanzon and Petersen (2008), Xiong et al. (2010), Mabrok et al. (2014), Liu and Xiong (2015) will not be applicable. To make them applicable, one has to design controllers such that the remaining part of the system is negative imaginary; see examples in Petersen and Lanzon (2010), Song, Lanzon, Patra, and Petersen (2012), Mabrok et al. (2015). The problem of designing controllers for

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non-negative imaginary systems such that the resulting closed-loop systems become negative imaginary is called the negative imaginary synthesis problem, and the designed controllers are called negative imaginary controllers. When the full system state is available, state feedback negative imaginary controllers can be designed, and the corresponding design conditions have been established in Petersen and Lanzon (2010), Song et al. (2012), Mabrok et al. (2015) for both minimal and nonminimal state-space realizations. However, in practice, the system state is often not available and only measurement output can be used when designing controllers. Also in many cases, the desired controllers have to meet structural constraints in the system design (Lin, Fardad, & Jovanović, 2011; Rubió-Massegú, Rossell, Karimi, & Palacios-Quiñonero, 2013; Siljak, 1991; Zečević & Šiljak, 2008). For example, the controllers have to be of a block diagonal structure in the decentralized control of large-scale systems. Therefore, the design of output feedback negative imaginary controllers with structural constraints is an appealing practice use of the stability results in negative imaginary systems theory. The design of output feedback controllers has been recognized as a hard problem in general (Abbaszadeh & Marquez, 2009; Dinh, Gumussoy, Michiels, & Diehl, 2012; Shu, Lam, & Xiong, 2010; Syrmos, Abdallah, Dorato, & Grigoriadis, 1997).

This paper studies the negative imaginary synthesis problem when designing output feedback controllers. Firstly, the design of static output feedback controllers is considered. A sufficient condition is established in terms of a linear matrix inequality and a linear matrix equality. For the established design condition, an arbitrarily structural constraint can be readily imposed on the controller to meet practical requirements. It deserves mentioning that the solvability of the condition depends on the choice of the right inverse of the measurement matrix. An iterative algorithm is proposed to search for an approximate right inverse. When the measurement output equals to the system state, our result recovers the available ones in Petersen and Lanzon (2010), Song et al. (2012). Then, the design condition is extended to design dynamic output feedback controllers and observer-based state feedback controllers. In particular, for observer-based control, the separation principle is shown to hold. Finally, three numerical examples are used to demonstrate the developed design theory. The first example demonstrates the application of the results to a robust stabilization problem where the uncertainty is modelled by a strictly negative imaginary system. The conservatism of the developed design condition is studied via the first example. The second example validates the applicability of the results to MIMO systems. The third example illustrates how structural constraints are imposed on the designed controllers. The designed controller is a decentralized reduced-order dynamic output feedback controller. In all, the contribution of this paper is that a systematic design theory for output feedback negative imaginary controllers is developed and controller structural constraints can be enforced.

Notation: Let $\mathbb{R}^{m\times n}$ and $\mathcal{R}^{m\times n}$ denote the set of $m\times n$ real matrices and real-rational proper transfer function matrices, respectively. A^{T} and A^* denotes the transpose and the complex conjugate transpose of a complex matrix A, respectively. $\Re[\cdot]$ is the real part of a complex number. The notation $X > 0$ or $X \geq 0$, where X is a real symmetric matrix, means that the matrix X is positive definite or positive semidefinite, respectively.

2. Problem formulation

Consider a linear time-invariant system

$$\begin{cases}\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t),\\ z(t) = C_1 x(t),\\ y(t) = C_2 x(t),\end{cases} \tag{1}$$

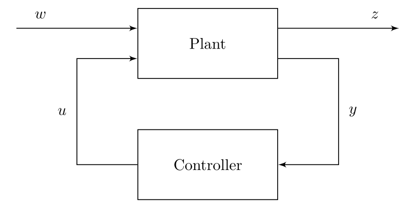

Fig. 1. Negative imaginary synthesis using output feedback control.

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^p$ is the control input, $y(t) \in \mathbb{R}^q$ is the measurement output, $w(t) \in \mathbb{R}^m$ is the system input, $z(t) \in \mathbb{R}^m$ is the system output. The matrices $A \in \mathbb{R}^{n\times n}$, $B_1 \in \mathbb{R}^{n\times m}$, $B_2 \in \mathbb{R}^{n\times p}$, $C_1 \in \mathbb{R}^{m\times n}$ and $C_2 \in \mathbb{R}^{q\times n}$ are known constant matrices. The measurement output matrix C_2 is assumed to be of full row rank without loss of generality. The system is chosen to be strictly proper to keep the results simple and tractable.

The objective of the paper is to design output feedback controllers, as shown in Fig. 1, such that the resulting closed-loop system is negative imaginary. The closed-loop system is negative imaginary if its transfer function is a negative imaginary transfer function. The following is the definition for negative imaginary transfer functions.

Definition 1 (*Xiong et al., 2010*)**.** A transfer function matrix $R(s) \in \mathcal{R}^{m\times m}$ is *negative imaginary* if

(1) $R(s)$ has no poles at the origin and in $\Re[s] > 0$;
(2) $j[R(j\omega) - R^*(j\omega)] \geq 0$ for all $\omega \in (0, \infty)$ except values of ω where $j\omega$ is a pole of $R(s)$;
(3) If $j\omega_0$, $\omega_0 \in (0, \infty)$, is a pole of $R(s)$, it is at most a simple pole, and the residue matrix $K_0 \triangleq \lim_{s\to j\omega_0}(s - j\omega_0)jR(s)$ is positive semidefinite Hermitian.

To determine whether a transfer function is negative imaginary or not, the negative imaginary lemma provides a necessary and sufficient condition in terms of the minimal state-space realization of the transfer function.

Lemma 1 (*Negative Imaginary Lemma (Xiong et al., 2010)*)**.** *Let (A, B, C, D) be a minimal state-space realization of a transfer function matrix $R(s) \in \mathcal{R}^{m\times m}$, where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$, $D \in \mathbb{R}^{m\times m}$. Then $R(s)$ is negative imaginary if and only if*

(1) $\det(A) \neq 0$, $D = D^{\mathsf{T}}$;
(2) *there exists a matrix $Y \in \mathbb{R}^{n\times n}$, $Y = Y^{\mathsf{T}} > 0$, such that*

$$AY + YA^{\mathsf{T}} \leq 0 \quad \text{and} \quad B + AYC^{\mathsf{T}} = 0.$$

When the realization (A, B, C, D) is not minimal, the conditions in Lemma 1 are only sufficient to test the negative imaginariness of $R(s)$; see Corollary 1 of Song et al. (2012).

Remark 1. The notation of negative imaginary systems has been extended to the cases where zero or infinite poles are allowed in the system (Ferrante, Lanzon, & Ntogramatzidis, 2016; Ferrante & Ntogramatzidis, 2013; Liu, J, & Xiong, 2016; Mabrok et al., 2014). New versions of negative imaginary lemmas have been reported in Mabrok et al. (2015). However, the results in Mabrok et al. (2015) cannot be considered as a generalization of Lemma 1. Lemma 1 is used in this paper to help the controller design, and the requirement of the closed-loop system having no poles at the origin is not a strong condition.

Before presenting the main results, some notation is defined to simplify the presentation. Let $C_2^{\perp} \in \mathbb{R}^{(n-q)\times n}$ such that $C_2 C_2^{\perp\mathsf{T}} = 0$ and $C_2^{\perp} C_2^{\perp\mathsf{T}} = I$; in other words, the rows of $C_2^{\perp}$ consist of the basis of the orthogonal complement subspace of the subspace spanned by the rows of C_2. One has that $[C_2^{\mathsf{T}}\ C_2^{\perp\mathsf{T}}]$ is invertible. Let $C_2^{+} \in \mathbb{R}^{n\times q}$