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Brief paper

Strong iISS for a class of systems under saturated feedback☆

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ABSTRACT

This paper proposes sufficient conditions under which nonlinear input-affine systems can be made Strongly iISS in the presence of actuator saturation. Strong iISS was recently proposed as a compromise between the strength of input-to-state-stability (ISS) and the generality of integral input-to-state stability (iISS). It ensures in particular that solutions are bounded provided that the disturbance magnitude is below a certain threshold, and that they tend to the origin in response to any vanishing disturbance. We propose a growth rate condition under which the bounded feedback law proposed by Lin and Sontag for disturbance-free nonlinear systems ensures Strong iISS in the presence of perturbations. We illustrate our findings on the angular velocity control of a spacecraft with limited-power thrusters. In the specific case of linear time-invariant systems with neutrally stable internal dynamics, we provide a simple static state-feedback that ensures Strong iISS in presence of actuator saturations. This second result is illustrated by the robust stabilization of the harmonic oscillator. In both cases, we provide an estimate of the maximum disturbance amplitude that can be tolerated without compromising solutions' boundedness.

1. Introduction

Actuator saturation exists in most practical control systems. The design of stabilizing feedback laws for such systems has attracted a lot of attention. The aim of this paper is to contribute to the investigations on what robustness properties can be achieved by bounded control. It is well known that a necessary and sufficient condition for the stabilizability of a linear time-invariant (LTI) plant by saturated feedback is that the internal dynamics have no pole with positive real part (Sontag, 1998b). Several studies in the literature have proposed bounded stabilizing feedback for particular classes of systems whose internal dynamics exhibit no exponential instability. For LTI systems whose system matrix eigenvalues have no positive real part, it has been shown in Sontag and Sussmann (1990) that global stabilization by bounded output feedback can be achieved if and only if the system is both detectable and stabilizable. For neutrally stable systems (meaning LTI systems whose internal dynamics exhibit no unbounded solutions), it is also known that stabilization can be achieved using a saturated linear static feedback: see e.g. Liu, Chitour, and Sontag (1996) and references therein. Nonetheless some classes of systems, although having no poles with positive real parts, cannot be stabilized by saturated linear static state-feedback; this class includes chains of three or more integrators (Fuller, 1969; Sussmann & Yang, 1991). Nested saturations have been proposed to stabilize such systems (Sussmann, Sontag, & Yang, 1993; Teel, 1992). Stabilization by bounded control has also proved useful for nonlinear dynamics, especially in the context of systems in feedforward form (Mazenc & Praly, 1996; Teel, 1996) or by relying on "universal constructions" (Lin & Sontag, 1991).

Beyond stabilization, it is often desirable to ensure some robustness properties in order to cope, for instance, with parameter uncertainty, measurement noise or exogenous disturbances. L_p-stabilization with respect to disturbances acting "inside" the saturation (meaning appearing linearly in the argument of the saturation function) was achieved in Saberi, Hou, and Stoorvogel (2000) based on the low-and-high gain control law introduced in Megretski (1996). This robust stabilization has been extended to systems with disturbances acting "outside" the saturation

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(meaning as an additive term in the dynamics, and thus acting in an unbounded way) in Wang, Saberi, Stoorvogel, and Grip (2012) for chains of integrators under matching conditions. Also, explicit estimates of L_p input/output gains have been obtained for neutrally stable systems based on a saturated linear static feedback (Liu et al., 1996). Another natural candidate for the evaluation of robustness to exogenous inputs is the framework of input-to-state stability (ISS, Sontag (1989a, 2006)) and its weaker variant integral ISS (iISS, Sontag (1998a)). In Angeli, Chitour, and Marconi (2005), a saturated linear state-feedback is proposed that ensures ISS with respect to sufficiently small disturbances despite parameter uncertainty for systems of dimension smaller than or equal to three, as well as feedforward systems. ISS of neutrally stable systems with respect to disturbances acting outside the saturation have been proposed in Arcak and Teel (2002) under matching conditions. Other approaches guarantee ISS and iISS with bounded control to nonlinear systems based on the aforementioned "universal constructions" (Liberzon, 2002).

Among other robustness features, ISS ensures a bounded response to any bounded disturbance. Intuitively, one may expect that bounded controls fail in general at guaranteeing the solutions' boundedness if the disturbance acts outside the saturation with a too large amplitude (unless matching conditions between the saturated actuator and the disturbance are imposed: see *e.g.* Arcak and Teel (2002); Wang et al. (2012)). At first sight, for these systems, nothing more than ISS with respect to small inputs can be established, thus providing no information on the system's behavior for larger inputs. In this note, we provide sufficient conditions under which a more interesting property, namely Strong iISS, can be achieved by saturated feedback. This property, introduced in Chaillet, Angeli, and Ito (2014a), not only guarantees ISS with respect to small inputs but also iISS. In particular, it ensures a bounded response to any disturbance whose amplitude is below a given threshold, but also the existence of solutions at all times even for disturbances above that threshold. It also guarantees that the state converges to zero in response to any vanishing disturbance, and it is known to be preserved under cascade interconnection (Chaillet, Angeli, & Ito, 2014b).

In this paper, we identify two classes of systems for which Strong iISS can be achieved by saturating feedback. The first one is made of input-affine systems that can be internally stabilized by the "universal construction" of Lin and Sontag (1991) and for which a specific growth rate condition holds. The second one is the class of neutrally stable LTI systems, for which a saturated linear state feedback is shown to ensure Strong iISS. In both cases, the considered disturbances act outside the saturation, and no matching condition between the actuation and the disturbances is assumed. We start by formulating the problem and motivating it through an example (Section 2). Then, in Section 3, we provide a sufficient condition under which Strong iISS is achieved by saturated feedback and provide growth rate conditions under which the "universal construction" originally proposed to ensure global asymptotic stability of the disturbance-free system also guarantees Strong iISS in presence of perturbations. We provide an explicit estimate of the maximal disturbance amplitude that can be tolerated without compromising solutions' boundedness. We illustrate our findings through the stabilization of the Euler equations of a rotating spacecraft. Finally, focusing on LTI systems whose internal dynamics are neutrally stable, we propose in Section 4 a simple linear state feedback that ensures Strong iISS to additive disturbance despite actuator saturation. Here also, we provide an explicit estimate of the maximum tolerable disturbance amplitude and compare it to numerical observations in an example. Proofs are provided in Section 5.

Notation. For a nondecreasing continuous function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, $\gamma(\infty) \in \mathbb{R}_{\geq 0} \cup \{\infty\}$ denotes the quantity $\lim_{s\to+\infty} \gamma(s)$. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{PD}$ if it is continuous and positive definite. It is of class $\mathcal{K}$ if, in addition, it is increasing. It is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and $\alpha(\infty) = \infty$. $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ belongs to class $\mathcal{KL}$ if, given any fixed $t \geq 0$, $\beta(\cdot, t) \in \mathcal{K}$ and, given any fixed $s \geq 0$, $\beta(s, \cdot)$ is continuous, nonincreasing and asymptotically goes to zero. Given $x \in \mathbb{R}^n$, $|x|$ denotes its Euclidean norm. Given a positive integer p, $\mathcal{U}^p$ denotes the set of all measurable locally essentially bounded functions $d : \mathbb{R}_{\geq 0} \to \mathbb{R}^p$. For a given $d \in \mathcal{U}^p$, $\|d\| := \operatorname{ess\,sup}_{t\geq 0} |d(t)|$. Given a constant $R > 0$, we let $\mathcal{U}^p_{<R}$ denote the set $\{d \in \mathcal{U}^p : \|d\| < R\}$. sat : $\mathbb{R}^n \to \mathbb{R}^n$ is the vector saturation function defined as $\text{sat}(x) = (\text{sat}^0(x_1), \ldots, \text{sat}^0(x_n))^T$, where $\text{sat}^0(s) := \min\{1; |s|\}\text{sign}(s)$ for each $s \in \mathbb{R}$. A function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is called a *storage function* if it is continuously differentiable and satisfies $V(0) = 0$ and $V(x) > 0$ for all $x \neq 0$. A storage function is said to be *proper* if, in addition, $\lim_{|x|\to\infty} V(x) = \infty$. Given a storage function V and a vector field $f : \mathbb{R}^n \to \mathbb{R}^n$, $L_f V(x) := \frac{\partial V(x)}{\partial x} f(x)$.

2. Problem statement

Consider a nonlinear system of the form $\dot{x} = f(x, u, d)$, where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $d \in \mathbb{R}^p$ the exogenous disturbance and $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R}^n$ denotes a locally Lipschitz function satisfying $f(0, 0, 0) = 0$. If the system is stabilized through a static state feedback $u = k(x)$, the system takes the form

$$\dot{x} = \tilde{f}(x, d), \tag{1}$$

where $\tilde{f}(x, d) := f(x, k(x), d)$. Given $x_0 \in \mathbb{R}^n$ and an input signal $d \in \mathcal{U}^m$, the solution of (1) starting at x_0 at time $t = 0$ is referred to as $x(\cdot, x_0, d)$ (or simply $x(\cdot)$) on the time domain where it is defined.

Assume that the state feedback $k(x)$ is nominally designed to ensure input-to-state stability (ISS, Sontag (1989a, 2006)) of the closed-loop system (1), namely

$$|x(t; x_0, d)| \leq \beta(|x_0|, t) + \mu(\|d\|), \quad \forall t \geq 0 \tag{2}$$

for all $x_0 \in \mathbb{R}^n$, all $d \in \mathcal{U}^p$ and all $t \geq 0$, where $\beta \in \mathcal{KL}$ and $\mu \in \mathcal{K}_\infty$. Such a control law may be designed using techniques from the literature, such as Krstic and Li (1998); Liberzon, Sontag, and Wang (2002); Malisoff, Rifford, and Sontag (2004); Sontag (1990); Teel and Praly (2000). Then, a natural question is to know what robustness can be guaranteed to (1) in the presence of actuator saturation. Intuitively, we can expect that the applied control input $u = \text{sat}(k(x))$ fails at guaranteeing a bounded state in response to disturbance of too large magnitude, thus compromising ISS. Nonetheless a weaker robustness property, namely iISS (Sontag, 1998a), can reasonably be expected, that is

$$|x(t; x_0, d)| \leq \beta(|x_0|, t) + \mu_1\left(\int_0^t \mu_2(|d(s)|)ds\right),$$

for all $x_0 \in \mathbb{R}^n$, all $d \in \mathcal{U}^p$, and all $t \geq 0$, where $\beta \in \mathcal{KL}$ and $\mu_1, \mu_2 \in \mathcal{K}_\infty$. Unfortunately, even if iISS systems prove robust with respect to classes of inputs with finite energy (in particular, as shown in Sontag (1998a), if the energy $\int_0^\infty \mu_2(|d(s)|)ds$ is finite, then the state converges to the origin), solutions can grow unbounded in the presence of arbitrarily small and even vanishing inputs (Chaillet et al., 2014a). Generically, we may expect a bounded state property at most for disturbances whose amplitudes are below a specific threshold. That is, we could investigate the property of ISS with respect to small inputs, meaning to require that (2) holds only for disturbances with sufficient low magnitude. However, with this property, no guarantee on the behavior of the system can be given when the disturbance magnitude gets larger; the very solution of the system may fail to exist. Hence, a good candidate to evaluate the robustness to exogenous disturbances of systems with saturated feedback seems to be the Strong iISS, recently introduced in Chaillet et al. (2014a).