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Brief paper

Consensusability of discrete-time linear multi-agent systems over analog fading networks[☆]

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ABSTRACT

This paper studies the consensusability problem of discrete-time linear multi-agent systems over analog fading networks. It aims to decide whether there exists a distributed controller such that the underlying multi-agent system can achieve mean square consensus over analog fading channels. Conditions to ensure mean square consensus are derived for the scenarios of undirected communication topologies with identical fading networks, balanced directed communication topologies with identical fading networks, and undirected communications topologies with non-identical fading networks, respectively. For scalar systems, the sufficient condition is shown to be necessary. The results indicate that the effect of fading networks on consensusability is determined by the statistics of channel fading. Finally, simulations are conducted to validate the theoretical results.

1. Introduction

Nowadays, single-agent systems are incapable of dealing with complex tasks, and cooperation among multi-agent systems becomes necessary. Among various cooperative tasks, consensus, which requires all agents to reach an agreement on certain quantity of common interest, builds the foundation of others (Olfati-Saber, Fax, & Murray, 2007; Ren & Beard, 2008). One question arises before control synthesis: whether there exist distributed controllers such that the multi-agent system can achieve consensus. This problem is usually referred to as consensusability of multi-agent systems. Several important results have been derived to answer this question, under an undirected/directed communication topology (Li, Duan, Chen, & Huang, 2010; Ma & Zhang, 2010; Trentelman, Takaba, & Monshizadeh, 2013; You & Xie, 2011). Ma and Zhang (2010) show that to ensure the consensus of a continuous-time linear multi-agent system, the LTI dynamics should be stabilizable and detectable, and the undirected communication topology should be connected. Furthermore, You and Xie (2011) show that for a discrete-time linear multi-agent system, the product of the unstable eigenvalues of the system matrix should additionally be upper bounded by a function of the eigenratio of the undirected graph. Extensions to directed graphs and robust consensus can be found in Li et al. (2010) and Trentelman et al. (2013). Most of the consensusability results discussed above are derived assuming perfect communication. However, in wireless networks, which are commonly used by most multi-agent systems nowadays, channel fading is unavoidable due to changing environments, and thus it is necessary to consider its impact on the consensusability of multi-agent systems.

Analog channel fading is usually modeled as a multiplicative noise, which can be used as well to describe the packet-loss phenomenon. As to the problem of networked control with communications corrupted by multiplicative noises of a single agent system, there exist plentiful results; see, e.g., Sinopoli et al. (2004), Elia (2005), Schenato, Sinopoli, Franceschetti, Poolla, and Sastry (2007), Xiao, Xie, and Qiu (2012). Sinopoli et al. (2004) consider the Kalman filtering problem over a packet-loss channel and it is shown that there exists a critical value for the pack-loss rate, above which the Kalman filter is unstable. The work (Elia, 2005) studies the networked stabilization problem over fading channels. It demonstrates that to ensure mean square stability, the mean square capacity of the fading channel should be greater than the

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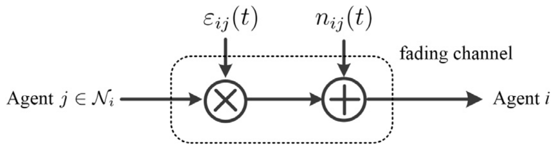

Fig. 1. Information transmission from agent j to agent i.

instability degree (Elia, 2004) of the SISO LTI dynamics. Xiao et al. (2012) further extend the results to MIMO systems with multiple fading channels. However, all the above results deal with single-agent systems only. When considering the consensus problem of multi-agent systems over fading networks, one needs to analyze the effect of the communication topology and this cannot be achieved directly using methods for control of a single agent system with multiplicative noises. Note that, fading factors in multi-agent systems can be regarded as coupling terms. The problem considered in the coupled multi-agent systems commonly assumes unknown constant couplings, e.g., Ma (2009) and Zhang and Zhang (2013). While in this paper, the fading factor is essentially stochastic, which makes the problem more difficult.

Recently, Xiao, Xie, Niu, and Hong (2014) consider the distributed estimation problem over analog fading networks using constant-gain estimators. Necessary and sufficient conditions on communication networks for bounded mean square estimation error covariance are given for continuous-time and discrete-time systems respectively, which reveal the fundamental limitation on distributed estimation induced by local communications, channel fading, and system dynamics. Xiao et al. (2014) deal with identical fading networks with undirected communication topologies only. To the best of authors' knowledge, there is no existing result on how non-identical fading networks impact the consensusability.

This paper focuses on the consensusability of multi-agent systems over identical and non-identical fading networks respectively. Sufficient conditions are derived under different communication environments and interaction topology settings. The derived results demonstrate how the system dynamics, the communication quality and the network topological structure interplay with each other to allow the existence of a linear distributed consensus controller. Specifically, the contributions of this paper are summarized as follows: (1) in the scenario of identical fading networks and undirected communication topologies, a sufficient condition is given to ensure mean square consensus, and it is shown that the sufficient condition is also necessary for scalar systems; (2) a sufficient condition which ensures mean square consensus under identical fading networks and balanced directed communication topologies is derived using Lyapunov methods; (3) edge Laplacian is adopted to analyze how non-identical fading networks affect mean square consensus, and sufficient conditions are derived under undirected tree communication topologies. Preliminary results on the case with identical fading networks have been reported in the conference paper (Xu, Xiao, & Xie, 2014). This paper contains new results on the case with non-identical fading networks and refined results on the case with identical fading networks. The rest of the paper is organized as follows. Section 2 provides background materials and problem formulation. Section 3 deals with the case of identical fading networks, and the mean square consensus problem over non-identical fading networks is discussed in Section 4. Simulations are given in Section 5. This paper ends with some concluding remarks in Section 6.

Notation: All matrices and vectors are assumed to be of appropriate dimensions that are clear from the context. $\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$ represent the sets of real scalars, n-dimensional real column vectors, and $m \times n$-dimensional real matrices, respectively. $\mathbf{1}$ denotes a column vector of ones. $I_N \in \mathbb{R}^{N\times N}$ represents the N by N identity matrix and the subscript N is dropped when the dimension is clear from the context. A', A^{-1}, $\rho(A)$ are the transpose, the inverse and the spectral radius of matrix A. $\otimes$, $\odot$ represent the Kronecker product and the Hadamard product, respectively. For a symmetric matrix A, $A \geq 0$ ($A > 0$) means that matrix A is positive semi-definite (definite). $\mathbb{E}\{\cdot\}$ denotes the expectation operator.

2. Preliminaries and problem formulation

Let $\mathcal{V} = \{1, 2, \ldots, N\}$ be the set of N agents with $i \in \mathcal{V}$ representing the i-th agent. Graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is used to characterize the interaction among agents, where $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the edge set with paired agents. An edge $(j, i) \in \mathcal{E}$ means that the i-th agent can receive information from the j-th agent. The neighborhood set $\mathcal{N}_i$ of agent i is defined as $\mathcal{N}_i = \{j \mid (j, i) \in \mathcal{E}\}$. The adjacency matrix is defined as $A_{adj} = [a_{ij}]_{N\times N}$, where $a_{ii} = 0$, $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise. The graph Laplacian matrix $\mathcal{L} = [\mathcal{L}_{ij}]_{N\times N}$ is defined as $\mathcal{L}_{ii} = \sum_{j\in\mathcal{N}_i} a_{ij}$, $\mathcal{L}_{ij} = -a_{ij}$ for $i \neq j$. A directed path on $\mathcal{G}$ from agent i_1 to agent i_l is a sequence of ordered edges in the form of $(i_k, i_{k+1}) \in \mathcal{E}$, $k = 1, 2, \ldots, l-1$. A graph contains a directed spanning tree if it has at least one agent with directed paths to all other agents. $\mathcal{G}$ is undirected if $A_{adj} = A'_{adj}$. An undirected graph is connected if there is a path between every pair of distinct nodes. A graph $\mathcal{G}$ is called balanced if and only if $\sum_{j=1}^N a_{ij} = \sum_{j=1}^N a_{ji}$ for all i.

The discrete-time dynamics of agent i has the following form

$$x_i(t+1) = Ax_i(t) + Bu_i(t), \qquad y_i(t) = Cx_i(t) \tag{1}$$

where $i = 1, 2, \ldots, N$, and $x_i \in \mathbb{R}^n$, $y_i \in \mathbb{R}^p$, $u_i \in \mathbb{R}^m$ represent the agent state, output and control input, respectively. Without loss of generality, assume B has full-column rank and C has full-row rank.

The agents communicate information to their neighbors through fading channels (see Fig. 1). Specifically, in this paper, we let the j-th agent send the information $Cv_j(t) - y_j(t)$ to the i-th agent at time t with $v_j \in \mathbb{R}^n$ representing the j-th agent's controller state as specified later. At the channel output side, the ith agent receives the deteriorated information

$$o_{ij}(t) = \varepsilon_{ij}(t)(Cv_j(t) - y_j(t)) + n_{ij}(t)$$

with ε_{ij} modeling the channel fading and n_{ij} denoting a zero-mean white communication noise with bounded variance. Depending on the particular propagation environment and communication scenario, different statistical models can be used for the channel fading ε_{ij} (e.g., Rayleigh, Nakagami, Rician) (Goldsmith, 2005). Combining all the received information from its neighbors, agent i generates the control input by using the following controller

$$\begin{aligned} v_i(t+1) &= (A+BK)\, v_i(t) \\ &\quad + F \sum_{j\in\mathcal{N}_i} \left[\varepsilon_{ij}(t)\,(Cv_i(t) - y_i(t)) - o_{ij}(t)\right] \\ u_i(t) &= Kv_i(t) \end{aligned} \tag{2}$$

where $i = 1, 2, \ldots, N$, and F and K are controller parameters to be designed.

Remark 1. Note that the above control protocol reduces to an observer based output feedback controller when a single agent system is concerned. In the paper, it is assumed that after each transmission, the instantaneous value of the fading ε_{ij} is known to the receiver, which is a reasonable assumption for a slowly varying channel with channel estimation (Goldsmith, 2005).

We aim to derive conditions on the fading statistics, the agent dynamics and the communication topology under which there exist F and K in the controller (2) such that the multi-agent system (1) can achieve mean square consensus. Let $s_i = [x_i', v_i']'$, $s = [s_1', s_2', \ldots, s_N']'$, and define the consensus error