Automatica 71 (2016) 300–307

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Minimum time control of heterodirectional linear coupled hyperbolic PDEs☆

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ARTICLE INFO

Article history:
Received 12 November 2015
Received in revised form
29 February 2016
Accepted 12 May 2016
Available online 10 June 2016

Keywords:
Hyperbolic partial differential equations
Stabilization
Backstepping

ABSTRACT

We solve the problem of stabilizing a general class of linear first-order hyperbolic systems. Considered systems feature an arbitrary number of coupled transport PDEs convecting in either direction. Using the backstepping approach, we derive a full-state feedback law and a boundary observer enabling stabilization by output feedback. Unlike previous results, finite-time convergence to zero is achieved in the theoretical lower bound for control time.

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1. Introduction

This article solves the problem of boundary stabilization of a general class of coupled heterodirectional linear first-order hyperbolic systems of Partial Differential Equations (PDEs) in minimum time, with arbitrary numbers m and n of PDEs in each direction and with actuation applied on only one boundary. First-order hyperbolic PDEs are predominant in modeling of traffic flow (Amin, Hante, & Bayen, 2008), heat exchanger (Xu & Sallet, 2002), open channel flow (Coron, d'Andréa Novel, & Bastin, 1999; de Halleux, Prieur, Coron, d'Andréa Novel, & Bastin, 2003) or multiphase flow (Di Meglio, 2011; Djordjevic, Bosgra, Van den Hof, & Jeltsema, 2010; Dudret, Beauchard, Ammouri, & Rouchon, 2012). Research on controllability and stability of hyperbolic systems has first focused on explicit computation of the solution along the characteristic curves in the framework of the C^1 norm (Greenberg & Tsien, 1984; Li, 1994; Qin, 1985). Later, Control Lyapunov Functions methods emerged, enabling the design of dissipative boundary conditions for nonlinear hyperbolic systems (Coron, 2009; Coron, Bastin, & d'Andréa Novel, 2008). In Coron, Vazquez, Krstic, and Bastin (2013) control laws for a system of two coupled nonlinear PDEs are derived, whereas in Castillo, Witrant, Prieur, and Dugard (2012), Coron et al. (2008), Prieur and Mazenc (2012), Prieur, Winkin, and Bastin (2008), Santos and Prieur (2008) sufficient conditions for exponential stability are given for various classes of quasilinear first-order hyperbolic system. These conditions typically impose restrictions on the magnitude of the coupling coefficients.

In Coron et al. (2013) a backstepping transformation is used to design a single boundary output-feedback controller. This control law yields H^2 exponential stability of closed loop 2-state heterodirectional linear and quasilinear hyperbolic system for arbitrary large coupling coefficients. A similar approach is used in Di Meglio, Vazquez, and Krstic (2013) to design output feedback laws for a system of coupled first-order hyperbolic linear PDEs with $m = 1$ controlled negative velocity and n positive ones. The generalization of this result to an arbitrary number m of controlled negative velocities is presented in Hu, Di Meglio, Vazquez, and Krstic (2015). There, the proposed control law yields finite-time convergence to zero, but the convergence time is larger than the minimum control time, derived in Li and Rao (2010) and Woittennek, Rudolph, and Knüppel (2009). This is due to the presence of non-local coupling terms in the targeted closed-loop behavior. The main contribution of this paper is a minimum time stabilizing controller. More precisely, a proposed boundary feedback law ensures finite-time convergence of all states to zero in minimum-time. This minimum-time, defined in Li and Rao (2010), Woittennek et al. (2009) is the sum of the two largest time of transport in each direction.

Our approach is the following. Using a backstepping approach (with a Volterra transformation) the system is mapped to a *target* system with desirable stability properties. This target system is a copy of the original dynamics with a modified in-domain coupling structure. More precisely, the target system is

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Rafael Vazquez under the direction of Editor Miroslav Krstic.

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http://dx.doi.org/10.1016/j.automatica.2016.05.030
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designed as an exponentially stable cascade. A full-state feedback law guaranteeing exponential stability of the zero equilibrium in the $\mathcal{L}^2$-norm is then designed. This full-state feedback law requires full distributed measurements. For this reason we derive a boundary observer relying on measurements of the states at a single boundary (the anti-collocated one). Similarly to the control design, the observer error dynamics are mapped to a target system using a Volterra transformation. Along with the full-state feedback law, this yields an output feedback controller amenable to implementation.

Technically, this design poses a novel challenge as far as proving the well-posedness of the Volterra transformation. The transformation kernels satisfy a system of equations with a cascade structure akin to the target system one. This structure enables a recursive proof of existence of the transformation kernels using tools similar to the ones presented in Hu et al. (2015).

The paper is organized as follows. In Section 2 we introduce the model equations and the notations. In Section 3 we present the stabilization result: the target system and its properties are presented in Section 3.1. In Section 3.2 we derive the backstepping transformation. Section 4 contains the main technical difficulty of this paper which is the proof of well-posedness of the kernel equations. In Section 4.1 we transform the kernel equations into an integral equation using the method of characteristics. In Section 4.2 we solve the integral equations using the method of successive approximations. In Section 5 we present the control feedback law and its properties. In Section 6 we present the uncollocated observer design. In Section 7 we give some simulation results. Finally in Section 8 we give some concluding remarks

2. Problem description

2.1. System under consideration

We consider the following general linear hyperbolic system which appears in Saint-Venant equations, heat exchangers equations and other linear hyperbolic balance laws (see Bastin & Coron, 2015).

$$u_t(t,x) + \Lambda^+ u_x(t,x) = \Sigma^{++}u(t,x) + \Sigma^{+-}v(t,x) \tag{1}$$

$$v_t(t,x) - \Lambda^- v_x(t,x) = \Sigma^{-+}u(t,x) + \Sigma^{--}v(t,x) \tag{2}$$

evolving in $\{(t,x)|\ t > 0,\ x \in [0,1]\}$, with the following linear boundary conditions

$$u(t,0) = Q_0 v(t,0), \qquad v(t,1) = R_1 u(t,1) + U(t) \tag{3}$$

where

$$u = (u_1 \dots u_n)^T, \qquad v = (v_1 \dots v_m)^T \tag{4}$$

$$\Lambda^+ = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}, \qquad \Lambda^- = \begin{pmatrix} \mu_1 & & 0 \\ & \ddots & \\ 0 & & \mu_m \end{pmatrix} \tag{5}$$

with constant speeds:

$$-\mu_m < \cdots < -\mu_1 < 0 < \lambda_1 \le \cdots \le \lambda_n \tag{6}$$

and constant real coupling matrices as well as the feedback control input

$$\Sigma^{++} = \{\sigma_{ij}^{++}\}_{1\le i\le n, 1\le j\le n} \qquad \Sigma^{+-} = \{\sigma_{ij}^{+-}\}_{1\le i\le n, 1\le j\le m} \tag{7}$$

$$\Sigma^{-+} = \{\sigma_{ij}^{-+}\}_{1\le i\le m, 1\le j\le n} \qquad \Sigma^{--} = \{\sigma_{ij}^{--}\}_{1\le i\le m, 1\le j\le m} \tag{8}$$

$$Q_0 = \{q_{ij}\}_{1\le i\le n, 1\le j\le m} \qquad R_1 = \{\rho_{ij}\}_{1\le i\le m, 1\le j\le n}. \tag{9}$$

The initial conditions denoted u_0 and v_0 are assumed to belong to $L^2([0,1],\Re)$.

Remark 1. The coupling terms are assumed constant here but the results of this paper can be adjusted for spatially-varying coupling terms.

2.2. Control problem

The goal is to design feedback control inputs $U(t) = (U_1(t), \dots, U_m(t))^T$ such that the zero equilibrium is reached in minimum time $t = t_F$, where

$$t_F = \frac{1}{\mu_1} + \frac{1}{\lambda_1}. \tag{10}$$

This problem is very similar to the one presented in Hu et al. (2015). The main difference is that the time proposed in this paper in which the controlled system is stabilized is much smaller.

3. Control design

The control design is based on the backstepping approach: using a Volterra transformation, we map the system (1)–(3) to a target system with desirable properties of stability.

3.1. Target system design

We map the system (1)–(3) to the following system

$$\alpha_t(t,x) + \Lambda^+\alpha_x(t,x) = \Sigma^{++}\alpha(t,x) + \Sigma^{+-}\beta(t,x) + \int_0^x C^+(x,\xi)\alpha(t,\xi)d\xi + \int_0^x C^-(x,\xi)\beta(t,\xi)d\xi \tag{11}$$

$$\beta_t(t,x) - \Lambda^-\beta_x(t,x) = \Omega(x)\beta(t,x) \tag{12}$$

with the following boundary conditions

$$\alpha(t,0) = Q_0\beta(t,0) \qquad \beta(t,1) = 0 \tag{13}$$

where C^+ and C^- are L^∞ matrix functions on the domain

$$\mathcal{T} = \{0 \le \xi \le x \le 1\} \tag{14}$$

while $\Omega \in L^\infty(0,1)$ is an upper triangular matrix with the following structure

$$\Omega(x) = \begin{pmatrix} \omega_{1,1}(x) & \omega_{1,2}(x) & \cdots & \omega_{1,m}(x) \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \omega_{m-1,m-1}(x) & \omega_{m-1,m}(x) \\ 0 & \cdots & 0 & \omega_{m,m}(x) \end{pmatrix}. \tag{15}$$

This system is designed as a copy of the original dynamics, from which the coupling terms of (2) are removed. The integral coupling appearing in (11) is added for the control design but does not have any incidence on the stability of the target system: since all the velocities are strictly positive the integral terms are feedforward terms.

Remark 2. This new target system is the main difference with Hu et al. (2015) and is the innovative aspect of this paper.

Remark 3. Without any loss of generality, one can assume that $\forall 1 \le i \le m, \sigma_{ii}^{--} = 0$ (such coupling terms can be removed using a change of coordinates as presented in, e.g., Coron et al., 2013). In this case, $\Omega(x)$ has exactly the same structure as the matrix $G(x)$ in Hu et al. (2015).

Besides, the following lemma assesses the finite-time stability of the target system.

Lemma 1. *The system (11), (12) reaches its zero equilibrium in finite-time* $t_F = \frac{1}{\mu_1} + \frac{1}{\lambda_1}$

Proof. The proof of this lemma is straightforward using the proof of Hu et al. (2015, Lemma 3.1). The system is a cascade of $\tilde{\alpha}$-system (that has zero input at the left boundary) into the β-system (that has zero input at the right boundary once $\tilde{\alpha}$ becomes null).

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