Automatica 71 (2016) 318–323

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Generalized hierarchical cyclic pursuit[☆]

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ARTICLE INFO

Article history:
Received 8 April 2015
Received in revised form
22 January 2016
Accepted 6 May 2016
Available online 11 June 2016

Keywords:
Autonomous system
Co-operative control
Cyclic pursuit
Hierarchy
Reachability

ABSTRACT

Cyclic pursuit has been an important connection topology in multi-agent systems. Many variants of the classical cyclic pursuit law have been analysed by researchers. Among these variants are homogeneous and heterogeneous cyclic pursuit, or single layer and hierarchical cyclic pursuit. In this paper hierarchical cyclic pursuit, with heterogeneous gains, has been considered. This paper generalizes existing results by allowing both heterogeneity in the gains, and extension of the single layer information flow graph to a multi-layer hierarchical structure. Some new results are presented about the reachability of agents in hierarchical cyclic pursuit. It is shown that the existing results may be obtained as special cases of the results obtained in this paper. Simulation results are provided to support the theoretical findings.

1. Introduction

Cyclic pursuit is a distributed control strategy in which agent i pursues agent $i+1$, modulo n, where n is the number of agents (Fig. 1), and has attracted the attention of many researchers (Juang, 2012; Kim & Sugie, 2007; Ma & Hovakimyan, 2013; Marshall, Broucke, & Francis, 2004, 2006; Ren, Beard, & Atkins, 2005; Sinha & Ghose, 2007). The vectors in Fig. 1 are the velocity directions of each agent. The equations of motion for agent i, along the x and y-directions, are:

$$\dot{x}_i = k_i(x_{i+1} - x_i); \qquad \dot{y}_i = k_i(y_{i+1} - y_i), \tag{1}$$

where, k_i is the gain for agent i and may be identical for all agents (homogeneous) or not (heterogeneous). The above equations can be extended to higher dimensions. When $k_i = k > 0, \forall i$, (1) reduces to the kinematics of homogeneous cyclic pursuit, and the agents converge to the centroid of their initial positions. For heterogeneous cyclic pursuit, the agents still achieve positional consensus, although not necessarily at the centroid, under some conditions on the gains (Sinha & Ghose, 2006). In hierarchical cyclic pursuit (Ding, Yan, & Lin, 2009; Shimizu & Hara, 2008; Smith, Broucke, & Francis, 2004), the first level consists of an agent in a group pursuing its leader within the group. In the second level, the weighted centroid of a group chases the weighted centroid of its leading group (modulo m) where m is the number of groups, each containing n agents (Fig. 2). Thus, in a two level cyclic pursuit, there are $N = n \times m$ agents. A similar extension holds for p levels. It was shown in Smith et al. (2004) that hierarchical cyclic pursuit leads to increased rate of convergence. However, all existing work related to hierarchical cyclic pursuit consider homogeneous gains.

In this paper, heterogeneous gains are considered in a hierarchical cyclic pursuit framework and its stability is examined. This heterogeneity, for a two level pursuit scheme, is shown to yield global reachability under certain initial configurations of the agents. The number of gains to be chosen is also reduced, thereby leading to ease of implementation. The results obtained here generalize the stability and convergence results in Smith et al. (2004). An analysis of the reachability set, which consists of the set of points where the agents can converge, is carried out. It is shown that heterogeneous hierarchical cyclic pursuit (HHCP) can achieve global reachability in some cases where single layer heterogeneous cyclic pursuit cannot.

2. Background and main results

With heterogeneous gains, as in (1), the point of convergence (X_f, Y_f) is given by Sinha and Ghose (2006)

$$X_f = \frac{\sum_{i=1}^{n} \frac{x_{i0}}{k_i}}{\sum_{i=1}^{n} \frac{1}{k_i}}, \qquad Y_f = \frac{\sum_{i=1}^{n} \frac{y_{i0}}{k_i}}{\sum_{i=1}^{n} \frac{1}{k_i}}, \tag{2}$$

☆ This work was supported by AOARD grant no. AOARD-11-4020 and NPMicAV. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Tamas Keviczky under the direction of Editor Christos G. Cassandras.

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http://dx.doi.org/10.1016/j.automatica.2016.05.023

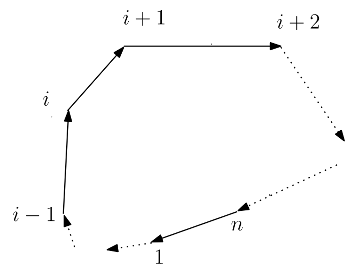

Fig. 1. Conventional cyclic pursuit.

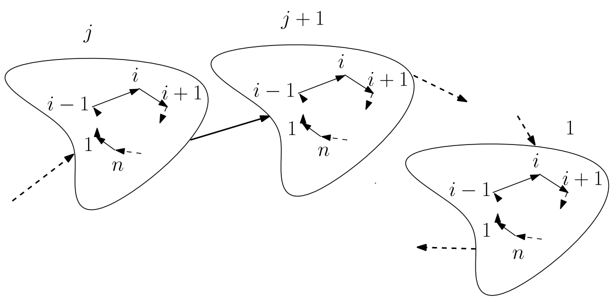

Fig. 2. Hierarchical cyclic pursuit.

where, (x_{i0}, y_{i0}) is the initial position of agent i. This reachability set does not span the whole of the 2-dimensional space (Mukherjee & Ghose, 2013). This paper shows that many points which are not reachable using conventional heterogeneous cyclic pursuit, are reachable using HHCP.

The stability analysis is different from Smith et al. (2004) as the system matrix is not circulant. Consider the two level HHCP (Fig. 2) governed by the linear equations,

$$\dot{z}_{i,j} = k_i(z_{i+1,j} - z_{i,j}) + g_j(z_{i,j+1} - z_{i,j}) \tag{3}$$

where, $z_{i,j} \in \mathbb{C}$ represents the position of agent i in group j. The total number of gains to be chosen is $n + m$, where $N = n \times m$ agents are split into m groups of n agents each. The input $u_{i,j}$ to agent i of group j, comprises two inputs, split up as $u_{i,j} = u_i + u_j$, where, $u_i = k_i(z_{i+1,j} - z_{i,j})$ and $u_j = g_j(z_{i,j+1} - z_{i,j})$. First consider the input u_i acting on agent i, and governing the intra-group dynamics, of each group. As shown in Sinha and Ghose (2006), at most one k_i can be negative. Next, consider the input u_j acting alone. One may consider another set of groups containing m agents each, where each group contains similarly indexed agents from each of the m original groups. For instance, the first such group contains agent 1 of each of the m groups. Hence, among the g_j's also, at most one may be negative (Sinha & Ghose, 2006). Thus, out of the $n + m$ gains to be chosen, two can be negative, thereby offering more flexibility in design while also dealing with a lesser number of design variables. Similarly, for p levels, at most p gains can be negative, each subject to a lower bound, which is dependent on the gains used in that level only. Thus, while choosing the lower limit for the negative g_j's, in the two level scheme, one need not take the k_i's into consideration. In the state space form, (3) can be written as $\dot{z} = Az$,

$$A = \begin{pmatrix} P - g_1 I_n & g_1 I_n & \cdots & 0 \\ 0 & P - g_2 I_n & g_2 I_n & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_m I_n & 0 & \cdots & P - g_m I_n \end{pmatrix} \tag{4}$$

$$P = \begin{pmatrix} -k_1 & k_1 & \cdots & 0 \\ 0 & -k_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ k_n & 0 & \cdots & -k_n \end{pmatrix} \tag{5}$$

where, I_n is the identity matrix of order n and $z \in \mathbb{C}^{mn}$ represents the position of each agent. Observe that the $mn \times mn$ matrix A has a null space spanned by the vector $[1\ 1\ \ldots\ 1] \in R^{mn}$. The nullity of A is unity, as shown in Theorem 1. Thus, stability of matrix A implies that the remaining $mn - 1$ eigenvalues of A lie in the open left half of the complex plane (Perko, 2001) leading to positional consensus such that all the agents converge to the same point. The stability of P leads to stable intra-group dynamics. However, the values of g_j, $\forall j$, affect the overall stability of the system. Consider that matrix P corresponds to a stable system. Now, for any group j (modulo m), choose the new state as:

$$z_j = \left(\sum_{i=1}^{n} \frac{z_{i,j}}{k_i}\right) \Big/ \left(\sum_{i=1}^{n} \frac{1}{k_i}\right). \tag{6}$$

This new state may be considered to be the equivalent of the weighted group centroid. It then follows that

$$\dot{z}_j = g_j(z_{j+1} - z_j). \tag{7}$$

Eq. (7) can be interpreted as centroid of group j chasing centroid of group $j + 1$, but unlike in Smith et al. (2004), the centroids are weighted. This can be written as:

$$\dot{p} = Qp; \quad p = [z_1\, z_2 \ldots z_m]^T \tag{8}$$

$$Q = \begin{pmatrix} -g_1 & g_1 & \cdots & 0 \\ 0 & -g_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_m & 0 & \cdots & -g_m \end{pmatrix}. \tag{9}$$

The stability analysis of Q is similar to that for P. Thus, among the g_j's, at most one (say l) may be negative subject to $g_l > -(\prod_{j \neq l} g_j)/(\sum_j \prod_{r \neq l,j} g_r)$. Hence, the stability analysis for level two involves only the gains in level two, provided the gains in level one satisfy stability conditions. The following theorem may now be stated.

Theorem 1. *Consider the two-level hierarchical cyclic pursuit with system matrices (4), (5). The system is stable if and only if at most one of the k_i's, say k_u, and one of the g_j's, say g_l, is negative and bounded as follows:*

$$k_u > -\frac{\prod_{i \neq u} k_i}{\sum_i \prod_{l \neq i,u} k_l}, \qquad g_l > -\frac{\prod_{j \neq l} g_j}{\sum_j \prod_{r \neq l,j} g_r}. \tag{10}$$

Proof. The proof uses the arguments based on Gershgorin's theorem (Horn & Johnson, 1990). The matrix A may be written as $A = I_m \otimes P + Q \otimes I_n$ where, $\otimes$ denotes the Kronecker product of two matrices, and I_m and I_n are the identity matrices of order m and n, respectively. Consider two square matrices T and R of appropriate dimensions which transform (via a similarity transformation) P and Q into block Jordan forms (not necessarily real Jordan blocks unless all the eigenvalues are real), J_P and J_Q. Since such transformation matrices always exist, $T^{-1}PT = J_P$, $R^{-1}QR = J_Q$, and $J_Q \otimes I_n + I_m \otimes J_P = (R^{-1}QR) \otimes (T^{-1}I_nT) + (R^{-1}I_mR) \otimes (T^{-1}PT) = (R^{-1} \otimes T^{-1})(Q \otimes I_n)(R \otimes T) + (R^{-1} \otimes T^{-1})(I_m \otimes P)(R \otimes T) = (R \otimes T)^{-1}(Q \otimes I_n + I_m \otimes P)(R \otimes T)$. So,

$$J_Q \otimes I_n + I_m \otimes J_P = (R \otimes T)^{-1} A (R \otimes T). \tag{11}$$

Thus, matrix A is similar to $J_Q \otimes I_n + I_m \otimes J_P$. Since J_P and J_Q are in block Jordan form, their diagonal entries indicate the eigenvalues of P and Q, respectively. Hence, it is clear from (11) that the eigenvalues of A are all the possible sums of eigenvalues of P and Q, or $\lambda_l(A) = \lambda_i(P) + \lambda_j(Q)$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, where $\lambda_s(.)$ is the sth eigenvalue of a matrix. Hence, if both the matrices P