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Brief paper

High-gain-predictor-based output feedback control for time-delay nonlinear systems☆

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ABSTRACT

This paper designs a high-gain predictor for output feedback control of nonlinear systems in the presence of input, output, and state delays. The high-gain predictor realizes the states appearing in the output feedback control in terms of predictive state, delayed state, and current state. The system includes internal and external dynamics, and the closed-loop system under state feedback is required to be asymptotically stable and locally exponentially stable. Positively invariant sets are found to verify boundedness, exponential stability, and performance recovery. In the simulation, a saturated sliding mode control is applied to demonstrate the performance recovery of the closed-loop system, and the fact that the high-gain-predictor parameter has a lower bound related to time delays.

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1. Introduction

In output feedback control of time-delay systems, state observers may be used to reconstruct the unmeasured states (e.g. Germani, Manes, & Pepe, 2012; Germani & Pepe, 2005). But observer design for nonlinear time-delay systems is challenging. Germani, Manes, and Pepe (2002) applied the Gronwall lemma to provide sufficient conditions on delay for a chain observer to prove exponential convergence of the estimation error. These conditions were relaxed in Kazantzi and Wright (2005). A Lyapunov–Krasovskii functional was introduced in Ahmed-Ali, Cherrier, and M'Saad (2009) such that a relationship between the delay and the number of cascade observers with specific vector gains was proposed. Many authors employed this method in the later work (e.g. Ahmed-Ali, Cherrier, & Lamnabhi-Lagarrigue, 2012; Ahmed-Ali, Van Assche, Massieu, & Dorleans, 2013; Farza, Sboui, Cherrier, & M'Saad, 2010; Ghanes, Leon, & Barbot, 2013). The aforementioned papers are devoted to proving the exponential convergence of the observer estimation errors. Recently, Karafyllis, Krstic, Ahmed-Ali, and Lamnabhi-Lagarrigue (2014) proved exponential stability of the closed-loop system in a disturbance free case. Those techniques for predictors (e.g., Germani et al., 2002; Karafyllis et al., 2014), work with time delays that are not necessarily small; however, they require exact models of the system. An advantage of high-gain observers is that we can dominate uncertain nonlinearities, while the price for that is restricting the delay to be small. Another advantage of using high-gain observers is that they can recover the performance of state feedback control.

In our previous work Lei and Khalil (2016), we used a high-gain predictor in feedback-linearization control. The system considered is nonlinear with time-varying input and output delays. To verify the performance recovery, we constructed a Lyapunov–Krasovskii functional, proved boundedness and exponential stability of the closed-loop system, and found a lower bound on the high-gain parameter, which relates to the maximum of involved delays. This work is an extension of Lei and Khalil (2016), which does the following: (1) we allow zero dynamics; (2) instead of feedback linearization, we allow any stabilizing state feedback control; (3) we allow state delay in addition to the input and output delays.

Notations. Throughout the paper, $|\cdot|$ means the absolute value; I denotes the identity matrix; λ denotes the eigenvalues, $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the minimum and maximum eigenvalues of the matrix respectively; the superscript of "T" stands for the transposition of a matrix; R^n denotes the n-dimensional Euclidean space and $\|\cdot\|$ denotes the Euclidean norm; R^+ represents the set of non-negative real numbers; given a positive constant $r \in R^+$, $L_2([-r,0];\ R^n)$ denotes the space of square integrable functions $\phi : [-r,0] \to R^n$; $\mathcal{C}^n$ denotes the space of absolutely continuous functions $\phi : [-r,0] \to R^n$, which have square integrable first-order derivatives; $x : [-r,\infty) \to R^n$ is a function; $x(t)$ means the

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value of x at t; x_t is an element of $\mathcal{C}^n$ defined by $x_t(\theta) = x(t+\theta)$, $-r \le \theta \le 0$ with the norm $\|x_t\|_s = \sup_{\theta\in[-r,0]} \|x(t+\theta)\| \le b$, where b is a positive constant; $x(t; t_0, \phi)$ represents the solution of the given time-delay system with initial time t_0 and initial condition ϕ; given a Lyapunov functional $V(x_t, \dot{x}_t, t) : \mathcal{C}^n \times L_2 \times R \to R^+$, $\dot{V}(x_t, \dot{x}_t, t) = \lim_{h\to 0^+} \sup \frac{1}{h}[V(x_{t+h}, \dot{x}_{t+h}, t+h) - V(x_t, \dot{x}_t, t)]$; a scalar continuous function $\alpha(d)$, defined for $d \in [0, \bar{d})$, belongs to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

2. System description

Consider a time-delay nonlinear system represented by

$$\begin{aligned}\dot{\eta}(t) &= \vartheta(\eta_t, \xi_t)\\ \dot{\xi}(t) &= A\xi(t) + B[\psi(\eta_t, \xi_t)u(t-\tau) + p(\eta_t, \xi_t)]\\ y(t) &= C\xi(t-s)\end{aligned} \tag{1}$$

where $\eta \in R^{\iota}$, $\xi \in R^{n-\iota}$ are the states, $u \in R$ is the control input, $y \in R$ is the output. The functions $\vartheta(\cdot),\ \psi(\cdot),\ p(\cdot)$ have multiple state delays, e.g.,

$$\begin{aligned}\vartheta(\eta_t, \xi_t) = \vartheta\big(&\eta(t), \eta(t-\bar{l}_1), \eta(t-\bar{l}_2), \ldots, \eta(t-\bar{l}_{m_1});\\ &\xi(t), \xi(t-l_1), \xi(t-l_2), \ldots, \xi(t-l_{m_2})\big)\end{aligned} \tag{2}$$

where $\bar{l}_j$ ($j = 1, 2, \ldots, m_1$) and l_i ($i = 1, 2, \ldots, m_2$) are state delays; $\psi(\cdot),\ p(\cdot)$ may have similar forms of state delays; τ and s are known sufficiently small input and output delays respectively; and $\tau,\ s \in [0, r]$, where $r = \tau + s$. The initial conditions η_{t_0} and ξ_{t_0} are bounded and $\|\xi_{t_0}\|_s \le \kappa_1$ with positive constant κ_1. The $(n-\iota)\times(n-\iota)$ matrix A, the $(n-\iota)\times 1$ matrix B, and the $1\times(n-\iota)$ matrix C represent a chain of $n-\iota$ integrators. System (1) is required to satisfy Assumption 1.

Assumption 1. (i) The nonlinear functions $\vartheta(\cdot),\ \psi(\cdot),\ p(\cdot)$ are locally Lipschitz.
(ii) $\psi(\cdot) \ne 0$ and $1/\psi(\cdot)$ is locally Lipschitz.
(iii) The origin $(\eta_t,\ \xi_t) = (0,\ 0)$ is an equilibrium point of the unforced system; that is, $p(0, 0) = 0$ and $\vartheta(0, 0) = 0$.

Consider a partial state feedback control of the form

$$\begin{aligned}u(t) = \gamma\big(&\xi(t+\tau),\ \xi(t+\tau-l_1),\ \xi(t+\tau-l_2), \ldots,\\ &\xi(t+\tau-l_{m_2})\big)\end{aligned} \tag{3}$$

so that

$$u(t-\tau) = \gamma\left(\xi(t),\ \xi(t-l_1),\ \xi(t-l_2), \ldots, \xi(t-l_{m_2})\right) \triangleq \gamma(\xi_t)$$

where the function $\gamma(\cdot)$ satisfies the following assumption.

Assumption 2. (i) $\gamma(\cdot)$ is a locally Lipschitz function in ξ_t.
(ii) $\gamma(\cdot)$ is globally bounded in ξ_t.
(iii) The origin of the closed-loop system

$$\begin{aligned}\dot{\eta}(t) &= \vartheta(\eta_t,\ \xi_t)\\ \dot{\xi}(t) &= A\xi(t) + B[\psi(\eta_t,\ \xi_t)\gamma(\xi_t) + p(\eta_t,\ \xi_t)]\end{aligned} \tag{4}$$

is asymptotically stable with the region of attraction $\mathcal{A} \subset \mathcal{C}^n$, and locally exponentially stable.
(iv) With $\chi = [\eta^{\mathrm{T}}, \xi^{\mathrm{T}}]^{\mathrm{T}}$, there exist a Lipschitz functional $V_1(\chi_t)$, and functions $\alpha_1, \alpha_2, \alpha_3$ of class $\mathcal{K}$ such that, for $\forall \chi_t \in \mathcal{A}$,

$$\begin{aligned}&\alpha_1(\|\chi(t_0)\|) \le V_1(\chi_t) \le \alpha_2\left(\|\chi_t\|_s\right),\\ &\dot{V}_1(\chi_t) \le -\alpha_3\left(\|\chi_t\|_s\right)\end{aligned} \tag{5}$$

and for any $c > 0$, $\Omega_c = \{V_1(\chi_t) \le c\}$ is a compact set of $\mathcal{A}$.

When the origin of the closed-loop system (4) is globally asymptotically stable, the existence of $V_1(\chi_t)$ satisfying (5) follows from the converse Lyapunov theorem of Karafyllis (2006).

Remark 1. Global boundedness of $\gamma(\cdot)$ is typically required in high-gain-observer designs to overcome the peaking phenomenon. It is usually achieved by saturating $\gamma(\cdot)$ outside a compact set of interest (see, Khalil, 2002).

Since the system (4) is locally exponentially stable, it is shown in Lemma 1 of the Appendix, with $\varepsilon = 1$, that there exist a constant $b > 0$ and a Lyapunov functional $V_2(\chi_t)$ that satisfies

$$\begin{aligned}&c_1\|\chi_t\|_s^2 \le V_2(\chi_t) \le c_2\|\chi_t\|_s^2, \qquad \dot{V}_2(\chi_t) \le -c_3\|\chi_t\|_s^2\\ &\left|V_2(\chi_t'') - V_2(\chi_t')\right| \le c_4\left(\|\chi_t''\|_s + \|\chi_t'\|_s\right)\ \|\chi_t'' - \chi_t'\|_s\end{aligned} \tag{6}$$

in the set $\mathcal{B}_b = \left\{\|\chi_t\|_s \le b\right\}$ for some positive constants $c_1,\ c_2,\ c_3$.

As an example of a control that satisfies Assumption 2, consider a feedback linearizable state-delay system without input and output delays as in Germani et al. (2012) and Germani and Pepe (2005):

$$\begin{aligned}\dot{\xi}(t) &= A\xi(t) + B[\psi(\xi_t)u(t) + p(\xi_t)]\\ y(t) &= C\xi(t).\end{aligned}$$

The feedback linearization control takes the form:

$$u(t) = \frac{-p(\xi_t) - K\xi(t)}{\psi(\xi_t)} \triangleq \gamma_0(\xi_t)$$

where K is a gain matrix such that $A - BK$ is Hurwitz. In view of Assumption 1, $\gamma_0(\xi_t)$ satisfies (i) in Assumption 2; condition (iii) of Assumption 2 is satisfied by design. Thus there exists a Lyapunov function $V_1(\xi) = \xi^{\mathrm{T}}(t)\bar{P}\xi(t)$ in which $\bar{P}$ is the solution of the Lyapunov equation $\bar{P}(A-BK) + (A-BK)^{\mathrm{T}}\bar{P} = -I$ so that the compact set $\Omega_c = \{V_1(\xi) \le c\} \subset \mathcal{A}$ is positively invariant. Then we saturate the control $\gamma_0(\xi_t)$ outside the set Ω_c in the way of

$$u(t) = \Xi\ \mathrm{sat}\left[\frac{\gamma_0(\xi_t)}{\Xi}\right] \triangleq \gamma(\xi_t)$$

where $\Xi \ge \max_{\xi\in\Omega_c}|\gamma_0(\xi_t)|$. Therefore, for every $\xi(0) \in \Omega_c$, $|\gamma(\xi_t)| \le \Xi$. Now the function $\gamma(\xi_t)$ satisfies all the conditions of Assumption 2.

3. High-gain predictor design

We will use $\hat{\xi}(t), \hat{\xi}(t-l_1), \hat{\xi}(t-l_2), \ldots, \hat{\xi}(t-l_{m_2})$ in the output feedback control in order to replace the states $\xi(t+\tau), \xi(t+\tau-l_1), \xi(t+\tau-l_2), \ldots, \xi(t+\tau-l_{m_2})$ in the state feedback (3). We now construct a high-gain predictor to generate $\hat{\xi}(t)$ and delay it in the observer equation according to the state delays.

The high-gain predictor takes the form

$$\begin{aligned}\dot{\hat{\xi}}(t) = &A\hat{\xi}(t) + B\left[\psi_0(\hat{\xi}_t)u(t) + p_0(\hat{\xi}_t)\right]\\ &+ H\left[y(t) - C\hat{\xi}(t-s-\tau)\right]\end{aligned} \tag{7}$$

with the initial condition $\hat{\xi}_{t_0}$ satisfying $\left\|\hat{\xi}_{t_0}\right\|_s \le \hat{\kappa}_1$, where $\hat{\kappa}_1$ is a positive constant. The functions ψ_0 and p_0 are nominal models of ψ and p; they are bounded functions of their arguments. The observer gain H is given by $H = [a_1/\varepsilon,\ a_2/\varepsilon^2,\ \ldots,\ a_{n-\iota}/\varepsilon^{n-\iota}]^{\mathrm{T}}$, in which ε is a small positive constant to be specified and a_j ($j = 1, 2, \ldots, n-\iota$) are positive constants chosen such that all the roots of the characteristic equation $\lambda^{n-\iota} + a_1\lambda^{n-\iota-1} + \cdots + a_{n-\iota-1}\lambda + a_{n-\iota} = 0$ have negative real parts.