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Denoising of Rician corrupted 3D magnetic resonance images using *tensor*-SVD

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ABSTRACT

In this paper, we propose a new method for denoising the volumetric magnetic resonance (MR) images degraded with Rician noise. Taking into account the multi-frame (multi-linear) nature, the proposed method formulates an unsophisticated approach by contemplating the MR data as third order tensor. Since the Rician noise is signal dependent, variance stabilization technique (VST) is applied to transform it as additive noise. The cubic patches extracted from 3D MR images are grouped as tensors which exhibit low-rank property. Thus, denoising problem is modelled as a low-rank tensor approximation of grouped tensors, solved by minimizing the tensor nuclear norm (TNN) in *tensor*-singular value decomposition (*t*-SVD) framework. Each denoised tensors are weighted-averaged to obtain the final denoised data. The efficiency of proposed algorithm is compared with the state of art techniques and has exhibited substantial improvement in terms of quality metrics such as PSNR, SSIM and EPI for synthetic MR images. The algorithm performance is assessed for real MR images with no reference quality metric viz. sharpness index (SI) and have shown superior results. Moreover, the effectiveness of proposed algorithm for MR image segmentation is evaluated. As observed from results, the accuracy of segmentation with regards to kappa coefficient is improved by 1–5% after applying the proposed denoising algorithm.

1. Introduction

Medical images are usually corrupted with random noise which reduces its accuracy in computer-aided diagnosis. This necessitates the requirement of denoising operation in medical image analysis [1]. In fact, MR images (MRI) are degraded due to hardware and scanning time limitations, motion artifacts, noise, etc. The term noise in MR image may refer to physiological or respiratory distortions, disturbance while producing pulse sequences in the magnet, thermal noise from the subject or electronics noise while obtaining the signal, etc. [2]. In modern MR imaging systems, sub-sampled *k*-spaces are acquired to hasten the acquisition. However, this sub-sampling may create aliasing artifacts. The techniques such as SENSitivity Encoding (SENSE) [3] and GeneRalized Autocalibrated Partially Parallel Acquisitions (GRAPPA) [4] for parallel MRI (pMRI), which are common in commercial devices, are used to rectify these artifacts to a certain extend.

An efficient MRI denoising algorithm aims to reduce the noise power while maintaining the resolution of useful features. Edge preservation is an important aspect to be preserved while processing a diagnostic image. Also, the algorithm must be robust to any artifacts. State of the art MR image denoising techniques include filtering methods, transform domain approaches, statistical techniques, algorithms utilizing sparsity and self-similarity, low-rank approximation, etc. [5–9].

1.1. Related works

Earlier, Henkelman et al. introduced spatial filtering approach to remove the Gaussian noise present in MR images [10]. To overcome the blurring issue in spatial filtering techniques, anisotropic diffusion filter was proposed [6]. Some modified and improved versions of anisotropic diffusion filters are available in literature. For example, Krissian et al. [11] proposed a noise-driven anisotropic diffusion filter based on partial difference equations (PDE) and linear minimum mean squared error estimation (LMMSE) techniques. Later, Hasanzadeh et al. [12] proposed LMMSE, which utilised the underlying self-similarity of MR data for denoising process. However, this approach failed to consider the data redundancy of 3D MR data. To overcome this issue, nonlocal LMMSE estimation was proposed in [7].

Even with these spatial domain techniques, some transform domain approaches were explored for denoising of Rician distributed noise. The wavelet sub-band coefficient mixing (WSM)

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proposed by Manjon et al. [13] was one of the pioneer approach. In [14], Buades et al. proposed non-local means (NLM) filter considering the natural redundancy of patterns within the images. Several variants of NLM filter such as Unbiased NLM (UNLM), Adaptive Blockwise NLM (ANLM), Optimized Blockwise NLM (OBNLM), Prefiltered Rotationally Invariant NLM for 3D MRI (PRINLM3D), etc. were proposed later [8,15–17], which produced state of the art results. Another approach based on unbiased kernel regression (UKR) was proposed by Rubio et al. [18], in which the noise-free image and its gradients were estimated by employing non-parametric second-order kernel regression. Later, another denoising approach based on the application of principal component analysis (PCA) over a set of similar patches was proposed by Coupe et al. in [9]. An improved bilateral filter design based on rough set theory was proposed in [19] for denoising of MR images. However, this approach was designed only for 2D MR images.

Recently, an effective non-local denoising approach known as block matching 3D (BM3D) filter was proposed for 2D images [20]. Here, similar image patches were grouped to 3D arrays and shrinking of coefficients in 3D transform-domain was performed for denoising operation. Foi et al. [21] extended BM3D to volumetric data, popularly known as BM4D technique. Later, the BM4D technique was used to denoise Rician noise using variance stabilisation technique (VST) as mentioned in [22]. An improved version of BM4D for non-uniform noise was proposed in [23]. However, both BM3D and BM4D approaches utilise orthogonal transforms and hence cannot adapt to varying image contents [24].

Nowadays, the low-rankness exhibited by natural signals have been exploited in many signal processing applications. The low-rankness of matrices formed by the vectorisation of non-local similar image patches motivates the use of low-rank approximation techniques for image analysis. In [25], natural image denoising was formulated as a low-rank matrix approximation problem, which was solved by weighted nuclear norm minimisation (WNNM) [26] producing state of the art results.

Later, for denoising of multi-dimensional (multi-linear) data, termed as *tensors*, such as video, MR images, seismic data, hyperspectral images, etc., many techniques were proposed in literature. These include tensor dictionary learning (Tensor-DL) [27], PARAFAC [28], higher order SVD (HOSVD) [24] and so on. In [29], the multichannel parallel MR image denoising was addressed using sparse and low-rank matrix decomposition method. This method considered the features such as rank deficiency of multichannel coil images and sparsity of artefacts. Later, Peng et al. [30] addressed MR spectroscopic image (MRSI) denoising with a low-rank approximation technique that exploits the low-rank structure of MR spectroscopic data due to partial separability and linear predictability. In [31], Dong et al. proposed a denoising technique based on Low-Rank Tensor Approximation (LRTA) with Laplacian Scale Mixture (LSM) which produced improved results for multispectral images as well as MR images. In their approach, the denoising task was formulated as a maximum a posteriori (MAP) estimation of core tensor coefficients. Moreover, similar 3D patches were grouped to form third order tensors, and HOSVD is applied to each grouped tensor to obtain the core coefficients. Some denoising algorithms based on HOSVD have appeared in [24,32,33]. An extension of [32] has appeared in [33] which involved a recursive regularisation stage to improve the denoising capability.

1.2. Contributions

An alternative representation known as *tensor*-singular value decomposition (t-SVD) has been proposed in [34,35] for building an approximation to a given tensor. The algorithm for computing t-SVD is based on fast Fourier transform (FFT) and hence is more computationally efficient compared to HOSVD [35]. These former methods [34,35] are focused on completion of missing data and removal of sparse noise in video applications, denoising of 5D seismic data and hyperspectral images, etc. The major contributions of this paper are as follows.

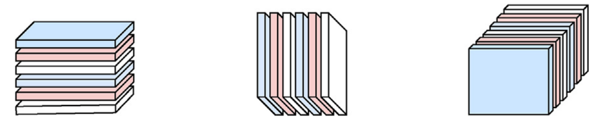

Fig. 1. Slices of third order tensor.

1. Inspired from the unsophisticated and efficient t-SVD framework [34,35], a new method for denosing MR images is designed for removing the conventional Rician noise by exploiting the low-rank property in the t-SVD framework. The proposed method shows an improvement in the accuracy of clinical diagnosis compared with state of the art techniques.
2. To the best of our knowledge, the proposed method is the first attempt to utilise t-SVD [34,35] for decomposition and denoising of MR images.

1.3. Paper organization

The organisation of the paper is as follows. In Section 2, we outline the advent of low-rank approximation techniques in MR image denoising and the preliminaries of tensor framework utilised in this paper. In Section 3, we discuss in detail the proposed t-SVD denoising algorithm for Rician noise removal. Section 4 covers an extensive analysis and its discussions on the proposed method. Section 5 discusses the effectiveness of the proposed method for tumor detection in MR images. In Section 6, we conclude the paper by summarising the implications and future directions.

2. Basic concepts and preliminaries

A Nth order tensor is a N-dimensional array of data. Thus, an order-1 tensor is a vector, order-2 tensor is a matrix and tensors of order three or higher are regarded as higher order tensors. For a third order tensor, *slices* are obtained by fixing all but two indices. If $\mathcal{T}$ is a third order tensor, then $\mathcal{T}(m, :, :)$ represents mth horizontal slice, $\mathcal{T}(:, m, :)$ is the mth lateral slice and $\mathcal{T}(:, :, m)$ represents mth frontal slice, as shown in Fig. 1. *Fibers* are the higher-order analogue of matrix rows and columns – defined by fixing every index but one. $\mathcal{T}(:, m, n)$, $\mathcal{T}(m, :, n)$ and $\mathcal{T}(m, n, :)$ denote the (m, n)th mode-1, mode-2 and mode-3 *fibers* of a third order tensor respectively.

2.1. Low rank approximation techniques for denoising MR images

In [30], MR image denoising was solved as low-rank matrix decomposition problem, where the observations were modelled as, $Z = X + N$. Here, X is the clean image and N is the Gaussian noise. The corresponding optimization problem is given by,

$$\hat{X} = \min_{X} \frac{1}{\sigma^2} \|Z - X\|_F^2 + \lambda \|X\|_{\omega,*} \quad (1)$$

where, $||X||_{\omega,*}$ denotes the weighted nuclear norm and is defined as $||X||_{\omega,*} = \sum_i |\omega_i \sigma_i(X)|$ where, $\omega = [\omega_1, \omega_2 \ldots \omega_n]$ is the weight assigned to ith singular value, $\sigma_i(X)$.