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On clustering based nonlinear projective filtering of biomedical signals

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ABSTRACT

We propose to modify the method of nonlinear state-space projections (NSSP) by application of the technique of k-means clustering. NSSP performs reconstruction of the state-space representation of the processed signals using the Taken's method of delays. Then it projects each state-space point on the appropriately constructed signal subspace and recovers the one-dimensional signal by averaging the results of all projections. The k-means clustering is applied to form so-called neighborhoods on the basis of which the signal subspaces are created. Within these neighborhoods, local density around each state-space point is estimated, to make construction of the signal subspaces more immune to high energy electromyographic noise.

The developed method is applied to process different types of ECG signals. For reference, the original NSSP method and its previously developed modifications are used. In different types of noise environment, the proposed method appears more effective than the original one and in most cases than the other reference methods. Moreover, visual results of fetal phonocardiogram and electronystagmogram processing show the wide range of its possible applications.

1. Introduction

Human body is a source of a variety of signals. Most of these signals contain valuable diagnostic information on the operation of different organs. When we want to get information on one selected organ, we must take into account that a mixture of interfering signals usually recorded. In such a mixture, one particular signal is regarded as the desired component, and the sum of the other ones forms the undesired noise. Suppression of noise is the primary operation performed by most modern systems for biomedical signal processing. To accomplish this operation, one needs a criterion allowing to distinguish the desired component from noise. One of the widely applied approaches uses spectral properties of the processed signals to achieve the goal. If the spectra of the desired component and noise do not overlap, the classical technique of linear filtering is often applied [1–5]. However, if the gap between the desired signal and the noise frequency bands is not sufficiently wide, a limited success can only be achieved, because it is impossible to design a filter with a very sharp cut-off, as the order would be nearly infinite. When the spectral criterion fails, the other ones must be used. A simple, yet very important one consists in the assumption of the desired component repeatability. Employing this criterion (complemented by a few additional conditions), we can use the technique of synchronous averaging to suppress noise [6–8]. Since the early 60s of the previous century, this approach has successfully been applied to ECG signals enhancement [9]. However, with more and more advanced methods of ECG interpretation, construction of the template showing the average morphology of the signal, discarding its subtle changes from beat-to-beat, appears to be a severe limitation of this technique.

With the progress in the field of nonlinear dynamical systems analysis, a very promising technique of nonlinear state-space projections (NSSP) emerged [10,11]. This technique has successfully been applied to suppress noise disrupting the ECG signal, and to enhance the desired component preserving its morphological variability [12]. It has also been applied to fetal ECG extraction from the maternal abdominal signals [13]. The other fields of NSSP application encompass, e.g. respiratory sound filtering [14], nuclear magnetic resonance (NMR) laser data [15], ballistocardiographic signal separation [16], electroencephalogram enhancement in a brain computer interface [17], analysis of voice signals [18,19] and

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even processing of the stellar light curves data [20,21]. Some new approaches to refine NSSP operation were proposed in [22–24].

The criterion used by NSSP to distinguish the desired signal and noise is associated with the state-space representation of both components. The method reconstructs this form of the processed signals using the Taken's embedding technique of delays [25]. One assumes that the state-space trajectory of the desired component lies on or is very near to a smooth nonlinear manifold in the embedding space. The equivalent form of the noise component is assumed to spread without a similar confinement. Therefore, depending on the noise level, the trajectory of the whole noisy signal is closer to or farther from that of the desired component, and by pushing it towards this desired one, we should be able to achieve noise suppression. To this end, for each point of the state-space trajectory of the processed noisy signal, NSSP performs analysis of the neighboring points to approximate linearly the globally nonlinear manifold, and then it projects the point on the determined linear subspace, to reduce its deviation from this manifold. Averaging of the results of locally linear projections leads to globally nonlinear noise reduction.

A very important operation, realized for each state-space point is construction of its neighborhood. Whereas in the original method [12], neighborhoods contain the points which are nearest in the embedding space, in [26] their location in time was used instead. Such approach could be applied to repeatable ECG signals, and after synchronization of the ECG beats, it was the location within a beat that was used as a criterion for neighborhoods construction. With this modification, a great reduction of computational costs was achieved, because instead of the computationally demanding comparison of the state-space distances, the fast operation of QRS detection was performed, and besides, all points having the same location within the respective beats were projected on the same subspace. In [27] this approach was modified by using the technique of dynamic time warping for neighborhoods determination. As a result both time and spatial location in the embedding space decided on the neighborhoods contents. With the decreased computational costs, it was possible to apply a robust technique for the linear subspaces construction, which resulted in great improvement of the method ability to suppress noise [28].

Construction of neighborhoods is somewhat similar to the operation of data clustering. The primary difference from clustering is that the number of groups (neighborhoods) is large and that they can overlap among each other (the formed groups can contain the same data points). The goal of this study is to apply the classical k-means clustering to form neighborhoods, and to take into account the state-space points density to develop an effective method of linear subspaces construction. The rest of the paper is organized as follows. In Section 2 an outline of the NSSP method is provided. In Section 3 the modifications proposed are described. Numerical experiments are presented in Section 4 and concluded in Section 5.

2. Nonlinear state-space projections

The method was developed for suppression of the measurement noise, corrupting deterministically chaotic signals [10,11]. For such signals the assumption concerning location of the desired component trajectory on a nonlinear manifold can strictly be satisfied: if a purely deterministic discrete signal is generated according to the nonlinear map $s(n+m-1)=f(s(n), s(n+1), \ldots, s(n+m-2))$, the nonlinear differentiable function $f(\cdot)$ forms $m-1$ dimensional nonlinear hypersurface (manifold) on which the trajectory evolves (or is confined to) [29]. It was however shown [29] that for many biomedical signals, the trajectories can also similarly be confined (although they can rather be located near but not precisely on such manifolds). Therefore the method was successfully applied, e.g. to ECG processing [12].

The primary operation is the reconstruction of the state-space representation of the observed noisy signal. To this end, the Takens embedding theorem [25] is applied, defining a point in the constructed space using the delayed signal values

$$\mathbf{x}^{(n)} = [x(n), x(n+\tau), \ldots, x(n+(m-1)\tau)]^\top, \tag{1}$$

where $x(n)$ is the processed signal, τ is the time lag ($\tau=1$ appeared advantageous in [12] and will be used in this study), m is the embedding dimension (the Takens embedding theorem [25] assures that for purely deterministic systems, under fairly broad conditions, the reconstructed state-space trajectory is equivalent to the one in the original phase space).

After the embedding operation has been accomplished, the nonlinear manifold, near which the desired component trajectory is assumed to be located, can locally linearly be approximated. In the original method [12], it is performed for each trajectory point, separately. Therefore, for each point, the distribution of the nearest points is analyzed. First, a set containing these points is formed: it is named simply as a neighborhood

$$\Gamma^{(n)} = \left\{ k : \|\mathbf{x}^{(k)} - \mathbf{x}^{(n)}\| \le \varepsilon \right\}, \tag{2}$$

where $\|\cdot\|$ denotes the Euclidean distance; ε is the assumed radius of neighborhoods.

We assume that embedding dimension m is greater than the dimension of the manifold, and that the level of noise is smaller than that of the desired signal. Therefore locally, within a neighborhood the directions along which the neighborhood points have maximal dispersion are used to span the desired signal subspace whose orthogonal complement is regarded as the noise subspace.

Thus, within a neighborhood, locally linear approximation of the globally nonlinear manifold can be realized by assuming that the neighborhood mass center

$$\bar{\mathbf{x}}^{(n)} = \frac{1}{|\Gamma^{(n)}|} \sum_{k \in \Gamma^{(n)}} \mathbf{x}^{(k)}, \tag{3}$$

($|\Gamma^{(n)}|$ denotes the neighborhood cardinality) forms the origin of the constructed linear subspace and that its axes correspond to the directions of the neighborhood points maximal dispersion. If this dispersion is measured by variance, the directions can be determined like in the classical principal component analysis [30], as the eigenvectors of the covariance matrix

$$\mathbf{C}^{(n)} = \frac{1}{|\Gamma^{(n)}| - 1} \sum_{k \in \Gamma^{(n)}} \left(\mathbf{x}^{(k)} - \bar{\mathbf{x}}^{(n)}\right) \left(\mathbf{x}^{(k)} - \bar{\mathbf{x}}^{(n)}\right)^\top, \tag{4}$$

and $\mathbf{x}^{(n)}$ can be projected on the subspace spanned by the eigenvectors associated with the largest eigenvalues. However, in [11] it appeared beneficial to limit corrections of the first and the last coordinates of the points under projection. To assure this, before eigendecomposition of the covariance matrix it is modified using a diagonal penalty matrix $\mathbf{R}$: $\mathbf{D}^{(n)} = \mathbf{R}\mathbf{C}^{(n)}\mathbf{R}$.

Finally, a correction of $\mathbf{x}^{(n)}$ is given by

$$\mathbf{x}'^{(n)} = \mathbf{R}^{-1}\mathbf{E}^{(n)}\mathbf{E}^{(n)\top}\mathbf{R}\left(\mathbf{x}^{(n)} - \bar{\mathbf{x}}^{(n)}\right) + \bar{\mathbf{x}}^{(n)}, \tag{5}$$

where $\mathbf{E}^{(n)} = \left[\mathbf{e}_1^{(n)}, \mathbf{e}_2^{(n)}, \ldots, \mathbf{e}_q^{(n)}\right]$; $\mathbf{e}_i^{(n)}$ is the eigenvector of $\mathbf{D}^{(n)}$ corresponding to the ith largest eigenvalue (it is the ith axis of the constructed signal subspace). All diagonal elements of $\mathbf{R}$ are equal to 1 except from $r_{11} = r_{mm} = r$. It is a large value of r that assures penalization of the corrections of both side coordinates.

The projections are performed for all points in the reconstructed state space. Since a signal sample occurs in m different points (at