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A larger family of nonlinear systems for the repetitive learning control[☆]

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ABSTRACT

A generalization of the PID$^{\rho-1}$ control has been recently presented in Marino et al. (2012), which guarantees the output tracking of smooth periodic reference signals (with known period) for uncertain minimum phase nonlinear systems in output feedback form with any relative degree $\rho \geq 1$. In this note, we show that the repetitive learning control proposed in Marino et al. (2012) also solves the same output tracking problem for a larger class of uncertain nonlinear systems in normal form, provided that the inverse system dynamics satisfy the Demidovich condition on suitable compact sets.

1. Introduction

Considerable research efforts have been spent in the last decades with the aim of addressing the existence of solutions to the output tracking problem for uncertain nonlinear systems.

In the unfavourable case in which the uncertainties are unstructured, the repetitive learning control approach (see Xu & Tan, 2003 and Ahn, Chen, & Moore, 2007) can be successfully followed when the output reference signals belong to the family of periodic time functions with known period.

In this respect, the most recent results for special classes of single-input single-output nonlinear systems can be found in (i) Marino and Tomei (2009) for systems which are partially feedback linearizable by state feedback with linear exponentially stable inverse system dynamics; (ii) Bifaretti, Tomei, and Verrelli (2012) for nonlinear systems with matching uncertainties; (iii) Marino, Tomei, and Verrelli (2012) for minimum phase systems in output feedback form with output dependent nonlinearities; (iv) Jin and Xu (2013) for nonlinear systems in normal form with no inverse system dynamics.

In this note, the above design techniques are used in conjunction with the notion of exponentially convergent systems (see Pavlov, van de Wouw, & Nijmeijer, 2006) in order to solve the output tracking problem for the larger class of uncertain nonlinear systems in normal form with inverse system dynamics satisfying the Demidovich condition on suitable compact sets. Differently from Jin and Xu (2013), the presence of uncertain inverse system dynamics is allowed, while in contrast to Marino et al. (2012), Marino and Tomei (2009), and Bifaretti et al. (2012) the inverse system dynamics are not restricted to be linear.

The resulting control design along with the related convergence analysis are not straightforward since nonlinearities appear and they are not exclusively output-dependent; nevertheless, the existence of a suitable periodic solution is no longer straightforward to be proved.

Those specific features appear in Consolini and Verrelli (2014) when a learning control for autonomous vehicles reproducing the human driver behaviour (which thus represents a special application of the general theory presented in this note) is designed to track planar curves with uncertain periodic curvature.[1] The first order inverse dynamics, which are expressed in the independent variable s (curvilinear abscissa), in the presence of the uncertain periodic s-function $\kappa(s)$ and of the positive real l, are $\eta'(s) = -\sin\left(\frac{\eta(s)+y(s)}{l}\right) - l\kappa(s)$. According to Theorem 2.41 and Definition 2.20 in Pavlov et al. (2006), the above dynamics represent a locally exponentially convergent system for the class of bounded inputs on $[0, +\infty)$ (i.e. there exists a neighbourhood $\mathcal{Z}$ of the origin and a positive real ρ such that the previous system is convergent in $\mathcal{Z}$

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[1] In particular, Consolini and Verrelli (2014) (see references therein for the general problem description) addresses the path-following problem for autonomous vehicles in which the goal is to steer the vehicle to reach and follow an uncertain periodic geometric path with specific constraints.

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for all inputs on $[0, +\infty)$ whose modulus is less than ρ for any s). Additional technical difficulties arise since an uncertain nonlinear state function (in contrast to Bifaretti et al., 2012; Marino & Tomei, 2009; Marino et al., 2012) is allowed, in this note, to multiply the control input.

2. Problem statement

We address the output tracking problem in which the output y of the nonlinear time-invariant single input-single output system ($f(\cdot)$ and $g(\cdot)$ are suitable uncertain smooth vector fields on $\mathbb{R}^n$, while $h(\cdot) : \mathbb{R}^n \to \mathbb{R}$ is a suitable uncertain smooth function)

$$\begin{aligned}\dot{x} &= f(x) + g(x)u \\ y &= h(x)\end{aligned} \tag{1}$$

is required to track a smooth periodic reference signal $y_*(t)$ (with known period T):

$$y_*(t+T) = y_*(t), \quad \forall\, t \geq -T. \tag{2}$$

In particular, we assume that the global relative degree $\rho \leq n$ is known and well defined for (1) and that system (1) is globally input–output linearizable (see Isidori, 2013) so that we may directly consider its normal form

$$\begin{aligned}\dot{z} &= \phi(z, \xi) \\ \dot{\xi}_j &= \xi_{j+1}, \quad j = 1, \ldots, \rho - 1 \\ \dot{\xi}_\rho &= L_f^\rho h(x) + L_g L_f^{\rho-1} h(x) u \doteq q(z, \xi) + b(z, \xi) u \\ y &= \xi_1\end{aligned} \tag{3}$$

in which $z = [z_1, \ldots, z_{n-\rho}]^{\mathrm{T}}, \xi = [\xi_1, \ldots, \xi_\rho]^{\mathrm{T}}$ are the new vector coordinates, with ξ being available for feedback.

Let $\overline{\mathcal{B}_{B_M}(0)}$ be the closed ball in $\mathbb{R}^n$ (with centre at the origin) of sufficiently large radius B_M; $\overline{\mathcal{B}_{B_{M*}}(0)}$ be the closed ball in $\mathbb{R}^{n-\rho}$ (with centre at the origin) of sufficiently large radius B_{M*}; $\overline{\mathcal{B}_{B_{M\xi}}(0)}$ be the closed ball in $\mathbb{R}^\rho$ (with centre at the origin) of sufficiently large radius $B_{M\xi} < B_M$ and containing the vector $\xi_*(t) = [y_*(t), y_*^{(1)}(t), \ldots, y_*^{(\rho-1)}(t)]$ for any $t \in \mathbb{R}$ (the components of the vector $\xi_*(t)$ are constituted by the time derivatives of the output reference signal $y_*(t)$, which are assumed to be available for feedback).

We assume that (i) for any $z \in \mathbb{R}^{n-\rho}$ and any $\xi \in \overline{\mathcal{B}_{B_{M\xi}}(0)}$ there exist symmetric positive definite matrices P and Q (possibly depending on $B_{M\xi}$) such that the Demidovich condition (see Pavlov et al., 2006)

$$P \frac{\partial \phi(z, \xi)}{\partial z} + \frac{\partial \phi^{\mathrm{T}}(z, \xi)}{\partial z} P \leq -Q \tag{4}$$

holds; (ii) for any $z \in \overline{\mathcal{B}_{B_{M*}}(0)}$ and any $\xi, \bar{\xi} \in \overline{\mathcal{B}_{B_{M\xi}}(0)}$ ($\bar{\xi} = [\bar{\xi}_1, \ldots, \bar{\xi}_\rho]^{\mathrm{T}}$) the following inequality

$$\|\phi(z, \xi) - \phi(z, \bar{\xi})\| \leq \alpha_\phi \|\xi - \bar{\xi}\| \tag{5}$$

holds in terms of the positive real α_ϕ (possibly depending on B_{M*} and $B_{M\xi}$); (iii) for any $(z, \xi), (\bar{z}, \bar{\xi}) \in \overline{\mathcal{B}_{B_M}(0)}$ the following inequalities

$$\begin{aligned}|q(z, \xi) - q(\bar{z}, \bar{\xi})| &\leq \alpha_q \|(z, \xi) - (\bar{z}, \bar{\xi})\| \\ |b(z, \xi) - b(\bar{z}, \bar{\xi})| &\leq \alpha_b \|(z, \xi) - (\bar{z}, \bar{\xi})\| \\ b(z, \xi) \geq b_m, \quad |q(0, 0)| &\leq \gamma_q, \; |b(0, 0)| \leq \gamma_b\end{aligned} \tag{6}$$

hold in terms of the positive reals α_q, α_b, b_m (possibly depending on B_M) and γ_q, γ_b.

In accordance with Section 1, condition (i) implies that the inverse system dynamics constitute a globally exponentially convergent system with the uniformly bounded steady-state property for the class of inputs $\overline{PC}(\overline{\mathcal{B}_{B_M}(0)})$ (see the subsequent Remark 1). For linear inverse system dynamics $\dot{z} = \Gamma z + \Theta \xi$, condition (i) is always satisfied when Γ is Hurwitz so that the results in Marino et al. (2012), Marino and Tomei (2009) for minimum phase systems are generalized in this note to the case of nonlinear inverse dynamics. The problem of the existence of a periodic solution for the tracking dynamics (which is implied by the minimum phase linear nature in Marino et al. (2012) (and related papers) and follows on the Brouwer's Fixed Point Theorem for the specific, first order example in Consolini and Verrelli (2014)), is thus here generally solved through the (more general) Demidovich condition. On the other hand, conditions (ii)–(iii) are rather standard in repetitive learning control scenarios (see Marino et al., 2012). In particular, the conditions (ii) and (iii) are "non-global" versions of the conditions in Marino and Tomei (2009), Bifaretti et al. (2012), Jin and Xu (2013) involving "global" bounding functions. Those "non-global" conditions, along with assumption (i), allow to prove that the subsequent simple straightforward generalization (7) of the PID$^{\rho-1}$ control solves the output tracking problem for the class of systems (3) for any initial condition with a properly dependent choice of the user-defined constant control gains k_i, $1 \leq i \leq \rho$.

Remark 1. Here we use the notation of Pavlov et al. (2006): (i) a function $w(\cdot) : \mathbb{R} \to W$ belongs to the class $\overline{PC}(W)$ if it is piecewise continuous and if there exists a compact set $K_W \subset W$ such that $w(t) \in K_W$ for all $t \in \mathbb{R}$; (ii) a system $\dot{z} = \phi(z, w)$ ($\phi(\cdot)$ is sufficiently smooth) is said to be globally exponentially convergent for the class of inputs $\overline{PC}(W)$ if it is globally exponentially convergent for every input $w \in \overline{PC}(W)$, i.e. if there exists a solution $\bar{z}_w(t)$ (depending on $w(t)$) that is defined and bounded for all $t \in \mathbb{R}$ and it is globally exponentially stable; such a system additionally possesses the uniform bounded steady-state property if for any compact set $K_W \subset W$ there exists a compact set $K_z \subset \mathbb{R}^{n-\rho}$ such that for any input $w \in \overline{PC}(W)$ the following property holds: "$w(t) \in K_W, \; \forall\, t \in \mathbb{R}$ implies $\bar{z}_W(t) \in K_z, \; \forall\, t \in \mathbb{R}$".

Remark 2. The approach presented in Xu, Huang, and Jiang (2013) is limited to the case in which the reference y_* is generated by finite-dimensional exosystems with the existence of solutions to the regulator equations being *a priori* assumed (along with the integral input-to-state stability property for the inverse tracking error system dynamics).

We now recall the notation in Marino et al. (2012), whose results are to be extended in this note. Let $\tilde{y} = y - y_*$ be the output tracking error; μ, M_u be positive control parameters; $\mathrm{sat}_{M_u}(\cdot) : \mathbb{R} \to [-M_u, M_u]$ be a continuous odd increasing function satisfying $\mathrm{sat}_{M_u}(q) = q$ for any $q \in (0, M_u]$ and $\mathrm{sat}_{M_u}(q) = M_u$ for any $q > M_u$; $\varphi(\cdot) : \mathbb{R}^+ \cup \{0\} \to [0, 1]$ be a continuous increasing function for $t \in [0, T]$ with $\varphi(0) = 0$ and $\varphi(t) = 1$ for any $t \geq T$.

The main purpose of this note is to prove that the straightforward generalization of the PID$^{\rho-1}$ control ($c(\rho) = 0$ if $\rho = 1$, $c(\rho) = 1$ if $\rho > 1$):

$$\begin{aligned}u(t) &= -k_\rho \left[c(\rho) \sum_{i=1}^{\rho-1} k_i \tilde{y}^{(i-1)}(t) + \tilde{y}^{(\rho-1)}(t) \right] + \hat{u}_*(t) \\ \hat{u}_*(t) &= \mathrm{sat}_{M_u}\left(\hat{u}_*(t-T)\right) \\ &\quad - \mu\varphi(t) \left[c(\rho) \sum_{i=1}^{\rho-1} k_i \tilde{y}^{(i-1)}(t) + \tilde{y}^{(\rho-1)}(t) \right] \\ \hat{u}_*(t) &= 0, \quad \forall\, t \leq 0,\end{aligned} \tag{7}$$

which has been recently presented in Marino et al. (2012) and adapted in Consolini and Verrelli (2014) to the case of spatial coordinate periodicity, solves (for any initial condition with a properly dependent choice of the constant user-defined control