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Robust adaptive attenuation of unknown periodic disturbances in uncertain multi-input multi-output systems☆

Saeid Jafari, Petros A. Ioannou

Department of Electrical Engineering, University of Southern California, Los Angeles, CA, USA

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ABSTRACT

In high-performance high-accuracy systems, the attenuation of vibrational disturbances is essential. In this paper, we design and analyze a robust output-feedback adaptive control scheme to attenuate noise-corrupted vibrational disturbances with unknown characteristics for multi-input multi-output linear time-invariant systems in the presence of plant unstructured modeling uncertainties. The conditions and the design parameters that contribute to the performance of the closed-loop system are investigated. The main ingredients of the proposed scheme include the use of a compensator to properly shape the singular values of the plant model, a robust adaptive law to handle unmodeled dynamics and output random noise, as well as an over-parametrization in the estimated parameters in order to provide flexibility for performance improvement. Analysis and simulations together with practical design considerations are presented to demonstrate the efficacy of the proposed scheme.

1. Introduction

Vibrations and unwanted sound suppression is a key enabling technology in a vast array of aerospace and industrial control applications. In many defense, industrial, and medical applications, unwanted sound and vibrations primarily caused by rotating components are dominantly periodic disturbances (Benesty, Sondhi, & Huang, 2008; Elliott & Nelson, 1993; Nelson & Elliott, 1992; Rudd, Lim, Li, & Lee, 2012). For example, high energy laser and optical communication systems can be very sensitive to platform vibrations and other jitter sources. Feedback adaptive control techniques have been proven to be quite effective in attenuating unknown periodic disturbances and enhancing performance (Perez-Arancibia, Gibson, & Tsao, 2009). Their application to advanced flight systems such as spacecraft (Lau, Joshi, Agrawal, & Kim, 2006), turboprop aircraft, and helicopters (Patt, Liu, Chandrasekar, Bernstein, & Friedmann, 2005) has also motivated a great deal of research in recent years (Aranovskiy & Freidovich, 2013; Basturk & Krstic, 2014; Bodson, 2005; Chen & Tomizuka, 2012; Isidori, Marconi, & Praly, 2012; Jafari, Ioannou, Fitzpatrick, & Wang, 2014; Kim, Kim, Chung, & Tomizuka, 2011; Landau, Alam, Martinez, & Buche, 2011; Landau, Alma, Constantinescu, Martinez, & Noe, 2011; Marino & Tomei, 2011, 2013; Nikiforov, 2004; Pigg & Bodson, 2010). The aforementioned papers are mainly for rejection of unknown periodic disturbances for single-input single-output (SISO) systems. For multi-input multi-output (MIMO) systems with strong coupling between the channels, however, the results for SISO systems are not directly applicable. In Pulido, Toledo, and Loukianov (2011), an estimator is designed to estimate the parameters of an unknown disturbance with a known number of distinct frequencies, and an adaptive internal-model based controller is then designed to suppress the effect of disturbances on the output of a MIMO plant. The limitations of internal-model based rejection of periodic disturbances with known time-varying frequencies have been studied in Kinney and de Callafon (2011). Recently, in Houtzager, van Wingerden, and Verhaegen (2013), a control method for MIMO systems has been proposed for rejection of periodic wind disturbances in wind turbines. These results are mostly for pure sinusoidal disturbances without considering the effect of low-amplitude broadband noise on the output of the plant. Moreover, it is mainly assumed that the plant model is perfectly known, without considering the effect of inevitable modeling uncertainties. The authors in Jafari and Ioannou (2013) proposed an adaptive scheme for rejection of unknown periodic disturbances with constant parametric characteristics for MIMO systems in the absence of plant modeling error. It is well known in robust adaptive control (Anderson, 2005; Ioannou & Sun, 1996) that adaptive

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E-mail addresses: sjafari@usc.edu (S. Jafari), ioannou@usc.edu (P.A. Ioannou).

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feedback systems designed for perfectly modeled plants could easily go unstable in the presence of small-size plant modeling uncertainties.

This paper presents the design and analysis of a robust adaptive controller for the attenuation of noise-corrupted vibrational disturbances with unknown characteristics in the presence of unstructured modeling uncertainties. In addition, the paper analyzes the trade-offs between performance and robustness which are used to choose certain design parameters. The main contributions of this paper are: (i) design and analysis of an output feedback adaptive controller for linear MIMO plants which is robust with respect to unmodeled dynamics, and additive output noise, (ii) design a pre-compensator to shape the singular values of the plant to have a high enough gain over the expected frequency range of the disturbance and low enough gain over the frequencies where the modeling error may be dominant. These modifications are beneficial to both performance and robustness with respect to noise and high-frequency unmodeled dynamics.

This paper is organized as follows: Section 2 presents some preliminaries and the notation used throughout the paper. The problem is defined and formulated in Section 3. Sections 4 and 5 contain the main results of the paper. The effectiveness of the proposed adaptive scheme is demonstrated in Section 6 using a simulation example. The concluding remarks are summarized in Section 7.

2. Preliminaries

An n_u-input, n_y-output finite-dimensional linear time-invariant system with real-rational transfer function matrix $H(z)$, input vector $u(k)$, and output vector $y(k)$ may be expressed as $y(k) = H(z)[u(k)]$, where $z = e^{j\omega T_s}$ is the operator of the z-transformation and T_s is the sampling period. For a signal x, the truncated signal x_k is defined as $x_k(i) = x(i)$, for $0 \le i \le k$ and $x_k(i) = 0$, for $i > k$. We use the $\mathcal{H}_\infty$-norm $\|H(z)\|_\infty = \max_{\theta\in[0,\pi]} \sigma_{\max}(H(e^{i\theta}))$, where $\sigma_{\max}$ denotes the maximum singular value, and the $\mathcal{L}_1$-norm $\|H(z)\|_1 = \max_{i=1}^{n_y} \sum_{j=1}^{n_u} \|h_{ij}\|_1$, where h_{ij} is the impulse response of the ij-element of $H(z)$, where $\|h_{ij}\|_1 = \sum_{k=0}^{\infty} |h_{ij}(k)|$ (Ioannou & Fidan, 2006; Ioannou & Sun, 1996; Skogestad & Postlethwaite, 2005). The $\mathcal{H}_\infty$ norm and the $\mathcal{L}_1$ norm are induced norms and satisfy the multiplicative property. The two norms are related as $\|H\|_\infty \le \sqrt{n_y}\|H\|_1 \le \sqrt{n_y n_u}(2n+1)\|H\|_\infty$, where n is the degree of a minimal realization of the corresponding system (Dahleh & Diaz-Bobillo, 1995).

The Swapping Lemma (Ioannou & Fidan, 2006) states that for signal vectors $x(k), y(k)$ and a proper stable rational scalar transfer function $H(z)$ with a minimal realization (A, B, C, D), i.e., $H(z) = C^T(zI - A)^{-1}B + D$, we have

$$H(z)[x^T(k)y(k)] = x^T(k)H(z)[y(k)] + H^c(z)[(H^b(z)[y^T(k)])\Delta x(k)],$$

where $\Delta x(k) = x(k+1) - x(k)$, $H^c(z) = -C^T(zI - A)^{-1}$, and $H^b(z) = z(zI - A)^{-1}B$.

We say that the signal x is μ-small in the mean square sense and write $x \in \mathcal{S}(\mu)$, when $\sum_{i=k}^{k+N-1} x(i)^T x(i) \le c_1 N\mu + c_2,\ \forall k, N,$ and some constant $c_1, c_2 \ge 0$ (Ioannou & Fidan, 2006; Ioannou & Sun, 1996).

A proper real-rational transfer function matrix $M(z)$ analytic in $|z| \ge 1$ is called *inner* if it satisfies $M^T(z^{-1})M(z) = I$ (an inner matrix is square or has more rows than columns), and is called *outer* if it is right invertible with a right inverse proper and analytic in $|z| > 1$ (an outer matrix is square or has more columns than rows). Any proper real-rational transfer function matrix $H(z)$ analytic in $|z| \ge 1$ with no zeros on the unit circle has an *inner-outer factorization* $H(z) = H_{\text{in}}(z)H_{\text{out}}(z)$, where $H_{\text{in}}(z)$ is inner, $H_{\text{out}}(z)$ is outer, and the right inverse of $H_{\text{out}}(z)$ is proper and analytic in $|z| \ge 1$ (Ionescu & Oara, 1996; Lin, Chen, Saberi, & Shamash, 1996). A proper real-rational transfer function matrix $M(z)$ analytic in $|z| \ge 1$ is called *all-pass* if it is square and satisfies $M^T(z^{-1})M(z) = I$; clearly a square inner function is all-pass.

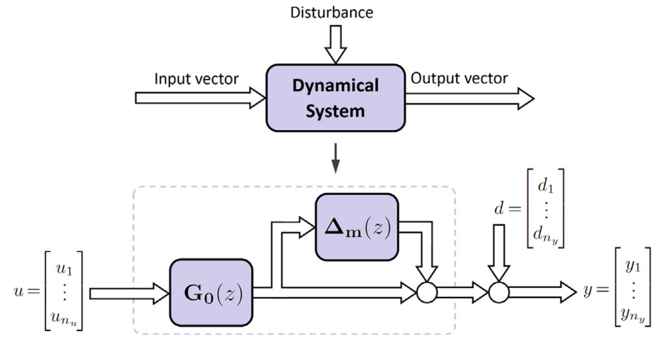

Fig. 1. The structure of the open-loop plant model with multiplicative output uncertainty.

3. Problem formulation

Consider an uncertain multi-input multi-output plant

$$\begin{aligned} y(k) &= G(z)[u(k)] + d(k) \\ &= (I + \Delta_m(z))G_0(z)[u(k)] + d(k), \end{aligned} \tag{1}$$

where $y \in \mathbb{R}^{n_y}$ is the measurable output, $u \in \mathbb{R}^{n_u}$ is the control input, $d \in \mathbb{R}^{n_y}$ is an unknown and unmeasurable bounded disturbance, $G(z) = (I + \Delta_m(z))G_0(z)$ is the $n_y \times n_u$ transfer matrix of the plant, $G_0(z) = [G_{0ij}(z)]_{n_y\times n_u}$ is the known modeled part of the plant which is bounded-input bounded-output (BIBO) stable and is possibly non-minimum phase, and $\Delta_m(z)$ is the multiplicative output uncertainty which is unknown but small in some norm sense in the low frequency range and may be relatively large at high frequencies (a qualitative assumption we later make more precise). We assume $\Delta(z) = \Delta_m(z)G_0(z)$ is proper and analytic in $|z| \ge 1$ which is a standard assumption in robust control theory for permissible perturbations. No knowledge about the structure and parameters of the unmodeled dynamics $\Delta_m(z)$ is assumed to be known. The structure of the open-loop plant model is shown in Fig. 1.

In many applications, the dominant part of the narrow-band vibrational disturbances is often modeled as a sum of sinusoidal terms with unknown frequencies, magnitude, and phases. For each output channel we assume the disturbance can be expressed as

$$\begin{aligned} d_j(k) &= d_{s_j}(k) + v_j(k) \\ &= \sum_{i=1}^{n_{f_j}} a_{ij}\sin(\omega_{ij}k + \varphi_{ij}) + v_j(k), \end{aligned} \tag{2}$$

for $j = 1, \dots, n_y$, where the sum of the periodic terms $d_{s_j}(k)$ is the dominant part of the disturbance, n_{f_j} is the number of distinct frequencies, $v_j(k)$ is a zero-mean bounded noise whose amplitude is much smaller than that of $d_{s_j}(k)$, and the parameters a_{ij}, ω_{ij} [rad/sample], and φ_{ij} [rad] are all unknown. The number of distinct frequencies in d_s whose corresponding amplitude is above the output noise level is bounded from above by a known bound. We express the additive output disturbance vector as

$$d(k) = d_s(k) + v(k), \tag{3}$$

where $d_s = [d_{s_1}, \dots, d_{s_{n_y}}]^T$ and $v = [v_1, \dots, v_{n_y}]^T$.

The objective is to design a control signal u to suppress the effect of d on y as much as possible. In this paper we use the controller