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Event-based control of linear hyperbolic systems of conservation laws ☆

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ABSTRACT

In this article, we introduce event-based boundary controls for 1-dimensional linear hyperbolic systems of conservation laws. Inspired by event-triggered controls developed for finite-dimensional systems, an extension to the infinite dimensional case by means of Lyapunov techniques, is studied. The main contribution of the paper lies in the definition of two event-triggering conditions, by which global exponential stability and well-posedness of the system under investigation is achieved. Some numerical simulations are performed for the control of a system describing traffic flow on a roundabout.

1. Introduction

Event-based control is a computer control strategy which aims to use communications and computational resources efficiently by updating control inputs aperiodically, only when needed. Several works have been developed in this area for finite dimensional networked control systems (see for instance the seminal work (Årzén, 1999; Åström & Bernhardsson, 1999) or the most recent ones (Heemels, Johansson, & Tabuada, 2012; Postoyan, Tabuada, Nesic, & Anta, 2015) and the references therein). Two components are essential in the framework of event-based control. The first one is a feedback control law which has been designed to stabilize the system. The second one is a triggering strategy which determines the time instants when the control needs to be updated. Usually, the triggering strategy guarantees that a Lyapunov function decreases strictly either by using an Input-to-State Stability (ISS) property (Tabuada, 2007) or by working directly on the time derivative of the Lyapunov function (Marchand, Durand, & Castellanos, 2013). Besides the interest of reducing communication and computational loads, event-based control is also known as a rigorous way to implement digitally continuous time controllers. In this work, such control strategies were developed for a class of infinite-dimensional systems of conservation laws, provided by linear hyperbolic partial differential equations (PDEs).

Hyperbolic systems of conservation laws stand out as having important applications in the modeling and control of physical networks: hydraulic (Bastin, Coron, & d'Andréa Novel, 2008), road traffic (Coclite, Garavello, & Piccoli, 2005), gas pipeline networks (Gugat, Dick, & Leugering, 2011), to name a few. Stability analysis and stabilization of such systems have attracted a lot of attention in the last decade. Two ways of acting on these systems exist: boundary and in domain control. For boundary control, backstepping (Coron, Vazquez, Krstic, & Bastin, 2013; Krstic & Smyshlyaev, 2008) and Lyapunov techniques (Coron, d'Andréa Novel, & Bastin, 2007; Fridman & Orlov, 2009; Martin & Michaël, 2011; Prieur & Mazenc, 2012) are the most commonly used. Several applications, in which control actions are on the boundary, can be found for instance in Bastin, Haut, Coron, and Novel (2007), Coron, Bastin, and d'Andréa Novel (2008), Dos Santos, Bastin, Coron, and d'Andréa Novel (2008) and Prieur, Winkin, and Bastin (2008) where the exponential stability of steady-states depends on the *dissipativity* of the boundary conditions. This paper focuses on boundary control using Lyapunov techniques where such a dissipativity property is an important issue to be taken into account.

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The design of event-based control strategies for distributed parameter systems is rarely treated in the literature. Extending existing results for ordinary differential equations (ODEs) to time-delay systems is proposed in Durand, Marchand, and Guerrero Castellanos (2014); however, this is quite far from the problem addressed in this paper. For parabolic PDEs, event-based control strategies are considered in Selivanov and Fridman (2015) and Yao and El-Farra (2013). Many difficulties that arise in the context of event-based control are due to the introduction of discontinuities when updating the control. Discontinuous output feedback controllers for infinite dimensional systems have been studied, for instance in Orlov (2009), where unit feedback controller and in turn global asymptotic stabilization are considered. Although, the framework of switched hyperbolic systems (Hante, Leugering, & Seidman, 2009; Lamare, Girard, & Prieur, 2015; Prieur, Girard, & Witrant, 2014) is highly inspiring – especially the work in Lamare et al. (2015) – for dealing with the well-posedness of the closed-loop solution of such systems under event-based control strategies. The main difference of Lamare et al. (2015) with respect to the current work is that in Lamare et al. (2015), no boundary control inputs are considered but rather switching boundary conditions governed by a switching signal, given as a output feedback, that imposes the mode that the system must evolve.

The main contribution of this paper is to propose a rigorous framework for event-based control of linear hyperbolic systems of conservation laws, as well as two event-based stabilization strategies based on the two aforementioned main triggering strategies developed for systems described by ODEs called ISS event-based stabilization and D^+V event-based stabilization in the sequel. The notion of existence and uniqueness of the solution is treated. It is also established that the number of events in a bounded time interval is necessarily bounded avoiding the well known Zeno phenomena. To the author's knowledge, this work is the first contribution to event-based control for hyperbolic PDE systems proposed in the literature. For PDEs, a well known approach for digital controller synthesis relies on numerical approximations by discretizing the space in order to get an ODE (see e.g. Djouadi, Camphouse, & Myatt, 2008) on which finite dimensional approaches can be applied. In this work, the method is completely different and addresses directly the boundary control without model reduction and the sampling in time of continuous controllers so that implementations on a digital platform may be carried out in an aperiodic fashion.

This paper is organized as follows. In Section 2, we introduce the class of linear hyperbolic system of conservation laws, and a sufficient condition for the existence of the solution is discussed. Section 3 contains the main results that are two strategies for event-based control. The existence and uniqueness of the solution as well as the stability of the closed loop systems are discussed. Section 4 provides a numerical example to illustrate the main results and to compare the two control strategies. Finally, conclusions and perspectives are given in Section 5.

Preliminary definitions and notation. $\mathbb{R}^+$ will denote the set of nonnegative real numbers. For any vector v, its dth component will be denoted v_d. Given a matrix A, its transpose will be denoted A^T and its component at row i and column j will be denoted by A_{ij}. 0 will denote the zero matrix of suitable dimension.

The usual Euclidean norm in $\mathbb{R}^n$ is denoted by $|\cdot|$ and the associated matrix norm is denoted $\|\cdot\|$. The set of all functions $\phi : [0,1] \to \mathbb{R}^n$ such that $\int_0^1 |\phi(x)|^2 < \infty$ is denoted by $L^2([0,1],\mathbb{R}^n)$ that is equipped with the norm $\|\cdot\|_{L^2([0,1],\mathbb{R}^n)}$. The restriction of a function $y : I \to J$ on an open interval $(x_1,x_2) \subset I$ is denoted by $y|_{(x_1,x_2)}$. Given an interval $I \subseteq \mathbb{R}$ and a set $J \subseteq \mathbb{R}^n$ for some $n \geq 1$, a *piecewise left-continuous function* (resp. a *piecewise right-continuous function*) $y : I \to J$ is a function continuous on each closed interval subset of I except maybe on a finite number of points $x_0 < x_1 < \cdots < x_p$ such that for all $l \in \{0,\ldots,p-1\}$ there exists y_l continuous on $[x_l,x_{l+1}]$ and $y_{l|(x_l,x_{l+1})} = y_{|(x_l,x_{l+1})}$. Moreover, at the points $x_0,\ldots,x_p$ the function is continuous from the left (resp. from the right). The set of all piecewise left-continuous functions (resp. piecewise right-continuous functions) is denoted by $\mathcal{C}_{lpw}(I,J)$ (resp. $\mathcal{C}_{rpw}(I,J)$). In addition, we have the following inclusions $\mathcal{C}_{lpw}([0,1],\mathbb{R}^n), \mathcal{C}_{rpw}([0,1],\mathbb{R}^n) \subset L^2([0,1],\mathbb{R}^n)$.

2. Linear hyperbolic systems

Let us consider the linear hyperbolic system of conservation laws (given in Riemann coordinates):

$$\partial_t y(t,x) + \Lambda \partial_x y(t,x) = 0 \quad x \in [0,1],\ t \in \mathbb{R}^+ \tag{1}$$

where $y : \mathbb{R}^+ \times [0,1] \to \mathbb{R}^n$, Λ is a diagonal matrix in $\mathbb{R}^{n\times n}$ such that $\Lambda = \mathrm{diag}(\lambda_1,\ldots,\lambda_n)$ with $0 < \lambda_1 < \lambda_2 < \cdots < \lambda_n$. We consider the following boundary condition:

$$y(t,0) = Hy(t,1) + Bu(t), \quad t \in \mathbb{R}^+ \tag{2}$$

where $H \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $u : \mathbb{R}^+ \to \mathbb{R}^m$.

In addition to the partial differential equation (1) and the boundary condition (2), we consider the initial condition given by

$$y(0,x) = y^0(x), \quad x \in [0,1] \tag{3}$$

where $y^0 \in \mathcal{C}_{lpw}([0,1],\mathbb{R}^n)$.

Remark 1. The results in this paper can be extended to first order linear hyperbolic systems with both negative and positive speeds ($\lambda_1 < \cdots \lambda_m < 0 < \lambda_{m+1} < \cdots < \lambda_n$) by defining the state description $y = [y_- \quad y_+]^T$, where $y_- \in \mathbb{R}^m$ and $y_+ \in \mathbb{R}^{n-m}$, and applying the change of variable $\tilde{y}(t,x) = [y_-(t,1-x) \quad y_+(t,x)]^T$. ○

We shall consider possibly discontinuous inputs $u \in \mathcal{C}_{rpw}(\mathbb{R}^+,\mathbb{R}^m)$, therefore solutions of (1)–(3) may not be differentiable everywhere. Thus, we introduce a notion of weak solutions (generalized ones) (in Section 2.1) as well as a sufficient condition for the existence and uniqueness of the solution for a class of discontinuous initial conditions and feedback laws (in Section 2.2).

2.1. Solution of the system

We consider solutions of (1)–(3) in the sense of characteristics Li (1994). For each component y_d of (1), one can define the characteristic curve solution of the differential equation $\dot{x}(t) = \lambda_d$ which is rewritten as $x(t) = x_0 + \lambda_d t$. By doing this, we obtain the following definition (see Lamare et al., 2015, Definition 4 for a more general case):

Definition 1. Let $y^0 \in \mathcal{C}_{lpw}([0,1],\mathbb{R}^n)$ and $u \in \mathcal{C}_{rpw}(\mathbb{R}^+,\mathbb{R}^m)$. A solution to (1)–(3) is a function $y : \mathbb{R}^+ \times [0,1] \to \mathbb{R}^n$ such that, for all t in $\mathbb{R}^+$ and $x_0 \in [-\lambda_d t, 1-\lambda_d t]$,

$$\frac{d}{dt} y_d(t, x_0 + \lambda_d t) = 0 \tag{4}$$

with the initial condition

$$y_d(0,x) = y_d^0(x), \quad \forall x \in [0,1] \tag{5}$$

and the boundary condition

$$y_d(t,0) = \sum_{j=1}^{n} H_{dj} y_j(t,1) + \sum_{j=1}^{m} B_{dj} u_j(t), \quad \forall t \in \mathbb{R}^+ \tag{6}$$

for all $d = 1,\ldots,n$.