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Brief paper

Robust controlled invariance for monotone systems: Application to ventilation regulation in buildings☆

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ABSTRACT

The robust controlled invariance describes the ability to maintain, using suitable control actions, the state of a system in a set for any value of the disturbances. By considering a class of monotone systems and a multidimensional interval as target set, we obtain a simple characterization of the robust controlled invariance. We then give a method to stabilize the state into a robust controlled invariant interval when it is initialized outside of the target set. These results are applied to a model for the temperature control in an intelligent building equipped with automated underfloor air distribution (UFAD) and implemented in a small-scale experimental UFAD flat.

1. Introduction

For autonomous systems, the notion of *positively invariant* set describes the property that trajectories initialized in a set remain inside this set forever. An extensive survey on the topic of invariance can be found in Blanchini (1999). When a control input is used to enforce the invariance, we talk about *controlled invariance*, independently introduced in Basile and Marro (1969) and Wonham and Morse (1970). An overview of the uses and results on controlled invariant sets for linear systems is given in Trentelman, Stoorvogel, and Hautus (2001). In this paper, we are interested in the study of *robust controlled invariance* where the robustness refers to bounded external disturbances.

In this paper, we deal with a class of nonlinear systems satisfying a monotonicity property. Monotone systems are systems which preserve partial orderings on the states, see Smith (1995) for autonomous systems and Angeli and Sontag (2003) for controlled systems. We show that this monotonicity property, associated with simple sets (multidimensional intervals), can be used to obtain a characterization for the robust controlled invariance, using only the extremal values of each state, control and disturbance input. We also show how these robust controlled invariant sets can be used to synthesize robust stabilizing controllers for monotone control systems. To the knowledge of the authors, there are very few works on (controlled) invariance of monotone nonlinear systems: invariance of intervals for autonomous monotone systems has been considered in Abate, Tiwari, and Sastry (2009); methods for approximating the maximal controlled invariant set for monotone discrete time systems without disturbance are presented in Lara, Doyen, Guilbaud, and Rochet (2007); a controller for reference tracking in a monotone SISO system is synthesized under state constraints in Chisci and Falugi (2006); finally, robust controlled invariance are considered for a class of monotone systems with planar outputs in Ghaemi and Del Vecchio (2014). Monotone systems can be found in numerous fields such as molecular biology (Sontag, 2007), chemical networks (Belgacem & Gouzé, 1999), multi-vehicle systems (Hafner, Cunningham, Caminiti, & Vecchio, 2013), or thermal dynamics in buildings, which is the application considered in this paper. We consider an *underfloor air distribution* (UFAD) system based on a 4-room small-scale experiment of a flat. We apply the results developed in the paper to that system and report the results obtained on our experimental platform.

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The paper is organized as follows. In Section 2, we introduce the class of systems we consider. In Section 3, we establish a certain number of results on robust invariance and robust controlled invariance. In Section 4, we show how our characterization of robust controlled invariant interval allows us to synthesize robust stabilizing controllers. Finally these methodological results are applied to the temperature control of a UFAD model and tested on a small-scale experimental flat in Section 5.

2. Monotone control systems

We consider a class of nonlinear systems given by:

$$\dot{x} = f(x, u, w), \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$ and $w \in \mathbb{R}^q$ denote the state, the control input and the disturbance input, respectively. The vector field f is assumed to be locally Lipschitz. The trajectories of (1) are denoted by $\Phi(\cdot, x_0, \mathbf{u}, \mathbf{w})$ where $\Phi(t, x_0, \mathbf{u}, \mathbf{w})$ is the state reached at time $t \in \mathbb{R}_0^+$ from the initial state $x_0 \in \mathbb{R}^n$, under control and disturbance inputs $\mathbf{u} : \mathbb{R}_0^+ \to \mathbb{R}^p$ and $\mathbf{w} : \mathbb{R}_0^+ \to \mathbb{R}^q$. When the control inputs of system (1) are generated by a state-feedback controller $\mathrm{u} : \mathbb{R}^n \to \mathbb{R}^p$, the dynamics of the closed-loop system is given by $\dot{x} = f_{\mathrm{u}}(x, w) = f(x, \mathrm{u}(x), w)$ and its trajectories are denoted by $\Phi_{\mathrm{u}}(\cdot, x_0, \mathbf{w})$.

2.1. Monotonicity

The subsequent developments of this paper require the system (1) to satisfy some monotonicity property and we particularly focus on the subclass of cooperative systems. For a variable $z \in \{x, u, w\}$ with $z \in \mathbb{R}^m$, the partial orderings $\succeq_z, \preceq_z, \gg_z$ and $\ll_z$ represent the classical componentwise inequalities $\geq, \leq, >$ and $<$ on $\mathbb{R}^m$. These orderings can be extended to functions $\mathbf{z}, \mathbf{z}' : \mathbb{R}_0^+ \to \mathbb{R}^m$ where $\mathbf{z} \succeq_z \mathbf{z}'$ if and only if $\mathbf{z}(t) \succeq_z \mathbf{z}'(t)$ for all $t \geq 0$. Given $\underline{z}$ and $\overline{z} \in \mathbb{R}^m$ with $\overline{z} \succeq_z \underline{z}$, $[\underline{z}, \overline{z}]$ denotes the interval such that $z \in [\underline{z}, \overline{z}]$ if and only if $\overline{z} \succeq_z z \succeq_z \underline{z}$. Following Angeli and Sontag (2003), we now introduce the notion of cooperative system using the partial orderings $\succeq_x$, $\succeq_u$ and $\succeq_w$.

Definition 1 (*Cooperative System*)**.** System (1) is cooperative if for all $x \succeq_x x'$, $\mathbf{u} \succeq_u \mathbf{u}'$, $\mathbf{w} \succeq_w \mathbf{w}'$, it holds for all $t \geq 0$, $\Phi(t, x, \mathbf{u}, \mathbf{w}) \succeq_x \Phi(t, x', \mathbf{u}', \mathbf{w}')$.

In a cooperative system, a variable (state or input) affects a state always in a positive way, as shown by the following characterization which is a generalization of the Kamke condition to systems with inputs.

Proposition 2 (*Angeli & Sontag, 2003*)**.** *System (1) is cooperative if and only if for all $i \in \{1, \dots, n\}$, for all $x \succeq_x x'$ with $x_i = x_i'$, $u \succeq_u u'$, $w \succeq_w w'$, it holds $f_i(x, u, w) \geq f_i(x', u', w')$.*

In the following, we shall make the following assumption for system (1).

Assumption 3. System (1) is cooperative with bounded control and disturbance inputs: $u \in [\underline{u}, \overline{u}]$ and $w \in [\underline{w}, \overline{w}]$.

Assumption 3 is crucial for our robustness analysis since we can focus on studying the behavior of the system only for the extremal values of the variables: all other behaviors are necessarily bounded by the extremal behaviors.

2.2. Additional assumptions

Some of the results presented in the following sections need additional requirements on system (1). The following assumption is necessary for all main results presented in this paper.

Assumption 4. System (1) satisfies the local control property: any component of the control input directly influences a single component of the state in (1).

With this assumption, system (1) can then be written as $\dot{x}_i = f_i(x, u_i, w)$ for all $i \in \{1, \dots, n\}$, where u_i represents all input components with a direct influence on x_i (i.e. u_i can be a vector, a scalar or the empty set).

We also extend the definition of a static input-state characteristic introduced in Angeli and Sontag (2003) to systems with both control and disturbance inputs. The following assumption is optional as it is only useful for secondary results: the main results can still be applied if it is not satisfied.

Assumption 5. System (1) has a static input-state characteristic $k_x : \mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^n$: for each pair (u, w) of constant control and disturbance inputs, (1) has a unique globally asymptotically stable equilibrium $k_x(u, w)$.

3. Robust invariance for monotone systems

In this section, we present and characterize several notions of robust invariance and focus on finding the associated inputs and state intervals. Some of the results of this section were previously presented in Meyer, Girard, and Witrant (2013) with less generality.

3.1. Robust invariance

A robust invariant is a set such that if the state of the system is initialized in this set then it remains in the set forever, for all values of the control and disturbance inputs. Restricting this notion to intervals, we have the following definition.

Definition 6 (*Robust Invariance*)**.** An interval $[\underline{x}, \overline{x}]$ is robust invariant if, for all $x_0 \in [\underline{x}, \overline{x}]$, $\mathbf{u} \in [\underline{u}, \overline{u}]$, $\mathbf{w} \in [\underline{w}, \overline{w}]$, it holds for all $t \geq 0$, $\Phi(t, x_0, \mathbf{u}, \mathbf{w}) \in [\underline{x}, \overline{x}]$.

Thus, if the initial state is in a robust invariant interval, this interval contains all reachable states from this initial condition. However, this does not mean that all points in this interval are reachable. In order to minimize the quantity of non-reachable states in the interval, one can look for the *minimal robust invariant interval* (where minimality refers to the set inclusion), which is useful in the subsequent study as we can restrict the analysis of system (1) to that region.

Theorem 7. *Under Assumption 3, $[\underline{x}, \overline{x}]$ is robust invariant if and only if $f(\overline{x}, \overline{u}, \overline{w}) \preceq_x 0$ and $f(\underline{x}, \underline{u}, \underline{w}) \succeq_x 0$. In addition, if Assumption 5 holds, then the minimal robust invariant interval is $[k_x(\underline{u}, \underline{w}), k_x(\overline{u}, \overline{w})]$.*

Proof. $[\underline{x}, \overline{x}]$ is robust invariant if and only if for any element x of the boundary of $[\underline{x}, \overline{x}]$, the flow $\Phi(t, x, \mathbf{u}, \mathbf{w})$ does not leave the interval. This is equivalent to having the vector field at x point inside the interval for all $u \in [\underline{u}, \overline{u}]$ and $w \in [\underline{w}, \overline{w}]$. By considering the elements of the boundary $\underline{x}$ and $\overline{x}$, it is clear that the conditions above are necessary. Let us show that they are also sufficient under Assumption 3. By Proposition 2, we have for all $i \in \{1, \dots, n\}$, $u \in [\underline{u}, \overline{u}]$, $w \in [\underline{w}, \overline{w}]$ and $x \in [\underline{x}, \overline{x}]$ with $x_i = \overline{x}_i$, $f_i(x, u, w) \leq f_i(\overline{x}, \overline{u}, \overline{w}) \leq 0$ and for all $i \in \{1, \dots, n\}$, $u \in [\underline{u}, \overline{u}]$, $w \in [\underline{w}, \overline{w}]$ and $x \in [\underline{x}, \overline{x}]$ with $x_i = \underline{x}_i$, $f_i(x, u, w) \geq f_i(\underline{x}, \underline{u}, \underline{w}) \geq 0$. Therefore, $[\underline{x}, \overline{x}]$ is robust invariant. Now, let us assume that Assumption 5 holds. By definition, we have $f(k_x(\overline{u}, \overline{w}), \overline{u}, \overline{w}) = 0$ and $f(k_x(\underline{u}, \underline{w}), \underline{u}, \underline{w}) = 0$. From what precedes, $[k_x(\underline{u}, \underline{w}), k_x(\overline{u}, \overline{w})]$ is robust invariant. Also, any robust invariant interval would contain $k_x(\underline{u}, \underline{w})$ and $k_x(\overline{u}, \overline{w})$ as these are globally asymptotically stable equilibria for constant inputs $\mathbf{u} = \underline{u}$, $\mathbf{w} = \underline{w}$ and $\mathbf{u} = \overline{u}$ and $\mathbf{w} = \overline{w}$, respectively. Hence, the robust invariant interval $[k_x(\underline{u}, \underline{w}), k_x(\overline{u}, \overline{w})]$ is minimal with respect to set inclusion. □