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Brief paper

Model reduction of switched affine systems[☆]

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ABSTRACT

This paper addresses model reduction and extends balanced truncation to the class of switched affine systems with endogenous switching. The switched affine system is rewritten as a switched linear one with state resets that account for the affine terms. Balanced truncation can then be applied to each mode dynamics, independently. As a result, different reduced state vectors are associated with the different modes, and reset maps are here appropriately redefined so as to account and compensate for this mismatch, possibly preserving the continuity of the output. The overall behavior of the reduced switched system is determined by both the selected reduction per mode and the adopted reset maps. In this paper, we consider a stochastic setting and propose a randomized method for the selection of the reduced order. The performance of the proposed approach is illustrated through a multi-room temperature control example.

1. Introduction

This paper addresses the design of an approximate model for a hybrid system (see e.g. Girard, Julius, & Pappas, 2008; Girard & Pappas, 2007; Girard, Pola, & Tabuada, 2010; Julius & Pappas, 2009; Mazzi, Sangiovanni Vincentelli, Balluchi, & Bicchi, 2008; Prandini, Garatti, & Vignali, 2014; Shaker & Wisniewski, 2012). The study of hybrid systems is typically challenging since they are characterized by intertwined continuous and discrete dynamics, Lunze and Lamnabhi-Lagarrigue (2009). Indeed, many problems that have been solved for purely discrete or purely continuous systems still lack an effective solution for hybrid systems. In particular, this is the case for the design of a reduced model.

In this paper, we focus on continuous-time Switched Affine (SA) systems with endogenous switching, and address the problem of obtaining a model that is simpler to analyze than the original system, and that is able to mimic its output behavior over a finite horizon $\mathcal{T}$. This is of interest when dealing with verification of properties that depend on the behavior of the system output over a finite horizon. Verification of properties related to the system response, like, e.g., safety and reach/avoid properties, is typically addressed in the literature through numerical methods in both the deterministic, Frehse (2005), Girard and Guernic (2008), Kurzhanski and Varaiya (2005) and Tomlin, Mitchell, Bayen, and Oishi (2003), and the stochastic, Abate, Amin, Prandini, Lygeros, and Sastry (2007) and Abate, Katoen, Lygeros, and Prandini (2010), settings. These methods scale exponentially with the dimension of the continuous state space component. One can then conceive a two-step procedure where an approximate abstraction with a reduced order continuous state space component is built first, and then a numerical verification method is applied to this abstraction in place of the original system.

When the input signal of the system is stochastic, the notion of approximate simulation introduced in Julius and Pappas (2009) for stochastic hybrid systems (Lygeros & Prandini, 2010) can be used to quantify the model performance over the output realizations. A randomized approach for assessing the performance of a given abstracted model according to this notion was proposed in Prandini et al. (2014). The approach also extends to model design. However, no constructive procedure is given on how to select and parameterize the model class. On the contrary, in this paper we provide a constructive procedure to build an approximate model of a SA system in the form of a reduced order Switched Linear (SL) system with appropriately defined state reset maps. The SA system is first rewritten as a SL one with state reset, and then Balanced Truncation (BT) (Antoulas, 2005) is adopted for reducing the order of the linear dynamics governing the evolution of the continuous

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state component in each mode. State reset maps are suitably redefined accounting for the mismatch in the continuous state vectors associated with different modes. A randomized method is also proposed to determine the order of the reduced linear dynamics in each mode, while accounting for the effect of discrete transitions and state resets on the hybrid system evolution. The overall methodology is extended to the case when a Dwell Time (DT) is present.

Note that BT is applied to switched linear systems in Petreczky, Wisniewski, and Leth (2013) which however deals with the case of externally induced switching. Our approach is inspired by Mazzi et al. (2008) which uses BT for hybrid systems with linear dynamics and endogenous switching. The main advances with respect to Mazzi et al. (2008) are the following: (1) the extension to the class of SA systems, (2) the introduction of novel state reset maps that provide better performance than the one adopted in Mazzi et al. (2008), and of variants of these maps able to preserve continuity. Correspondingly, different initializations of the approximate model are derived based on the same logic underlying the reset maps definition, (3) the introduction of a randomized approach to select the order of the reduced linear dynamics in each mode, when the input is stochastic, and (4) the extension to the case of SA systems with DT. As a matter of fact, mode transitions and resets may strongly affect the system evolution. Indeed, the state reset map determines the new value of the continuous state after a discrete transition between modes has just occurred; while for a linear asymptotically stable system the contribution of the initial state becomes negligible in the long run, in a SA system this is generally not the case. One would in fact need to guarantee that the time between discrete transitions is sufficiently large to make the zero-input response (ZIR) vanish, which cannot be guaranteed a-priori, unless a suitable DT triggering the discrete transitions is enforced.

The choice of the order of the approximate model should then account for the influence of the state reset map on the quality of the approximation. Hence it cannot be based only on the analysis of the Hankel Singular Values (HSVs) of the linear dynamics in each mode, as suggested in Mazzi et al. (2008). The proposed randomized approach serves this purpose, since it accounts for the hybrid evolution of the candidate approximate model including mode transitions and resets. The quality of the approximation is determined also by the domains triggering the mode transitions of the SA system. Notably, redesigning the domains is quite a complex issue, Geyer, Torrisi, and Morari (2008), and it is not addressed in this paper but left for further investigation.

A preliminary version of this work appeared in Papadopoulos and Prandini (2014). Additional contributions are the introduction of reset maps that preserve the output continuity, the initialization of the approximate model derived from these maps, the extension to the case of SA systems with DT, and a more thoughtful benchmark example that includes the analysis of the new reset maps and the effect of the DT.

The scope of this work does not include the problem of minimal realization. To the best of our knowledge, minimal realization theory has been mainly developed for linear and bilinear switched and hybrid systems with externally induced switching, while it is still an open problem for continuous-time hybrid systems with endogenous switching (Petreczky, 2015).

2. Switched affine systems modeling framework

A SA system is an instance of a hybrid system, whose dynamics are characterized through a discrete state component q_a (mode) taking values in $Q = \{1, 2, \dots, m\}$ and a continuous component $\xi_a \in \Xi_a = \mathbb{R}^n$ evolving according to affine dynamics that depend on the value taken by q_a. The output $y_a \in Y_a = \mathbb{R}^p$ of the systems is an affine function of the state and of the input $u \in U = \mathbb{R}^m$ that depends on q_a as well. The continuous dynamics of a SA system within a given mode $q_a \in Q$ are given by

$$\mathcal{S}_a : \begin{cases} \dot{\xi}_a(t) = \mathcal{A}_{q_a}\xi_a(t) + \mathcal{B}_{q_a}u(t) + f_{q_a} \\ y_a(t) = \mathcal{C}_{q_a}\xi_a(t) + g_{q_a}. \end{cases} \tag{1}$$

Assumption 1. For any $i \in Q$, matrix $\mathcal{A}_i$ is Hurwitz, $(\mathcal{A}_i, \mathcal{B}_i)$ is controllable, and $(\mathcal{A}_i, \mathcal{C}_i)$ is observable. □

As for the discrete state evolution, a collection of polyhedra $\{Dom_{a,i} \subseteq Y_a \times U,\ i \in Q\}$ is given, which covers the whole set $Y_a \times U$, i.e., $\cup_{i\in Q} Dom_{a,i} = Y_a \times U$. $Dom_{a,i}$ is defined through r_i linear inequalities, i.e., $Dom_{a,i} = \{(y_a, u) \in Y_a \times U : G_i^{y_a} y_a + G_i^u u \le G_i\}$, with $G_i^{y_a} \in \mathbb{R}^{r_i \times p}$, $G_i^u \in \mathbb{R}^{r_i\times m}$ and $G_i \in \mathbb{R}^{r_i}$.

Mode $i \in Q$ is active as long as (y_a, u) keeps evolving within $Dom_{a,i}$ and a transition to mode $j \neq i \in Q$ occurs as soon as (y_a, u) exits $Dom_{a,i}$ and enters into $Dom_{a,j}$ (endogenous switching).

Assumption 2. The switched affine system (1) admits a unique solution from any initial state. □

Note that the considered switched system can be rephrased in the hybrid automata framework described in Tomlin, Lygeros, and Sastry (2000), where a precise notion of execution is given and conditions for well-posedness (existence and uniqueness) are mentioned. Moreover, if the collection $\{Dom_{a,i}, i \in Q\}$ is a polyhedral subdivision of $Y_a \times U$,[2] then the SA system reduces to a standard piecewise affine system.

Remark 1. If the transition condition depends on the state ξ_a, then one can include ξ_a as output variable to get back to the considered modeling framework where domains are defined as a function of the output (and input).

3. System reduction: an approach based on BT

The proposed procedure unfolds into the following steps: (1) the SA system is rewritten as a SL system with state reset (Section 3.1); (2) a reduced order model of the SL system is introduced by first applying BT to the continuous dynamics in each mode (Section 3.2), and then introducing appropriate maps for the reset of the reduced continuous state component when a mode transition occurs (Section 4); (3) the output of the SA system is reconstructed based on the reduced SL system output (Section 3.3).

3.1. Reformulation as a SL system with state reset

We next build a SL system with state reset that is equivalent to the original SA system, in that (ξ_a, q_a) and y_a can be recovered exactly from the state and output variables of the SL system.

Let $y \in Y = Y_a$, and $\xi \in \Xi = \Xi_a$ evolve according to linear dynamics that depend on the operating mode $q \in Q$ as follows:

$$\mathcal{S} : \begin{cases} \dot{\xi}(t) = \mathcal{A}_q\xi(t) + \mathcal{B}_q u(t) \\ y(t) = \mathcal{C}_q\xi(t). \end{cases} \tag{2}$$

Set $\bar{y}_{a,q} = \mathcal{C}_q\bar{\xi}_{a,q} + g_q$, where $\bar{\xi}_{a,q} = -\mathcal{A}_q^{-1} f_q$, with $\mathcal{A}_q$ invertible by Assumption 1. A transition from mode $i \in Q$ to mode $j \in Q$ occurs as soon as $(y + \bar{y}_{a,i}, u)$ exits Dom_i and enters Dom_j, where $Dom_q = Dom_{a,q}$, $q \in Q$.

[2] This requires that each polyhedron $Dom_{a,i}$ is of dimension $p + m$, and the intersection $Dom_{a,i} \cap Dom_{a,j}$, $i \neq j$, is either empty or a common proper face of both polyhedra.