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Brief paper

Extremum seeking control of a nonholonomic system with sensor constraints[☆]

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ABSTRACT

This paper presents an extremum seeking control (ESC) scheme for mobile robots with nonholonomic constraints. Many sensors, such cameras, have a limited field of view. If a target function is based on these sensors, correct robot orientation is critical for maximizing the target function. Our approach is novel in that it will maximize a target function that is a function of robot position and orientation, while overcoming the nonholonomic constraints that prevent simple motion along the gradient of all degrees of freedom. Stability analysis proves our ESC scheme is well behaved and the robot will settle in the neighborhood of a maximum of the target function. Simulation and experiment results elucidate performance for different tasks.

1. Introduction

Mobile robots can be used for many automatic searching tasks, such as seeking resources, search and rescue, surveillance, and exploration. Many of these tasks can be modeled as an optimization problem. The search area is the feasible range of the problem, and the mobile robot acts as an observer, measuring some physical variable related with the searching task. The measurement at every point in the feasible range constitutes a measurement-to-point mapping or a target function. The mobile robot then seeks the point where the measurement is at its maximum. For example, in the source seeking task, a mobile robot is equipped with a sensor to measure the strength of a scalar signal emitted by a source and seeks a path to this source (Cochran & Krstic, 2009; Liu & Krstic, 2010b; Matveev, Teimoori, & Savkin, 2011; Zhang, Arnold, Ghods, Siranosian, & Krstic, 2007).

In many cases there is little or no a priori information of the target function, nor is there absolute information of the robot's location. In these situations, extremum seeking control (ESC) algorithms are a strong option (Cochran & Krstic, 2009; Dürr, Stanković, Ebenbauer, & Johansson, 2013; Fu & Özgüner, 2011; Ghaffari, Krstić, & Nešić, 2012; Haring, Van De Wouw, & Nešić, 2013; Khong, Nešić, Tan, & Manzie, 2013; Krstić & Wang, 2000; Liu & Krstic, 2010b; Matveev et al., 2011; Moase & Manzie, 2012; Nešić, Nguyen, Tan, & Manzie, 2013; Porat & Nehorai, 1996; Rotea, 2000; Stanković & Stipanović, 2010; Tan, Nesic, & Mareels, 2006; Zhang et al., 2007). Many of ESC algorithms attempt to find the maximum value of the target function by adjusting the control inputs according to a gradient estimation algorithm implemented in real time. The gradient is estimated by using an external, periodic perturbation and a series of filtering and modulation operations. The estimated gradient is integrated to generate the control inputs, which constitute an estimate of the optimal set of variables. If the ESC loop is stable, the inputs to the integrators will vanish when the system reaches steady state. This results in a zero gradient, which is a necessary condition for unconstrained optimization. Thus, in steady state, the system is at a local maximum.

For nonholonomic mobile robots, the ESC problem is particularly difficult, as nonholonomic constraints prevent the system from actuating along all degrees of freedom. It is therefore impossible to move along an arbitrary gradient. Unique ESC algorithms were presented in Cochran and Krstic (2009), Ghods and Krstic (2010), Liu and Krstic (2010b), Matveev et al. (2011) and Zhang et al. (2007) to search for the maximum of a scalar measurement in a 2D plane. In these works, the functions being maximized were

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functions of position, but not orientation. That is, the maximum of the target function is invariant to robot orientation.

When the target function is dependent on the robot orientation, the problem becomes more challenging. A constant angular velocity, as used in Zhang et al. (2007), may no longer be applicable, as the robot cannot settle at an orientation that will maximize the signal. On the other hand, if a robot keeps a constant linear velocity as in Cochran and Krstic (2009), Liu and Krstic (2010b) and Matveev et al. (2011), the robot must maintain a curved trajectory in which the orientation sweeps through all values (e.g. circle or rose curve) to stay in a bounded neighborhood around the maximum point. Thus the robot cannot settle at any fixed orientation. In the case that the angular velocity is tuned by ESC and the linear velocity is regulated by a derivative control law as in Ghods and Krstic (2010), the robot is able to settle at a fixed orientation, but this settled orientation is somewhat arbitrary and has no relation with the target function. The problem becomes more difficult still for robots with sensors that have limited field of view (FOV), since the robot has to keep its target in the FOV (Gans & Hutchinson, 2007) or it will lose the measurement. Examples of such kind of tasks that depend on orientation and FOV include mobile robots equipped with forward facing camera, marine vehicles equipped with active sonar, or vehicles equipped with active radar or lidar.

In this paper, we propose an ESC scheme for nonholonomic mobile robots in SE(2), that is, optimal pose seeking including both the optimal position and optimal orientation. We are motivated by tasks in which a robot equipped with a camera seeks to maximize the value of the image information in the current view (Yinghua Zhang & Gans, 2011; Yinghua Zhang, Shen, & Gans, 2011; Zhang & Gans, 2013). A related problem would be searching for a target via template matching, in which a mobile robot equipped with a camera continuously computes the correlation function between the current image and a preloaded image and moves to a position where the correlation function achieves its maximum. Another motivating problem is a mobile robot seeking to maximize communication signal strength by properly aiming its antenna.

In the following sections, we first introduce the model of nonholonomic mobile robot, and the definition of averaged system. Then, the proposed SE(2) ESC algorithm is presented in detail, and its stability is proved under the assumption of quadratic target function. In the end, simulations and experiments are shown to demonstrate its performance.

2. Background

2.1. Nonholonomic mobile robots

We adopt the kinematic unicycle model for our nonholonomic mobile robot. The coordinates of the robot state are $[x, y, \theta]^T$, where x and y are the position coordinates of the robot in a 2D plane and θ is the orientation angle. The equations of state can be written in matrix form as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w \qquad (1)$$

where v and w are linear and angular velocity, respectively, and are inputs to the system.

2.2. Averaged system

If a system is given by $\dot{\mathbf{x}} = \varepsilon \mathbf{f}(\mathbf{x}, t, \varepsilon)$, where $\mathbf{x} \in \mathbb{R}^n$, ε is a small positive parameter, and $\mathbf{f}(\mathbf{x}, t, \varepsilon)$ is T-periodic in t, the averaged system is

$$\dot{\mathbf{x}}^a = \varepsilon \bar{\mathbf{f}}(\mathbf{x}^a)$$

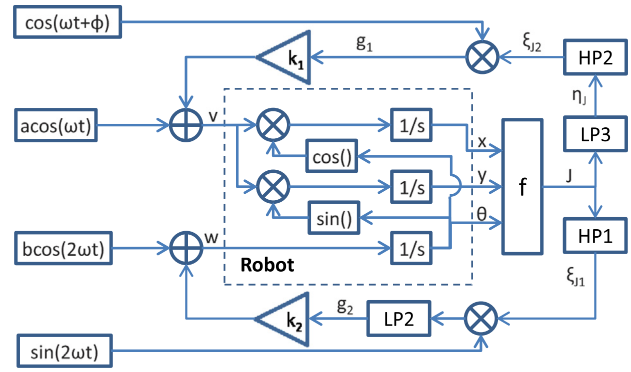

Fig. 1. Our proposed ESC scheme for mobile robots.

where $\bar{\mathbf{f}}(\mathbf{x}^a) = \frac{1}{T}\int_0^T \mathbf{f}(\mathbf{x}^a, t, 0)dt$. See Khalil (2002) for details of averaging theory and methods.

3. System model

The proposed ESC scheme for nonholonomic mobile robot is shown in Fig. 1. The target function we are going to explore is represented by the block f, and the function value $J = f(x, y, \theta)$. A group of linear filters are used in this scheme: the low pass filters LP2, LP3 and high pass filters HP1, HP2. We implemented them as 1st order filters. The transfer functions of LP2 and LP3 are respectively $\frac{\omega_2}{s+\omega_2}$ and $\frac{s(1-k_3)+\omega_1}{s+\omega_1}$, and the transfer function of HP1 and HP2 are respectively $\frac{s}{s+\omega_1}$ and $\frac{s}{s+\omega_2}$, where the ω_1 and ω_2 are cutoff frequencies for the filters, and the constant $k_3 \neq 1$. The signals η_J, ξ_{J2} and ξ_{J1} are outputs of LP3, HP2 and HP1 respectively. The inputs $a\cos(\omega t)$ and $b\cos(2\omega t)$ serve as dithers for v and w respectively, and $\cos(\omega t+\phi)$ and $\sin(2\omega t)$ serve as demodulating signals, where the phase ϕ is constant. The signals g_1 and g_2 are the signals of estimated gradients of the target function with respect to position and orientation. They are multiplied by constants k_1 and k_2 before being applied as input to the robot in (1).

In contrast with typical multi-dimensional ESC, an additional low-pass filter LP3 is necessary in the linear velocity feedback loop. The linear and angular velocity inputs have different dither frequencies. LP3 discriminates the components from each input in the output signal to facilitate the demodulation and estimation process. Considering the system bandwidth, the frequency difference cannot be too large. Thus, we suggest 2ω for the angular dither frequency.

To facilitate the proof of system stability, we transform the system block diagram to an equivalent form shown in Fig. 2, in which the target function f, dither signals, and constant gains are same as Fig. 1. The first difference is that we transform LP3 in Fig. 1 to the form of an input minus a high pass filter. Define ξ as the outputs of HP3

$$\dot{\xi} = -\omega_1 \xi + k_3 \dot{J}. \qquad (2)$$

The second difference is that we transform the two high pass filters in Fig. 1 to the form of input minus a low pass filter. Define η_1, η_2 as the outputs of LP1 and the LP2 in the lower branch respectively

$$\dot{\eta}_1 = -\omega_1 \eta_1 + \omega_1 J$$
$$\dot{\eta}_2 = -\omega_2 \eta_2 + \omega_2 (J - \xi). \qquad (3)$$

As illustrated in Fig. 2, the robot position dynamics are

$$\dot{x} = \cos(\theta)[a\cos(\omega t) + k_1(J - \xi - \eta_2)\cos(\omega t + \phi)]$$
$$\dot{y} = \sin(\theta)[a\cos(\omega t) + k_1(J - \xi - \eta_2)\cos(\omega t + \phi)] \qquad (4)$$