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Brief paper

Predictor-based networked control under uncertain transmission delays ☆

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ABSTRACT

We consider state-feedback predictor-based control of networked control systems with large time-varying communication delays. We show that even a small controller-to-actuators delay uncertainty may lead to a non-small residual error in a networked control system and reveal how to analyze such systems. Then we design an event-triggered predictor-based controller with sampled measurements and demonstrate that, depending on the delay uncertainty, one should choose various predictor models to reduce the error due to triggering. For the systems with a network only from a controller to actuators, we take advantage of the continuously available measurements by using a continuous-time predictor and employing a recently proposed switching approach to event-triggered control. By an example of an inverted pendulum on a cart we demonstrate that the proposed approach is extremely efficient when the uncertain time-varying network-induced delays are too large for the system to be stabilizable without a predictor.

1. Introduction

In networked control systems (NCSs), which are comprised of sensors, controllers, and actuators connected through a communication medium, transmitted signals are sampled in time and are subject to time-delays. Most existing papers on NCSs study robust stability with respect to small communication delays (see, e.g., Antsaklis & Baillieul, 2004; Fridman, Seuret, & Richard, 2004; Gao, Chen, & Lam, 2008; Liu & Fridman, 2012). To compensate large transport delays, predictor-based approach can be employed. So far this was done only for sampled-data control with *known constant delays* (Karafyllis & Krstic, 2012; Mazenc & Normand-Cyrot, 2013). In this paper we develop predictor-based sampled-data control for *unknown time-varying delays*.

There are several works that study robustness (w.r.t. delay uncertainty) of a predictor-based *continuous-time* controller (Bekiaris-Liberis & Krstic, 2013; Karafyllis & Krstic, 2013; Li, Zhou, & Lin, 2014; Yue & Han, 2005). In these works the residual error that appears due to delay uncertainty can be made arbitrary small by reducing the upper bound of the uncertainty. However, this is not true for *sampled-data systems*, where an arbitrary small delay uncertainty may lead to a non-vanishing error (because the terms that appear in the residual error may belong to different sampling intervals).

In this work we study an NCS with two networks: from sensors to a controller and from the controller to actuators. Both networks introduce large time-varying delays. We assume that the messages sent from the sensors are time stamped (Zhang, Branicky, & Phillips, 2001). This allows to calculate the sensors-to-controller delay. The controller-to-actuators delay is assumed to be unknown but belongs to a known delay interval. We use a state-feedback predictor, which is calculated on the controller side, to partially compensate both delays. By extending the time-delay modeling of NCSs (Fridman, 2014; Fridman et al., 2004; Gao et al., 2008), we present the system in a form suitable for analysis. Using a proper Lyapunov–Krasovskii functional, we derive LMI-based conditions for the stability analysis and design that guarantee the desired decay rate of convergence.

As the next step we introduce an event-triggering mechanism (Heemels, Johansson, & Tabuada, 2012; Tabuada, 2007) into predictor-based networked control. The event-triggering condition is checked on a controller side and allows to reduce the amount of control signals sent through a controller-to-actuators network. We demonstrate that it is reasonable to choose different predictor models for a zero and non-zero controller-to-actuators delay uncertainty. Finally, we consider predictor-based event-triggered control with continuous-time measurements and

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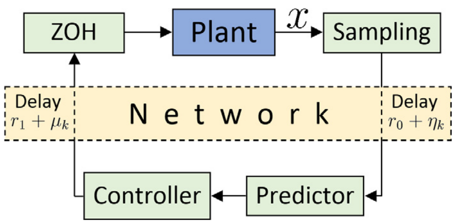

Fig. 1. NCS with a predictor.

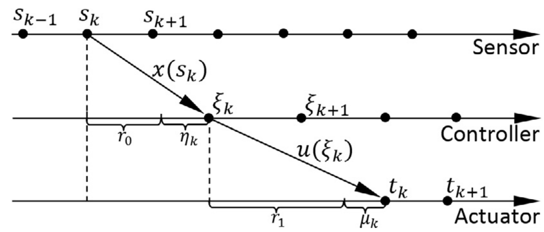

Fig. 2. Time-delays and updating times.

sampled control signals sent through a controller-to-actuators network. Such systems naturally appear when a visually observed vehicle is controlled through a wireless network. To take advantage of the continuously available measurements, we use a continuous-time predictor (Artstein, 1982; Kwon & Pearson, 1980; Mazenc & Normand-Cyrot, 2013) and a recently proposed switching approach to event-triggered control (Selivanov & Fridman, in press).

By an example of an inverted pendulum on a cart we demonstrate that the proposed approach is extremely efficient when the uncertain time-varying network-induced delays are too large for the system to be stabilizable without a predictor. Moreover, the considered event-triggering mechanism allows to significantly reduce the network workload.

2. Networked control employing predictor

Consider the linear system

$$\dot{x}(t) = Ax(t) + Bu(t), \quad t \geq 0 \tag{1}$$

with the state $x \in \mathbb{R}^n$, control input $u \in \mathbb{R}^m$, and constant matrices A, B of appropriate dimensions for which there exists $K \in \mathbb{R}^{m\times n}$ such that $A+BK$ is a Hurwitz matrix. Let $\{s_k\}$ be sampling instants such that

$$0 = s_0 < s_1 < s_2 < \cdots, \qquad \lim_{k\to\infty} s_k = \infty, \qquad s_{k+1} - s_k \leq h.$$

At each sampling time s_k the state $x(s_k)$ is transmitted to a controller, where a control signal is calculated and transmitted to actuators (see Fig. 1). We assume that the controller and the actuators are event-driven (update their outputs as soon as they receive new data). Both state and control signals are subject to network-induced delays $r_0+\eta_k$ and $r_1+\mu_k$, respectively. Thus, the controller updating times are $\xi_k = s_k + r_0 + \eta_k$ and the actuators updating times are $t_k = \xi_k + r_1 + \mu_k$, where $k \in \mathbb{Z}_+$, $\mathbb{Z}_+ = \{0, 1, 2, \ldots\}$ (see Fig. 2). Here r_0 and r_1 are known constant transport delays, η_k and μ_k are time-varying delays such that

$$0 \leq \eta_k \leq \eta_M, \quad 0 \leq \mu_k \leq \mu_M, \quad \xi_k \leq \xi_{k+1}, \quad t_k \leq t_{k+1}. \tag{2}$$

We assume that the sensors and controller clocks are synchronized and together with $x(s_k)$ the time stamp s_k is transmitted so that the value of $\eta_k = \xi_k - s_k - r_0$ can be calculated on the controller side at time ξ_k. Delay uncertainty μ_k is assumed to be unknown. Note that we do not require $\eta_k + \mu_k$ to be less than the sampling interval but the sequences $\{\xi_k\}$ and $\{t_k\}$ of updating times should be increasing.

Define $u(\xi) = 0$ for $\xi < \xi_0$. Then (1) transforms to

$$\begin{aligned} \dot{x}(t) &= Ax(t), & & t \in [0, t_0), \\ \dot{x}(t) &= Ax(t) + Bu(\xi_k), & & t \in [t_k, t_{k+1}),\ k \in \mathbb{Z}_+. \end{aligned} \tag{3}$$

To construct a predictor-based controller for (3), define

$$v(\xi) \triangleq \begin{cases} 0, & \xi < \xi_0, \\ u(\xi_k), & \xi \in [\xi_k, \xi_{k+1}),\ k \in \mathbb{Z}_+ \end{cases} \tag{4}$$

and consider the change of variable (Artstein, 1982; Kwon & Pearson, 1980)

$$z(t) \triangleq e^{A(r_0+r_1)}x(t) + \int_{t-r_1}^{t+r_0} e^{A(t+r_0-\theta)}Bv(\theta)\,d\theta, \tag{5}$$

where $t \geq 0$. We set $z(t) = 0$ for $t < 0$. If $\mu_M = 0$, i.e. controller-to-actuators delay is constant, (4), (5) is the state prediction, namely, $z(t) = x(t + r_0 + r_1)$. If $\mu_k \not\equiv 0$ to obtain the precise state prediction one needs to integrate (3), where $t_k = \xi_k + r_1 + \mu_k$ depends on μ_k. Since μ_k is unknown, we use the prediction (4), (5) that is imprecise for $\mu_k \not\equiv 0$. By substituting (3) for $\dot{x}(t)$ we obtain

$$\begin{aligned} &\dot{z}(t) = Az(t) + Bv(t+r_0) - e^{A(r_0+r_1)}Bv(t-r_1), \quad t \in [0, t_0), \\ &\dot{z}(t) = Az(t) + Bv(t+r_0) + e^{A(r_0+r_1)}B[u(\xi_k) - v(t-r_1)], \\ &\quad t \in [t_k, t_{k+1}),\ k \in \mathbb{Z}_+. \end{aligned} \tag{6}$$

Consider the following control law

$$\begin{aligned} u(\xi_k) \triangleq Kz(s_k) = K\Bigg[& e^{A(r_0+r_1)}x(s_k) \\ & + \int_{\xi_k-\eta_k-r_0-r_1}^{\xi_k-\eta_k} e^{A(\xi_k-\eta_k-\theta)}Bv(\theta)\,d\theta\Bigg], \quad k \in \mathbb{Z}_+. \end{aligned} \tag{7}$$

Since η_k is available to the controller at time ξ_k, the control signal (7) can be calculated. Moreover, no numerical difficulties arise while calculating the integral term in (7) with a piecewise constant $v(\theta)$ given by (4).

We analyze (4)–(7) using the time-delay approach to NCSs (Fridman, 2014; Fridman et al., 2004; Gao et al., 2008). According to (4), (7), $v(t + r_0) = Kz(s_k)$ whenever $t + r_0 \in [\xi_k, \xi_{k+1})$, that is, when $t \in [\xi_k - r_0, \xi_{k+1} - r_0)$. If $t < \xi_0 - r_0$ then $v(t + r_0) = 0 = Kz(t - \eta_0)$. Therefore,

$$v(t + r_0) = Kz(t - \tau(t)), \quad t \in \mathbb{R}, \tag{8}$$

where

$$\tau(t) = \begin{cases} \eta_0, & t < \xi_0 - r_0, \\ t - s_k, & t \in [\xi_k - r_0, \xi_{k+1} - r_0),\ k \in \mathbb{Z}_+. \end{cases}$$

Note that for $t \geq \xi_0 - r_0$

$$0 \leq \tau(t) \leq \max_k\{(s_{k+1} + r_0 + \eta_{k+1}) - r_0 - s_k\} \leq h + \eta_M.$$

By similar reasoning we obtain

$$\begin{aligned} \dot{z}(t) &= Az(t) + BKz(t-\tau(t)) \\ &\quad + e^{A(r_0+r_1)}BK[z(t-\tau_2(t)) - z(t-\tau_1(t))], \quad t \geq 0, \end{aligned} \tag{9}$$

with

$$z(0) = e^{A(r_0+r_1)}x(0), \qquad z(t) = 0 \quad \text{for } t < 0, \tag{10}$$

$$\begin{aligned} &\tau(t) \triangleq \begin{cases} \eta_0, & t < \xi_0 - r_0, \\ t - s_k, & t \in [\xi_k - r_0, \xi_{k+1} - r_0),\ k \in \mathbb{Z}_+, \end{cases} \\ &\tau_1(t) \triangleq \begin{cases} r_1 + r_0 + \eta_0, & t \in [0, t_0 - \mu_0), \\ t - s_k, & t \in [t_k - \mu_k, t_{k+1} - \mu_{k+1}),\ k \in \mathbb{Z}_+, \end{cases} \\ &\tau_2(t) \triangleq \begin{cases} r_0 + r_1 + \eta_0 + \mu_0, & t \in [0, t_0), \\ t - s_k, & t \in [t_k, t_{k+1}),\ k \in \mathbb{Z}_+, \end{cases} \\ &0 \leq \tau(t) \leq \bar{\tau} \triangleq h + \eta_M, \\ &r_0 + r_1 \leq \tau_1(t) \leq \tau_2(t) \leq \tau_M \triangleq r_0 + r_1 + h + \eta_M + \mu_M. \end{aligned} \tag{11}$$