Automatica 70 (2016) 140–147

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Extended sliding mode observer based control for Markovian jump linear systems with disturbances☆

Jinhui Zhang [a,1], Peng Shi [b,c], Weiguo Lin [d]

[a] School of Electrical Engineering and Automation, Tianjin University, Tianjin, 300072, China

[b] School of Electrical and Electronic Engineering, The University of Adelaide, SA 5005, Australia

[c] College of Engineering and Science, Victoria University, Melbourne, 8001 VIC, Australia

[d] College of Information Science & Technology, Beijing University of Chemical Technology, Beijing 100029, China

ARTICLE INFO

Article history:
Received 24 April 2015
Received in revised form
5 November 2015
Accepted 7 March 2016
Available online 16 April 2016

Keywords:
Extended sliding mode observer
Disturbance rejection
Markovian jump linear systems

ABSTRACT

This paper addresses the problem of disturbance rejection control for Markovian jump linear systems with matched and mismatched disturbances. Based on the state and disturbance estimates obtained by the proposed discontinuous or continuous extended sliding mode observers, the composite controllers are designed to actively reject the disturbance. Moreover, the problem of stochastic stability analysis for the estimation error systems and the closed-loop systems are also performed respectively. Finally, a numerical example is provided to illustrate the efficiency and advantage of the proposed methods.

1. Introduction

It is well known that many dynamical systems having variable structures subject to random abrupt changes can be modeled by hybrid systems with both time-evolving and event-driven mechanisms. The special class of hybrid systems are the so-called Markovian jump linear systems (MJLSs). The MJLSs are linear systems with randomly jumping parameters, where the jumps are governed by a Markov process chain. It is worth mentioning that the well-known networked control systems (NCSs) can be modeled as MJLSs if the packet dropouts and channel delays are modeled by Markov chains. Over the past decades, MJLSs have been widely investigated and research topics on MJLSs include stability analysis, stabilization and filtering problems, see for example Basin, Gonzalez, Acosta, and Fridman (2005), Basin, Ferreira, and Fridman (2007), Basin and Rodriguez-Ramirez (2011, 2014), de Farias, Geromel, do Val, and Costa (2000), Dong and Yang (2008), de Souza, Trofino, and Barbosa (2006), Huang, Long, and Li (2015), Kang, Zhang, and Ge (2008), Karana, Shi, and Kaya (2006), Shi, Boukas, and Agarwal (1999), Shi and Yu (2009), Zhang and Boukas (2009), Zhang, Boukas, and Lam (2008), and the references therein. For more details, please refer to the monograph (Boukas, 2008; Costa, Fragoso, & Marques, 2005).

In practical control systems, due to the friction and load variation, environment noises, unmodeled dynamics, or errors caused by sensors and actuators, various types of disturbances or uncertainties are unavoidable and can severely degrade the control performance. Therefore, disturbance attenuation and rejection becomes a crucial problem to achieve stability and pursue better control performances. During the past decades, several disturbance attenuation and rejection approaches have been established for MJLSs, such as H_∞ control (Boukas & Liu, 2001; Kang et al., 2008), sliding mode control (Basin & Rodriguez-Ramirez, 2014; Shi, Xia, Liu, & Rees, 2006; Wang & Fei, 2015; Wu & Shi, 2010; Wu & Ho, 2010) and disturbance observer based control (DOBC) (Yao & Guo, 2013, 2014). The H_∞ control technique possessing advantages over classical control techniques is an effective disturbance attenuation method and has already been successfully applied in practice. However, the robustness against disturbance achieved by the H_∞ control approach is guaranteed at the price of degraded nominal performance, and the disturbance is assumed to belong to $\mathcal{L}_2[0,\infty)$, i.e., the disturbance has finite energy. While the

☆ This work was partially supported by the National Natural Science Foundation of China (Grant Numbers 61473024, 61403018 and 61573112), Joint Key Grant from National Natural Science Foundation of China, and Zhejiang Province Government (Grant Number U1509217), and Australian Research Council (Grant Numbers DP140102180, LP140100471, LE150100079). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Michael V. Basin under the direction of Editor Ian R. Petersen.

E-mail addresses: zhangjinh@gmail.com (J. Zhang), peng.shi@adelaide.edu.au (P. Shi), linwg@mail.buct.edu.cn (W. Lin).

[1] Tel.: +86 22 27406130; fax: +86 22 27406272.

http://dx.doi.org/10.1016/j.automatica.2016.03.020
0005-1098/

sliding mode control technique has the robustness with respect to the so-called matched disturbances or plant uncertainties. However, when the system is switching stochastically among different subsystems, the trajectories of the system cannot stay on each sliding surface of subsystems forever, therefore, it is difficult to determine whether the closed-loop system is stochastically stable (Xia, Fu, Shi, Wu, & Zhang, 2009). Different from the aforementioned two control approaches, the DOBC can provide an active and effective way to handle disturbances and achieve a good disturbance-rejection performance without scarifying the nominal performance (Li, Yang, Chen, & Chen, 2014). More recently, the composite DOBC and H_∞ control technique has been proposed for MJLSs with nonlinearity and multiple disturbances in Yao and Guo (2013). However, the disturbance is assumed to be generated by an exogenous system, which may limit the practical applications of the proposed control approaches since it may be a challenging problem to identify the unknown disturbances.

In this paper, we focus on the disturbance rejection control problem for MJLSs with matched and mismatched disturbances. Motivated by the DOBC approaches (Yao & Guo, 2013, 2014), both the discontinuous and continuous extended sliding mode observers are developed to estimate the state and disturbance simultaneously, and the stochastic stability analysis problem of the estimation error systems are also performed respectively. With the state and disturbance estimates, the composite controllers are designed for systems with matched and mismatched disturbance, respectively. It can be shown that, by introducing the disturbance estimate in the controller, the disturbance can be actively rejected effectively. There are three main features of the proposed approaches being worth mentioning,

- Different from the disturbance attenuation approaches, such as H_∞ control and sliding mode control, the proposed control approaches are essentially the active disturbance rejection control approaches.
- Compared with the existing DOBC approaches (Yao & Guo, 2013, 2014), the disturbances considered in this paper are not generated from an exogenous system, but assumed to be unknown and bounded.
- The composite controllers are designed based on the state and disturbance estimates, which are more effective than the state-based controllers (Shi et al., 2006; Xia et al., 2009; Yao & Guo, 2013, 2014) because of the system states are not always available due to the limit of physical condition or expense to measure.

Finally, a numerical example is included to demonstrate the effectiveness of the theoretical results obtained.

2. Problem formulation

Fix a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. With the probability space $(\Omega, \mathcal{F}, \mathcal{P})$, we consider the following Markovian jump linear systems,

$$\begin{aligned}\dot{x}(t) &= A(r_t)x(t) + B(r_t)u(t) + B_w(r_t)w(t)\\ y(t) &= C(r_t)x(t) + D_w(r_t)w(t)\end{aligned}\tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state variable, $u(t) \in \mathbb{R}^m$ is the control input and $w(t) \in \mathbb{R}^p$ is the external disturbance. The jumping process $(r_t, t \geq 0)$ taking values in a finite set $\mathcal{S} \triangleq \{1, 2, \ldots, N\}$, governs the jumping among the different system modes, and $(r_t, t \geq 0)$ is a continuous-time, discrete-state homogeneous Markov process and has the following mode transition probabilities:

$$\mathbf{Pr}(r_{t+\delta} = j | r_t = i) = \begin{cases} \lambda_{ij}\delta + o(\delta) & \text{if } j \neq i \\ 1 + \lambda_{ij}h + o(\delta) & \text{if } j = i \end{cases}$$

where $\delta > 0$, $\lim_{\delta\to 0} \frac{o(\delta)}{\delta} = 0$, and $\lambda_{ij} \geq 0$ for $i \neq j$, $i, j \in \mathcal{S}$ denotes the transition probability from mode i at time t to mode j at time $t + \delta$, and $\lambda_{ii} = -\sum_{j=1, j\neq i}^{N} \lambda_{ij}$ for all $i \in \mathcal{S}$, and the Markov process transition rates matrix Λ is defined by:

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1N} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{N1} & \lambda_{N2} & \cdots & \lambda_{NN} \end{bmatrix}.$$

Assumption 1. The external disturbance satisfies the following that $\|w(t)\| \leq \varrho$ and $\|\dot{w}(t)\| \leq \rho$.

For any function $V(t, x, r_t)$, the weak infinitesimal operator $\mathfrak{F}_i^x[\cdot]$ of the process $\{x(t), r_t, t \geq 0\}$ at the point $\{t, x, i\}$

$$\mathfrak{F}_i^x[V] = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}\dot{x}(t) + \sum_{j=1}^{N} \lambda_{ij} V(x, j).$$

For each possible value $r_t = i$, $i \in \mathcal{S}$, the matrix $A(r_t)$ will be denoted by A_i for the sake of simplicity.

The following definitions about stability are presented.

Definition 1. The equilibrium $x(t) = 0$ of system (1) is said to be

- (weakly) stable in probability (for $t > 0$) if, for every $\varepsilon > 0$ and $\delta > 0$, there exists an $r > 0$ such that if $t > 0$ and $\|x_0\| < r$, then $\mathbf{Pr}\{\|x(t)\| > \varepsilon\} < \delta$.
- asymptotically stable in probability if it is stable in probability and, for each $\varepsilon > 0$, $x_0 \in \mathbb{R}^n$ and $i_0 \in \mathcal{S}$ there is $\lim_{t\to\infty} \mathbf{Pr}\{\|x(t)\| > \varepsilon\} = 0$.

Definition 2. A stochastic process $x(t)$ is said to be bounded in probability if the random variables $\|x(t)\|$ are bounded in probability uniformly in t, i.e., $\lim_{R\to\infty} \sup_{t>0} \mathbf{Pr}\{\|x(t)\| > R\} = 0$.

3. Design of extended sliding mode observers

Similar to Yao and Guo (2013), define

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}, \qquad \bar{A}(r_t) = \begin{bmatrix} A(r_t) & B_w(r_t) \\ 0 & 0 \end{bmatrix},$$

$$\bar{B}(r_t) = \begin{bmatrix} B(r_t) \\ 0 \end{bmatrix}, \qquad \bar{D} = \begin{bmatrix} 0 \\ I_p \end{bmatrix},$$

$$\bar{C}(r_t) = \begin{bmatrix} C(r_t) & D_w(r_t) \end{bmatrix},$$

thus, MJLS (1) can be rewritten into the following extended form,

$$\begin{aligned}\dot{\bar{x}}(t) &= \bar{A}(r_t)\bar{x}(t) + \bar{B}(r_t)u(t) + \bar{D}\dot{w}(t)\\ y(t) &= \bar{C}(r_t)\bar{x}(t).\end{aligned}\tag{2}$$

3.1. Discontinuous extended SMO

For MJLS (2), consider the following discontinuous mode-dependent extended SMO,

$$\dot{\hat{\bar{x}}}(t) = \bar{A}(r_t)\hat{\bar{x}}(t) + \bar{B}(r_t)u(t) + L(r_t)\tilde{y}(t) + \bar{D}F(r_t)v_1(t)\tag{3}$$

where $\tilde{y}(t) = y(t) - \bar{C}(r_t)\hat{\bar{x}}(t)$, and

$$v_1(t) = \begin{cases} \eta \dfrac{\tilde{y}(t)}{\|\tilde{y}(t)\|} & \text{if } \|\tilde{y}(t)\| \neq 0 \\ 0 & \text{if } \|\tilde{y}(t)\| = 0 \end{cases}$$