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Brief paper

Event-triggered leader-following tracking control for multivariable multi-agent systems☆

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ABSTRACT

The paper considers event-triggered leader–follower tracking control for multi-agent systems with general linear dynamics. For both undirected and directed follower graphs, we propose event triggering rules which guarantee bounded tracking errors. With these rules, we also prove that the systems do not exhibit Zeno behavior, and the bounds on the tracking errors can be tuned to a desired small value. We also show that the combinational state required for the proposed event triggering conditions can be continuously generated from discrete communications between the neighboring agents occurring at event times. The efficacy of the proposed methods is discussed using a simulation example.

1. Introduction

Cooperative control of multi-agent systems has received increasing attention in the past decade, see Ren and Beard (2008) and references therein. However, many control techniques developed so far rely on continuous communication between agents and their neighbors. To address this concern, several approaches have been proposed in recent years. One approach is to apply sampled control Xie, Liu, Wang, and Jia (2009). However in sampled data control schemes control action updates continue periodically with the same frequency even after the system has reached the control goal with sufficient accuracy and no longer requires intervention from the controller. Efforts to overcome this shortcoming have led to the idea of triggered control. Self-triggered control strategies (Dimarogonas, Frazzoli, & Johansson, 2012; Heemels, Johansson, & Tabuada, 2012; Mazo, Anta, & Tabuada, 2010) employ a triggering mechanism to proactively predict the next time for updating the control input ahead of time, using the current measurements. On the other hand, event-triggered controllers (Dimarogonas et al., 2012; Dimarogonas & Johansson, 2009; Heemels & Donkers, 2013; Lunze & Lehmann, 2010; Tabuada, 2007; Wang & Lemmon, 2011) trigger control input updates by reacting to excessive deviations of the decision variable from an acceptable value, i.e., when a continuously monitored triggering condition is violated. This latter approach is the main focus in this paper.

The development of event-triggered controllers remains challenging, because the agents in a multi-agent system do not have access to the complete system state information required to make decisions about control input updates. To prove the concept of event-triggering, the early work was still assuming continuous communication between the neighboring agents (Dimarogonas et al., 2012; Dimarogonas & Johansson, 2009). To circumvent this limitation, several approaches have been proposed, e.g., see Adaldo et al. (2014), Fan, Feng, Wang, and Song (2013) Seyboth, Dimarogonas, and Johansson (2013), Garcia, Cao, and Casbeer (2014), Liuzza, Dimarogonas, di Bernardo, and Johansson (2013), Meng and Chen (2013) and Zhu, Jiang, and Feng (2014).

All the papers mentioned above considered the event-triggered control problem for leaderless systems. The leader-following control is one of the important problems in cooperative control of multi-agent systems (Hong, Hu, & Gao, 2006; Jadbabaie, Lin, & Morse, 2003; Ren & Atkins, 2007; Ren & Beard, 2008), and the interest in event-based solutions to this problem is growing (Hu, Chen, & Li, 2011; Hu, Geng, & Zhu, 2015; Li, Liao, Huang, & Zhu, 2015; Zhang & Hong, 2012). General multidimensional leader following problems still remain technically challenging, and the development is often restricted to the study of single or double integrator dynamics (Hu et al., 2011, 2015; Li et al., 2015; Zhang & Hong, 2012). Zeno behavior presents another challenge, and is not

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always excluded (Hu et al., 2011; Zhang & Hong, 2012). Excluding Zeno behavior is an important requirement on control protocols since excessively frequent communications reduce the advantages of using the event-triggered control.

In this paper, we also consider the event-triggered leader-following control problem for multi-agent systems. Unlike (Hu et al., 2011, 2015; Li et al., 2015; Zhang & Hong, 2012), the class of systems considered allows for general linear dynamics. Also, the leader can be marginally stable or even unstable. For both undirected and directed system interconnections, we propose sufficient conditions for the design of controllers which guarantee that the leader tracking errors are contained within certain bounds; these bounds can be optimized by tuning the parameters of the design procedure. We also show that with the proposed event-triggered control protocols, the system does not exhibit Zeno behavior. These results are the main contribution of the paper.

Its another contribution is the event-triggered control protocols that do not require the neighboring agents to communicate continuously. Instead, the combinational state to be used in the event triggering condition is generated continuously within the controllers, by integrating the information obtained from the neighbors during their communication events. The idea is inspired by Fan et al. (2013), however, the procedure in Fan et al. (2013) developed for single integrator systems cannot be applied to multi-agent systems with general linear dynamics considered here, since in our case dynamics of the measurement error depend explicitly on the combinational state. Also different from Fan et al. (2013), the proposed algorithm involves one-way communications between the neighboring agents. The combinational state is computed continuously by each agent and is broadcast to its neighbors *only at the time when the communication event is triggered* at this node and *only in one direction*. The neighbors then use this information for their own computation, and do not send additional requests to measure the combinational state. This is an important advantage of our protocol compared with event-triggered control strategies proposed in Fan et al. (2013), Hu et al. (2011, 2015), Li et al. (2015) and Zhu and Jiang (2015). In these references, when an event is triggered at one agent, it must request its neighbors for additional information to update the control signals. Owing to this, our scheme is applicable to systems with a directed graph which only involves one way communications.

In comparison with the recent work on event-triggered control for general linear systems (Garcia et al., 2014; Liu, Cao, De Persis, & Hendrickx, 2013; Zhu & Jiang, 2015; Zhu et al., 2014), the main distinction of our method is computing the combinational state directly using the neighbors' information. This allowed us to avoid additional sampling when checking event triggering conditions, cf. Zhu and Jiang (2015); Zhu et al. (2014). In contrast in Garcia et al. (2014), to avoid continuous transmission of information, each agent was equipped with models of itself and its neighbors. In Liu et al. (2013), estimators were embedded into each node to enable the agents to estimate their neighbors' states. Both approaches make the controller rather complex, compared with our controller which does not require additional models or estimators. The leader–follower context and the treatment of both directed and undirected versions of the problem are other distinctions.

The paper is organized as follows. Section 2 includes the problem formulation and preliminaries. The main results are given in Sections 3 and 4. In Section 3 we consider the case when the system of followers is connected over a directed graph. Although these results are applicable to systems connected over an undirected graph as well, the symmetry of the graph Laplacian makes it possible to derive an alternative control design scheme in Section 4. In Section 5, the generation of the combinational state is discussed. Section 6 provides an illustrative example. The conclusions are given in Section 7.

Throughout the paper, $\Re^n$ and $\Re^{n\times m}$ are a real Euclidean n-dimensional vector space and a space of real $n \times m$ matrices. $\otimes$ denotes the Kronecker product of two matrices. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ will denote the largest and the smallest eigenvalues of a real symmetric matrix. For $q \in \Re^n$, $\mathrm{diag}\{q\}$ denotes the diagonal matrix with the entries of q as its diagonal elements. I_N is the $N \times N$ identity matrix. When the dimension is clear from the context, the subscript N will be suppressed.

2. Problem formulation and preliminaries

2.1. Communication graph

Consider a communication graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$, where $\bar{\mathcal{V}} = \{0, \dots, N\}$ is a finite nonempty node set, $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ is an edge set of pairs of nodes, and $\bar{\mathcal{A}}$ is an adjacency matrix. Without loss of generality, node 0 will be assigned to represent the leader, while the nodes from the set $\mathcal{V} = \{1, \dots, N\}$ will represent the followers.

The (in general, directed) subgraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ obtained from $\bar{\mathcal{G}}$ by removing the leader node and the corresponding edges describes communications between the followers; the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the communication links between them, with the ordered pair $(j, i) \in \mathcal{E}$ indicating that node i obtains information from node j; in this case j is the neighbor of i. The set of neighbors of node i in the graph $\mathcal{G}$ is denoted as $N_i = \{j|(j, i) \in \mathcal{E}\}$. Following the standard convention, we assume that $\mathcal{G}$ does not have self-loops or repeated edges. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \Re^{N\times N}$ of $\mathcal{G}$ is defined as $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. Let $d_i = \sum_{j=1}^{N} a_{ij}$ be the in-degree of node $i \in \mathcal{V}$ and $\mathcal{D} = \mathrm{diag}\{d_1, \dots, d_N\} \in \Re^{N\times N}$. Then $\mathcal{L} = \mathcal{D} - \mathcal{A}$ is the Laplacian matrix of the graph $\mathcal{G}$, it is symmetric when $\mathcal{G}$ is undirected.

We assume throughout the paper that the leader is observed by a subset of followers. If the leader is observed by follower i, then the directed edge $(0, i)$ is included in $\bar{\mathcal{E}}$ and is assigned with the weighting $g_i = 1$, otherwise we let $g_i = 0$. We refer to node i with $g_i \neq 0$ as a pinned node. Let $G = \mathrm{diag}\{g_1, \dots, g_N\} \in \Re^{N\times N}$. The system is assumed to have at least one follower which can observe the leader, hence $G \neq 0$.

In addition, we assume the graph $\mathcal{G}$ contains a spanning tree rooted at a pinned node i_r, i.e., $g_{i_r} > 0$. Then, $-(\mathcal{L}+G)$ is a Metzler matrix. According to Hu and Hong (2007), the matrix $-(\mathcal{L} + G)$ is Hurwitz stable,[1] which implies that $-(\mathcal{L} + G)$ is diagonally stable (Kaszkurewicz & Bhaya, 2000). That is, there exists a positive definite diagonal matrix $\Theta = \mathrm{diag}\{\vartheta_1, \dots, \vartheta_N\}$ such that $H = \Theta^{-1}(\mathcal{L} + G) + (\mathcal{L} + G)'\Theta^{-1} > 0$. We will also use the following notation: $\alpha = \frac{1}{2}\lambda_{\min}(H)$, $\vartheta_{\min} = \min_i(\vartheta_i)$, $\underline{\vartheta} = \min_i(\vartheta_i^{-1})$, $P = \Theta^{-1}(\mathcal{L} + G)(\mathcal{L} + G)'\Theta^{-1}$ and $F = (\mathcal{L} + G)'(\mathcal{L} + G)$.

2.2. Problem formulation

Consider a multi-agent system consisting of a leader agent and N follower agents. Dynamics of the ith follower are described by the equation

$$\dot{x}_i = Ax_i + Bu_i, \tag{1}$$

where $x_i \in \Re^n$ is the state, $u_i \in \Re^p$ is the control input. Also, the dynamics of the leader agent are given by

$$\dot{x}_0 = Ax_0. \tag{2}$$

[1] These properties of the matrix $\mathcal{L} + G$ can be guaranteed under weaker assumptions on the graph $\mathcal{G}$ (Hu & Hong, 2007).