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Brief paper

A direct design procedure for linear state functional observers ☆

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ABSTRACT

We propose a constructive procedure to design a Luenberger observer to estimate a linear multiple linear state functional for a linear time-invariant system. Among other features the proposed design algorithm is not based on the solution of a Sylvester equation nor on the use of canonical state space forms. The design is based on the solution set of a linear equation and a realization method. The consistency of this equation and the stability of the observer can be used as a functional observability test.

1. Introduction

Since Luenberger's works (Luenberger, 1963, 1964, 1966) a significant amount of research has been devoted to the problem of observing a linear functional of the state of a linear time-invariant system. The main developments are detailed in O'Reilly (1983), in Aldeen and Trinh (1999), Trinh and Fernando (2007) and Tsui (1985, 1998) and, in the recent books Korovin and Fomichev (2009) and Trinh and Fernando (2012) and the reference therein. The problem at first glance can be formulated as follows. For the linear state-space model

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t),\\ y(t) &= Cx(t),\end{aligned} \tag{1}$$

where, for every time t in $\mathbb{R}^+$, $x(t)$ is the n-dimensional state vector, $u(t)$ the p-dimensional measured input, $y(t)$ the m-dimensional measured output, and, A, B and C are constant matrices of adapted dimensions, the objective is to get,

$$v(t) = Lx(t), \tag{2}$$

where L is a constant $(l \times n)$ matrix. The observation of $v(t)$ can be carried out with the design of a Luenberger observer

$$\begin{aligned}\dot{z}(t) &= Fz(t) + Gu(t) + Hy(t),\\ w(t) &= Pz(t) + Vy(t),\end{aligned} \tag{3}$$

where $z(t)$ is a q-dimensional state vector. Constant matrices F, G, H, P and V are determined such that

$$\lim_{t\to\infty} (v(t) - w(t)) = 0.$$

We know from Fortmann and Williamson (1972) and Fuhrmann and Helmke (2001) that the observable linear functional observer (3) exists if and only if there exists a $(q \times n)$ matrix T such that:

$$G = TB,$$

$$TA - FT = HC, \tag{4}$$

$$L = PT + VC, \tag{5}$$

where F is an Hurwitz matrix, namely, when all the real parts of the eigenvalues of F are strictly negative. When these conditions are fulfilled we have $\lim_{t\to\infty}(z(t) - Tx(t)) = 0$. It is also well known that when rank $\left(\begin{bmatrix} L^\top & C^\top \end{bmatrix}\right) = m + l$ the order q of the multiple state functional observer is such that $q \geq l$ (Roman & Bullock, 1975; Sirisena, 1979). Darouach in Darouach (2000) has then proposed existence conditions for a Luenberger observer of the functional (2) with a minimum order l. Moreover, when the model (1) is detectable we have $q \leq n - m$. Indeed, $n - m$ is the order of the reduced-order observer or Cumming–Gopinath observer (Cumming, 1969; Gopinath, 1971) which can be built to observe $x(t)$ and, consequently, $v(t)$.

Until now the direct design of a minimal observer of a given linear functional is an open question. Since Fortmann and Williamson (1972), a lot of design schemes have been proposed to reduce the order of the observer (3). One way is to determine the matrices

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T and F such that the Sylvester equation (4) is fulfilled (Trinh, Nahavandi, & Tran, 2008; Tsui, 2004). Unfortunately, F and T are unknown in (4) and some conditions are added to get a solution (e.g. fixed eigenvalues for the observer, canonical state space forms). Another way, implied by the functional observability notion (Fernando, Jennings, & Trinh, 2010a,b; Fernando, Trinh, & Jennings, 2010; Fernando & Trinh, 2013; Jennings, Fernando, & Trinh, 2011), consists in expanding the matrix L with a matrix R such that there exists a s-order Luenberger observer of the linear state functional $Sx(t)$ where

$$S = \begin{bmatrix} R \\ L \end{bmatrix},$$

and $s = \mathrm{rank}(S)$. It is underlined in Trinh and Fernando (2012, p. 66) that "how to find a matrix R with the smallest number of rows is an intriguing and challenging problem". Nevertheless, based on singular value decompositions (Fernando et al., 2010b), on eigenspace projections (Jennings et al., 2011), on canonical forms (Korovin, Medvedev, & Fomichev, 2010), or on computations for row-space extensions (Fernando & Trinh, 2013), some procedures have been proposed to tackle the observer design where the matrix F can have arbitrary eigenvalues. Indeed, the design of an observer must be thought in two different frameworks: on the one hand, the fixed-pole observer problem where the poles are fixed at the outset and, on the other hand, the stable observer problem where the poles are permitted to lie anywhere in the left half-plane. The main contributions on the design of functional observers tackle the first problem.

In the opposite, in order to seek for minimality of the observer order we focus here on the stable observer problem and develop a constructive procedure to design a Luenberger observer of the functional (2) for the system (1). Our algorithm is based on the solution set of a linear equation and linear algebraic operations in a state space setting. It can be seen as an extension of the algorithm proposed in Rotella and Zambettakis (2011) for single linear functional observers. With respect to other procedures the design procedure does not require the solution of the Sylvester equation. Moreover, the proposed solution exhibits free design parameters in the candidate observer to achieve asymptotic stability. Let us mention that we do not suppose any canonical form for the system neither for the observer. The main objectives of the paper are thus to provide a simple test for functional observability and a constructive procedure to design a stable Luenberger observer of a linear functional for a linear system. The paper is organized as follows. In the first section the procedure to design the observer structure is detailed. From the consistency condition of a linear equation are deduced a minimal index and the state space equation of the observer. The second section is devoted to analyze the stabilizability of the matrix F. The proposed procedure is exemplified in a third section. Finally, a method is proposed in the fourth section to reduce the order of the stable observer and to get it minimal. It has been underlined in Tsui (1998), that the calculus of the matrix T is not a necessary step. This point is a specific feature of the procedure we propose. Indeed, we exhibit the closed-form of this matrix as a collateral result.

2. A constructive procedure

Let us suppose that there exists an integer ν such that

$$\mathrm{rank}\left(\begin{bmatrix} LA^{\nu} \\ \Sigma_{\nu} \end{bmatrix}\right) = \mathrm{rank}\ (\Sigma_{\nu}), \qquad (6)$$

where the matrix Σ_{ν} is defined as $\Sigma_{\nu} = C$ when $\nu = 0$ and,

$$\Sigma_{\nu} = \begin{bmatrix} CA^{\nu} \\ LA^{\nu-1} \\ CA^{\nu-1} \\ \vdots \\ LA \\ CA \\ L \\ C \end{bmatrix}, \qquad (7)$$

when $\nu > 0$. In other words, the linear equation

$$LA^{\nu} = \Phi \Sigma_{\nu}, \qquad (8)$$

is consistent, namely, there exist matrices $F_{L,i}$, $i = 0$ to $\nu - 1$, and $F_{C,i}$, $i = 0$ to ν, such that

$$LA^{\nu} = \sum_{i=0}^{\nu-1} F_{L,i} LA^{i} + \sum_{i=0}^{\nu} F_{C,i} CA^{i}. \qquad (9)$$

Remark 1. Due to Cayley–Hamilton theorem the hypothesis (6) can always be fulfilled.

Let us notice as well that we have, for $k = 0, 1, \ldots$

$$v^{(k)}(t) = LA^{k}x(t) + \sum_{i=0}^{k-1} LA^{k-1-i}Bu^{(i)}(t),$$

so, from (9) we can write

$$v^{(\nu)}(t) = \sum_{i=0}^{\nu-1} F_{L,i}LA^{i}x(t) + \sum_{i=0}^{\nu} F_{C,i}CA^{i}x(t) + \sum_{i=0}^{\nu-1} LA^{\nu-1-i}Bu^{(i)}(t). \qquad (10)$$

2.1. Structural design of the observer

This section is devoted to the determination of matrices F, G, H, P and, V in (3) from the existence of the relationship (9). Firstly, to eliminate $x(t)$ in (10) we use

- for $i = 1$ to $\nu - 1$, $v^{(i)}(t) = \sum_{j=0}^{i-1} LA^{j}Bu^{(i-1-j)}(t) + LA^{i}x(t)$, thus

$$LA^{i}x(t) = v^{(i)}(t) - \sum_{j=0}^{i-1} LA^{i-1-j}Bu^{(j)}(t); \qquad (11)$$

- for $i = 1$ to ν, $y^{(i)}(t) = \sum_{j=0}^{i-1} CA^{j}Bu^{(i-1-j)}(t) + CA^{i}x(t)$, thus

$$CA^{i}x(t) = y^{(i)}(t) - \sum_{j=0}^{i-1} CA^{i-1-j}Bu^{(j)}(t). \qquad (12)$$

Taking into account (11) and (12) in (10) we get then

$$v^{(\nu)}(t) = \sum_{i=0}^{\nu-1} F_{L,i}v^{(i)}(t) + \sum_{i=0}^{\nu} F_{C,i}y^{(i)}(t) + \sum_{i=0}^{\nu-1} G_{i}u^{(i)}(t), \qquad (13)$$

where $G_{\nu-1} = \left(L - F_{C,\nu}C\right)B$ and, for $\nu \geq 2$ and $j = 0$ to $\nu - 2$,

$$G_{j} = \left(LA^{\nu-1-j} - \sum_{i=j+1}^{\nu-1} F_{L,i}LA^{i-1-j} - \sum_{i=j+1}^{\nu} F_{C,i}CA^{i-1-j}\right)B. \qquad (14)$$