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Brief paper

A unified time-varying feedback approach and its applications in adaptive stabilization of high-order uncertain nonlinear systems[☆]

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ABSTRACT

We are concerned with two key problems of adaptive control design: one is to find the feasible conditions and the valid boundary of the time-varying feedback scheme; another one lies in the improved generalization of the homogeneous idea in the global adaptive stabilization of high-order uncertain nonlinear systems. The distinguished feature of the system to be investigated is the serious coexistence between unknown time-varying parameters and unknown time-varying control coefficients. Design procedures of the continuous controller are provided based on the sign function technique and the delicate search of the time-varying function and a Lyapunov function. Finally, the control of uncertain single-link robotic manipulator demonstrates the application of design scheme.

1. Introduction

Uncertainty has a potential tendency to deteriorate system performance or even destabilize control systems, so its research is a long-standing active subject in the field of nonlinear control (Karafyllis & Jiang, 2011; Khalil, 2002; Krstić, Kanellakopoulos, & Kokotović, 1995). In many applications, the structure of the system model may be known, but its parameters may be unknown and change with time because of changes in operating conditions, aging of the equipment, and so on. Examples include an aircraft wing rock and a machine with friction. It should be emphasized that parameter uncertainty is usually compensated by an on-line estimate. Particularly, in Lin and Qian (2002), an improved adaptive scheme was proposed to deal with nonlinear parameterization, which promoted the advancement of the compensation technique.

In the past decades, with the aid of adding a power integrator method, control design has been intensively investigated for high-order nonlinear systems, see e.g., Lin and Qian (2001, 2006), Liu (2013), Sun and Liu (2015), Sun, Liu, and Zhang (2015), Sun, Song, Li, and Yang (2015), Sun, Xue, and Zhang (2015) and Yan and Liu (2010) and references therein. It should be noticed that the study generalized the form of the strict feedback nonlinear systems by developing the approaches of feedback linearization (Khalil, 2002) and backstepping (Krstić et al., 1995). With the increasing uncertainties of high-order nonlinear systems to be investigated, adding a power integrator method failed to meet the requirement of the control objective. Fortunately, in combination with the homogeneous domination idea, it accelerated the solution to the problems of adaptive stabilization and tracking control (Li, Jing, & Zhang, 2011; Lin & Pongvuthithum, 2003; Liu, 2014; Lv, Sun, & Xie, 2015; Polendo & Qian, 2007; Zhang, Fidan, & Ioannou, 2003; Zhang, Liu, & Mu, 2015). Moreover, it also enhanced the attractiveness in dealing with unmeasurable states, for example, the constant growth rate and the polynomial growth rate were considered in Lei and Lin (2009) and Li, Qian, and Frye (2009), respectively. So far even the state feedback control design of high-order uncertain nonlinear systems has been heavily blocked by the unknown growth rate, not to mention the output one. To address this issue, with the fact that the on-line estimates of unknown parameters can be viewed as time-varying functions (after all time is implicit) in mind, a time-varying scheme is implanted to the homogeneous domination idea and adding a power integrator method. In other words, a constant gain or a dynamic gain is introduced to make up the shortage of traditional adaptive technique and create a flexible freedom in control design, see Li and Liu (2015), Lv et al. (2015) and Qian and Du (2012) and the references therein. In view of above discussions, it is not hard to see that the choice of desired gains is solvable only in some special cases, dependent on the structure

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of the systems to be investigated. As a result, an interesting question is proposed spontaneously. *Is there a unified method to guarantee the feasibility of time-varying scheme for general control problems?* The problem will be transformed into an inequality with constraint conditions in this paper. To some extent, its satisfactory solution will encourage the advancement of the methodology for homogeneous domination idea.

In retrospect of the existing results of high-order uncertain nonlinear systems (Gao & Yuan, 2015; Lei & Lin, 2009; Li et al., 2011; Li & Liu, 2015; Li et al., 2009; Liu, 2014; Lv et al., 2015; Polendo & Qian, 2007; Qian & Du, 2012; Sun, Zhang, & Xie, 2014; Wang & Lin, 2015; Yan & Liu, 2011; Zhang et al., 2015; Zhu, Wen, Su & Liu, 2014), nonlinear functions are divided into three categories according to the power constraints: linear, high-order and low-order terms. For instance, (Lei & Lin, 2009; Li et al., 2011; Polendo & Qian, 2007; Zhang et al., 2015) included high-order and linear terms, and low-order and linear terms could be seen in Liu (2014) and Lv et al. (2015). Despite the hypothesis in Lei and Lin (2009), Li et al. (2009) and Sun et al. (2015) allowed nonlinear functions to encompass three situations, there were still strong limitations that the growth rates of the nonlinear functions must be precisely known. In consequence, another focal point of this paper is to *develop a new adaptive feedback design approach for high-order uncertain nonlinear systems with unknown time-varying parameters*, under the assumptions that the growth rates are unknown and there exist three types of nonlinear terms. This task is interesting in its own right, and also provides a permissible controller which is capable of handling a large family of nonlinearities. As a matter of fact, following the feedback technique in Liu (2013, 2014), we propose a time-varying design scheme to deal with the serious unknowns. Then, we successfully construct a continuous state-feedback controller on basis of adaptive compensation technique, sign function technique and the delicate choice of a Lyapunov function, and it ensures the global boundedness of the closed-loop system states and the asymptotic convergence of the original system states. Last but not least, an affirmative answer will be given to the practical application of the proposed scheme through the effective control of an uncertain single-link robotic manipulator system.

Now, we are ready to highlight the contributions or difficulties of this paper from four aspects. (i) The valid boundary of a time-varying scheme is strictly proved at the theoretical level, which is shown in Theorem 1. The current work gives an insight into better understanding of adaptive control problems for uncertain nonlinear systems. (ii) The homogeneous idea is generalized to investigate high-order nonlinear systems with unknown control coefficients for the first time, and the inherent difficulty is to find an appropriate function of time, which can capture the information of all the possible unknown time-varying uncertainties. (iii) In contrast to those in Gao and Yuan (2015), Lei and Lin (2009), Li et al. (2011), Li and Liu (2015), Li et al. (2009), Liu (2014), Lv et al. (2015), Yan and Liu (2011), Polendo and Qian (2007), Qian and Du (2012), Wang and Lin (2015), Sun et al. (2014), Zhang et al. (2015) and Zhu et al. (2014), the system in this paper has two essential features: the upper bounds of nonlinear functions with unknown growth rates consist of high-order, low-order and linear terms (see Assumption 2), and no prior knowledge on the upper bounds of the time-varying control coefficients need to be required. Hence, there is no denying that some new mathematical tools have to be found to suit current circumstances, such as the introduction of sign function, the establishment of Lemma 6 and the maneuverable transformation skill. (iv) This paper introduces a continuous unified strategy for global adaptive stabilization of a significant class of high-order uncertain nonlinear systems.

2. Preliminaries

The following notations will be used throughout this paper. $\mathbb{R}^{\geq 1}_{odd}$ denotes a real number set, whose element is the form of $\frac{q_1}{q_2}$, where q_1 and q_2 are positive odd integers and $q_1 \geq q_2$. For a real vector $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$, $\bar{x}_i \triangleq [x_1, \ldots, x_i]^T \in \mathbb{R}^i$, $i = 1, \ldots, n$, and we let $\bar{x}_n = x$; the norm of $x \in \mathbb{R}^n$ is defined by $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$; the arguments of functions are sometimes simplified, for instance, a function $f(x(t))$ is denoted by $f(x)$, $f(\cdot)$ or f. For any $y \in \mathbb{R}$, a sign function $\mathrm{sign}(y)$ satisfies: $\mathrm{sign}(y) = 1$ if $y > 0$, $\mathrm{sign}(y) = 0$ if $y = 0$, and $\mathrm{sign}(y) = -1$ if $y < 0$. For a given positive constant a, $\lceil y \rceil^a \triangleq |y|^a \mathrm{sign}(y)$, $\forall y \in \mathbb{R}$. A continuous function $h : [0, b) \to [0, \infty)$ belongs to class $\mathcal{K}$, if it is strictly increasing and $h(0) = 0$. It belongs to class $\mathcal{K}_\infty$, if $b = \infty$ and $h(y) \to \infty$ as $y \to \infty$.

Then, we list six lemmas which play an important role in constructing the global stabilizer, and the proofs of Lemmas 1–5 can be found in Sun et al. (2014).

Lemma 1. *For given $m > 0$, $n > 0$ and $a(x, y)$, and every $x \in \mathbb{R}$, $y \in \mathbb{R}$, there holds*

$$|a(x,y)x^m y^n| \leq c(x,y)|x|^{m+n} + \frac{n}{m+n}\left(\frac{m}{(m+n)c(x,y)}\right)^{\frac{m}{n}} \cdot |a(x,y)|^{\frac{m+n}{n}} |y|^{m+n},$$

where $c(x, y) > 0$.

Lemma 2. *For given $r \geq 0$ and every $x \in \mathbb{R}$, $y \in \mathbb{R}$, there holds $|x + y|^r \leq c_r(|x|^r + |y|^r)$, where $c_r = 2^{r-1}$ if $r \geq 1$, and $c_r = 1$ if $r < 1$.*

Lemma 3. *The function $f(x) = \lceil x \rceil^a$ $(a \geq 1)$ is continuously differentiable on $(-\infty, +\infty)$, and its derivative satisfies $\dot{f}(x) = a|x|^{a-1}$.*

Lemma 4. *Let $f : [a, b] \to \mathbb{R}$ $(a < b)$ be a continuous function that is monotone and satisfies $f(a) = 0$, then $\left|\int_a^b f(x)dx\right| \leq |f(b)| \cdot |b - a|$.*

Lemma 5. *Suppose $\frac{a}{b} \in \mathbb{R}^{\geq 1}_{odd}$, $b \geq 1$, then*

$$\left|x^{\frac{a}{b}} - y^{\frac{a}{b}}\right| \leq 2^{1-\frac{1}{b}} \left|\lceil x \rceil^a - \lceil y \rceil^a\right|^{\frac{1}{b}}, \quad \forall x \in \mathbb{R},\ \forall y \in \mathbb{R}.$$

Lemma 6. *For given $0 \leq r \leq 1$, $\omega \leq 1$, and every $x \in \mathbb{R}$, $y \in \mathbb{R}$, there hold*

$$\left|\lceil x - y \rceil^r + \lceil y \rceil^r\right| \leq 2^{1-r}|x|^r, \tag{1}$$

$$\lceil y \rceil^{2-r-\omega} \lceil x - y \rceil^r \leq -\frac{1}{2}|y|^{2-\omega} + c|x|^{2-\omega}, \tag{2}$$

where $c = \frac{r}{2-\omega}\left(\frac{2(2-w-r)}{2-\omega}\right)^{\frac{2-\omega-r}{r}} 2^{\frac{(1-r)(2-\omega)}{r}}$.

Proof. According to the definition of $\lceil \cdot \rceil$, one can deduce

$$\begin{aligned}
&\lceil \lceil x - y \rceil^r \rceil^{\frac{1}{r}} - \lceil \lceil -y \rceil^r \rceil^{\frac{1}{r}} \\
&= \lceil |x - y|^r \mathrm{sign}(x - y) \rceil^{\frac{1}{r}} - \lceil |-y|^r \mathrm{sign}(-y) \rceil^{\frac{1}{r}} \\
&= \left||x - y|^r \mathrm{sign}(x - y)\right|^{\frac{1}{r}} \mathrm{sign}(|x - y|^r \mathrm{sign}(x - y)) \\
&\quad - \left||-y|^r \mathrm{sign}(-y)\right|^{\frac{1}{r}} \mathrm{sign}(|-y|^r \mathrm{sign}(-y)) \\
&= \lceil x - y \rceil - \lceil -y \rceil = x.
\end{aligned}$$

With this fact and Lemma 5 in mind, one has

$$\begin{aligned}
&\left|\lceil x - y \rceil^r + \lceil y \rceil^r\right| = \left|\lceil x - y \rceil^r - \lceil -y \rceil^r\right| \\
&\leq 2^{1-r}\left|\lceil \lceil x - y \rceil^r \rceil^{\frac{1}{r}} - \lceil \lceil -y \rceil^r \rceil^{\frac{1}{r}}\right|^r = 2^{1-r}|x|^r.
\end{aligned}$$