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A method for sub-sample computation of time displacements between discrete signals based only on discrete correlation sequences

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ABSTRACT

In this paper, we propose a new method for sub-sample computation of time displacements between two sampled signals. The new algorithm is based on sampled auto- and cross-correlation sequences and takes into account only the sampled signals without the need for the customary interpolation and fitting procedures. The proposed method was evaluated and compared with other methods, in simulated and real signals. Four other methods were used for comparison: two based on cross-correlation plus fitting, one method based on spline fitting over the input signals, and another based on phase demodulation.

With simulated signals, the proposed approach presented similar or better performance, concerning bias and variance, in almost all the tested conditions. The exception was signals with very low SNRs (<10 dB), for which the methods based on phase demodulation and spline fitting presented lower variances. Considering only the two methods based on cross-correlation, our approach presented improved results with signals with high and moderate noise levels. The proposed approach and other three out of the four methods used for comparison are robust in real data. The exception is the phase demodulation method, which may fail when applied to signals collected from real-world scenarios because it is very sensitive to phase changes caused by other oscillations not related to the main echoes.

This paper introduced a new class of methods, demonstrating that it is possible to estimate sub-sample delay, based on discrete cross-correlations sequences without the need for interpolation or fitting over the original sampled signals. The proposed approach was robust when applied to real-world signals and presented a moderated computational complexity when compared to the other tested algorithms. Although the new method was tested using ultrasound signals, it can be applied to any time-series with observable events.

1. Introduction

The precise estimation of the temporal displacement between two discrete signals beyond the sample period, i.e., sub-sample time-delay estimation (SS-TDE), is crucial in signal processing. One field where SS-TDE is needed is biomedical signal processing for medical ultrasound for non-invasive temperature estimation [10,12,13], elastography [8,18], and blood velocity estimation [3]. When estimating blood velocity temporal displacements are assumed to be proportional to the local blood velocity obtained from pulsed Doppler signals [3]. Elastograms are obtained by computing temporal shifts between echoes created by the compression and decompression of the medium under analysis. The rational is that different tissues present different elastic properties giving rise to different temporal shifts [8]. One of the effects of media temperature variation is the changing of the speed of sound. Changes in the speed of sound are caused by temporal displacements between echoes measured at different temperatures and originating in the same interface. Thus, in past decades, methodologies based on temporal echo-shifts have been proposed for non-invasive temperature estimation [10,12,13].

A straightforward means of estimating displacements above the sampling period is by interpolation of the discrete signals. SS-TDE precision is improved; however, besides the increasing computational cost, it remains restricted by the new sampling period. An alternative to interpolation that aims to reduce computational cost and discard the restriction of the sampling period, SS-TDE can be obtained by computing a so-called pattern-matching function

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(PMF). The PMF, commonly a cross-correlation function, is a function computed to compare signals. The usual procedure is to consider some samples of the PMF to find the parameters of an analytic function. This function is then subjected to analytical operations (e.g., derivative to find the maximum) to achieve the SS-TDE. A widely applied method is to use the cross-correlation function as a PMF and then consider a parabola or cosine fitting around its peak. Finally, SS-TDE is obtained by computing the time-lag where the maximum of the parabola or cosine occurs. The maximum is determined by determining the zero of the derivative of the analytic fitted function [3–5]. In the previously described method the PMF is sampled at the original sampling frequency and usually suffers from relatively high bias and variance [16]. A different approach is to perform a fitting over the original signals, instead of over the PMF. A method proposed by Viola and Walker [16] implemented a cubic spline representation of just one signal, and then computed an analytical PMF that described the sum of the squared errors between both signals. The sub-sample displacement was obtained by computing the minimum of the PMF. The authors of the spline-fitting claimed that their approach surpassed other algorithms in terms of jitter and variance over a wide range of conditions, and an improvement in performance was obtained at a reasonable computational cost. In other research, time displacements were interpreted as equivalent to linear phase additions, and were obtained by phase demodulation without the need for any interpolation or fitting [14]. The central frequency of a transducer was considered as the modulation carrier, and only the media effect changed phase.

We describe in this paper a new method for sub-sample time displacement estimation between two discrete signals. Our method is based only on the discrete auto-correlation and cross-correlation sequences as in [14], without the need for any interpolation or fitting. We compared the performance of our method with the performance of four algorithms: one proposed in [16] and based on the spline fitting of one of the signals; two algorithms based on the computation of the discrete cross-correlation function followed by a parabola or cosine fitting [3,4]; and one based on phase demodulation as described in [14]. As performance indicators, we considered both bias (accuracy) and standard deviation of jitter errors (precision).

2. Methods

Two morphologically similar discrete time signals $s_1[n]$ and $s_2[n]$ with length N, sampling frequency of $fs = 1/Ts$, and delayed by a given real-valued amount of time (δ), were considered. The first stage of the method was the computation of the discrete-value time displacement as presented in Fig. 1. This was achieved by computing the normalized cross-correlation sequence ($\hat{R}_{s_1,s_2}[m]$):

$$\hat{R}_{s_1,s_2}[m] = \begin{cases} \dfrac{1}{N}\displaystyle\sum_{k=0}^{N-m-1} s_1[k+m]s_2[k] & \text{if } m \geq 0 \\ \hat{R}^*_{s_2,s_1}[-m] & \text{if } m < 0 \end{cases}. \quad (1)$$

The integer sample delay estimate (ID) is given by:

$$ID = \underset{m}{\operatorname{argmax}}\, \hat{R}_{s_1,s_2}[m]. \quad (2)$$

Using ID, it is possible to align (synchronize) the signals. This was the first step of the second phase of the method, which aimed to assess the remaining part to estimate δ that referred to the sub-sample time displacement. The synchronization consisted of the circular shifting of one of the signals by ID samples so that they became apparently in phase. The computation of the sub-sample displacement encompassed: (1) the evaluation of the normalized autocorrelation sequence of one of the signals, known as the reference signal, for example, $s_1[k]$, (2) the computation of the normalized cross-correlation sequence between the synchronized signals, and (3) the difference between these two computed sequences. Mathematically:

$$\hat{R}_{s_1,s_1}[m] = \begin{cases} \dfrac{1}{N}\displaystyle\sum_{k=0}^{N-m-1} s_1[k+m]s_1[k] & \text{if } m \geq 0 \\ \hat{R}^*_{s_1^s,s_1^s}[-m] & \text{if } m < 0 \end{cases} \quad (3)$$

$$\hat{R}_{s_1,s_2^s}[m] = \begin{cases} \dfrac{1}{N}\displaystyle\sum_{k=0}^{N-m-1} s_1[k+m]s_2^s[k] & \text{if } m \geq 0 \\ \hat{R}^*_{s_2^s,s_1^s}[-m] & \text{if } m < 0 \end{cases} \quad (4)$$

$$\hat{Rd}[m] = \hat{R}_{s_1,s_1}[m] - \hat{R}_{s_1,s_2^s}[m], \quad (5)$$

where s_2^s is the synchronized version of s_2 by implementing a circular shift of ID samples, and $\hat{Rd}[m]$ is the difference between the reference signal autocorrelation ($\hat{R}_{s_1,s_1}[m]$) and the synchronized signals cross-correlation ($\hat{R}_{s_1,s_2^s}[m]$). We verified that the maximum or the minimum values of $\hat{Rd}[m]$ were proportional to the sub-sample displacement. Fig. 2 shows the maximum and minimum variation of $\hat{Rd}[m]$ as a function of the sub-sample delay. This figure shows a clear linear relationship. The development of an analytical expression for $\hat{Rd}[m]$ ($rd(\tau)$) in continuous-time signals is provided in Annex A, as well as the computation of the relationship between its maximum and minimum with the sub-sample delay.

The total delay between the discrete signals can be found by evaluating Eq. (6), i.e. by summing the integer delay multiplied by the sampling period, with the maximum or the minimum of $\hat{Rd}[m]$ multiplied by a scaling factor. In this paper, we always considered the maximum:

$$\hat{\delta} = ID * Ts + \max(\hat{Rd}[m]) * factor. \quad (6)$$

The scaling factor was computed *a-priori* by introducing an artificial delay on the reference signal. The rational is that as the signals were artificially delayed, we knew exactly the delay that corresponded to the maximum of $\hat{Rd}[m]$. In fact, the computation of the scaling factor was a calibration action performed with a reference signal before any delay computation. The computation steps were: (1) delay of the reference signal by one sample, (2) computation of the auto-correlation sequence of the original reference signal and computation of the cross-correlation sequence between the original and delayed reference signal, (3) determination of the maximum of the difference between the auto- and cross-correlation sequences, and (4) computation of the factor by dividing the sample period by the maximum of $\hat{Rd}[m]$, as described in Eq. (7):

$$factor = \frac{T_s}{\max(\hat{R}_{s_1,s_1}[m] - \hat{R}_{s_1,s'_1}[m])}, \quad (7)$$

where Ts is the sampling period, s_1 is the reference signal and s'_1 is the reference signal delayed by one sample. For multiple delay estimations using the same reference signal the factor and the auto-correlation sequence ($\hat{R}_{s_1,s_1}[m]$) needed to be computed only the first time.