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Brief paper

Stability analysis of some classes of input-affine nonlinear systems with aperiodic sampled-data control[☆]

Hassan Omran[a], Laurentiu Hetel[b], Mihaly Petreczky[b], Jean-Pierre Richard[b,c], Françoise Lamnabhi-Lagarrigue[d]

[a] Ecole des Mines de Douai, Department of Computer Science and Automatic Control, F-59500 Douai, France

[b] Centre de Recherche en Informatique, Signal et Automatique de Lille (CRIStAL) UMR CNRS 9189, Ecole Centrale de Lille, 59651 Villeneuve d'Ascq, France

[c] INRIA, Project NON-A, Lille, France

[d] L2S (CNRS UMR 8506), European Embedded Control Institute (EECI), SUPELEC, 3 rue Joliot Curie, 91192 Gif-sur-Yvette, France

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ABSTRACT

In this paper we investigate the stability analysis of nonlinear sampled-data systems, which are affine in the input. We assume that a stabilizing controller is designed using the emulation technique. We intend to provide sufficient stability conditions for the resulting sampled-data system. This allows to find an estimate of the upper bound on the asynchronous sampling intervals, for which stability is ensured. The main idea of the paper is to address the stability problem in a new framework inspired by the dissipativity theory. Furthermore, the result is shown to be constructive. Numerically tractable criteria are derived using linear matrix inequality for polytopic systems and using sum of squares technique for the class of polynomial systems.

1. Introduction

Recent years have witnessed an increasing number of works on stability analysis of nonlinear sampled-data systems. This challenging problem is of great interest since in applications practical controllers are often implemented digitally.

When implementing a controller digitally, the *emulation* approach is often considered (Nešić, Teel, & Carnevale, 2009). In this approach, a continuous-time controller is designed, next it is implemented using a sample-and-hold device. However, the digital implementation must preserve the stability of the continuous-time system. Intuitively, the sampling interval must be sufficiently small to ensure the stability (Burlion, Ahmed-Ali, & Lamnabhi-Lagarrigue, 2006; Hsu & Sastry, 1987). Still, in practice it is difficult to maintain a constant sampling period during real-time control and the variations of the sampling interval may have a destabilizing effect (Fiter, Omran, Hetel, & Richard, 2014; Fridman, 2014), even for small sampling intervals. A quantitative estimation of the so-called *Maximum Sampling Interval* MSI that ensures stability (under time-varying sampling intervals) is very important from the practical point of view. Several works in the literature target this problem (see for example Karafyllis & Kravaris, 2009, Karafyllis & Krstic, 2012, Mazenc, Malisoff, & Dinh, 2013, and Nešić et al., 2009).

The case of linear sampled-data systems has been extensively studied. For the input delay approach, see Fridman (2010), Fridman, Seuret, and Richard (2004), Mazenc and Normand-Cyrot (2012) and Seuret (2012) where stability conditions are derived based on Lyapunov–Krasovskii functionals (Seuret & Gouaisbaut, 2013). The works in Fujioka (2009) and Mirkin (2007) use tools from robust control theory. A polytopic approximation of the discrete-time model is used in Fiter, Hetel, Perruquetti, and Richard (2012) and Hetel, Kruszewski, Perruquetti, and Richard (2011) to handle the sampling effect based on Lyapunov–Razumikhin functions. In the hybrid systems modeling approach (Goebel, Sanfelice, & Teel, 2009), the sampled-data system is represented as an impulsive system (Forni, Galeani, Nešić, & Zaccarian, 2013) and

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E-mail addresses: hassan.samir.omran@gmail.com (H. Omran), laurentiu.hetel@ec-lille.fr (L. Hetel), mihaly.petreczky@ec-lille.fr (M. Petreczky), jean-pierre.richard@ec-lille.fr (J.-P. Richard), francoise.lamnabhi-lagarrigue@lss.supelec.fr (F. Lamnabhi-Lagarrigue).

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stability is investigated using Lyapunov functions with discontinuities at the impulse times (Briat & Seuret, 2012; Naghshtabrizi, Hespanha, & Teel, 2008). The previous works provide constructive methods to estimate the MSI, such as Linear Matrix Inequalities (LMIs) based criteria.

The problem is more challenging in the nonlinear case (Laila, Nešić, & Astolfi, 2006; Monaco & Normand-Cyrot, 2007). We cite as follows some recent works. In Nešić et al. (2009), the authors specialized the results on generic Networked Control Systems (NCSs) for the particular case of sampled-data systems; stability conditions are presented based on the hybrid systems theory. In Bauer, Maas, and Heemels (2012), asymptotic stability of NCSs is studied using the same hybrid systems formulation; the Lyapunov functions are constructed with a sum of squares (SOS) techniques. The input delay approach is explored in Karafyllis and Kravaris (2009) for the nonlinear case, where Razumikhin functions together with theory of vector Lyapunov functions have been used. The work in Mazenc et al. (2013) also considers the input delay approach and it investigates the robustness of nonlinear systems, with respect to both sampling and delay. The approach is inspired by the Lyapunov–Krasovskii functional method.

Here, we investigate a new research direction for nonlinear affine systems. The considered approach is inspired by the notion of exponential dissipativity (Haddad & Sadikhov, 2013). This notion was initiated by Willems (1972). Since its introduction, it has been attracting an increasing attention. Dissipativity can be used to study stability, passivity, robustness and it is useful in a large variety of analysis and design problems. It was motivated by passivity properties of electrical circuits and it can be seen as a generalized notion of abstract energy for dynamical systems. Recently, local asymptotic stability of bilinear sampled-data systems controlled by a linear state feedback has been considered in Omran, Hetel, Richard, and Lamnabhi-Lagarrigue (2014) using the analysis of contractive invariant sets and dissipativity theory. The obtained results are promising, but the extension for generic nonlinear systems is not trivial.

The purpose of this work is to extend our previous result in Omran et al. (2014), concerning the analysis of bilinear sampled-data systems, to the case of input-affine nonlinear sampled-data systems. Dissipativity based conditions are used to estimate the MSI. The robustness with respect to variations of the sampling intervals is considered. The results are shown to be applicable for local and global analyses. Additionally, in order to show the effectiveness of the results, we study the particular cases of polytopic systems and polynomial systems. We apply the result to a benchmark example from the literature to show the usefulness of the proposed stability conditions.

The remainder of the paper is organized as follows: the problem under study is introduced in Section 2; in Section 3 the system is represented by an equivalent model which is useful for our analysis; the main result is given in Section 4; case studies are presented in Section 5, where the main result is applied to the cases of polytopic and polynomial systems; finally, illustrative examples are presented in Section 6.

Notation: $\mathbb{R}$ is the set of real numbers and $\mathbb{R}^+$ is the set of positive real numbers. $\mathbb{R}^n$ is the n-dimensional Euclidean space. The space of functions $f : [a, b) \to \mathbb{R}^n$ which are quadratically integrable over the interval $[a, b)$ is $L_2^n[a, b)$. The set of real matrices of dimension $n \times m$ is denoted by $\mathbb{R}^{n\times m}$. The transpose of a matrix M is denoted by M^T. For $P \in \mathbb{R}^{n\times n}$, $P > 0$ (resp. $P \geq 0$) means that it is a positive definite (resp. positive semi-definite) matrix. The identity matrix is I, the zero matrix is 0, both with appropriate dimensions. For a given $M^T = M \geq 0$, the weighted inner product is denoted by $\langle x, y\rangle_M = x^T My$, and the corresponding norm by $\|x\|_M = \sqrt{\langle x, x\rangle_M}$. The Euclidean norm is denoted by $|x|$. The convex hull is denoted by $conv\{\cdot\}$. The notation $p(\chi) \in \mathbb{R}[\chi]$ with $\chi \in \mathbb{R}^n$, denotes that $p(\chi)$ belongs to the set of polynomials in the variables $\{\chi_1, \chi_2, \ldots, \chi_n\}$ with coefficients in $\mathbb{R}$. For $x_1, x_2 \in \mathbb{R}^n$, (x_1, x_2) denotes $[x_1^T, x_2^T]^T$. A function $\beta : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{K}$ if it is continuous, zero at zero and strictly increasing. It is said to be of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$, and it is unbounded. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{KL}$ if $\beta(\cdot, t)$ is of class $\mathcal{K}$ for each $t \geq 0$, and $\beta(s, \cdot)$ is non-increasing and satisfies $\lim_{t\to\infty} \beta(s, t) = 0$ for each $s \geq 0$. Recall that a function $f : \mathbb{R} \to \mathbb{R}^n$ is said to be piecewise-continuous on an interval $J \subset \mathbb{R}$ if for every bounded subinterval $J_0 \subset J$, f is continuous for all $t \in J_0$ except, possibly, at a finite number of points where f may have discontinuities. It is right-continuous at t if $f(t) = \lim_{\theta\uparrow t} f(\theta) \triangleq \lim_{\theta\to t,\theta>t} f(\theta)$.

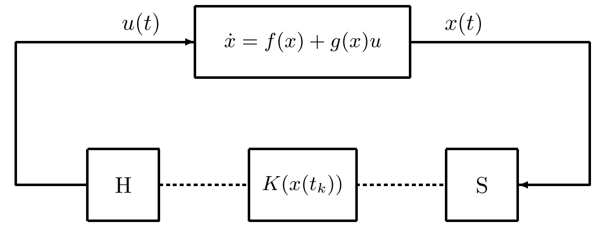

Fig. 1. Sampled-data feedback control of an affine nonlinear system.

2. Problem formulation

Consider the affine nonlinear control system given by

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

where $f(\cdot)$, $g(\cdot)$ are sufficiently smooth functions on a neighborhood of the origin x = 0 denoted by $\mathcal{D}$, $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the state and the input, respectively. Suppose that there exists a sufficiently smooth function $K(\cdot)$ which defines the continuous-time stabilizer $u = K(x)$. The interconnection between the previous controller and the continuous-time system (1) yields

$$\dot{x} = f_n(x) := f(x) + g(x)K(x). \tag{2}$$

H.1 We suppose that system (2) has a well-defined solution $x(t) \in \mathcal{D}$ on the interval $[t_0, +\infty)$ for any initial condition $x(t_0) = x_0 \in \mathcal{D}$.

Note that $\mathcal{D}$ is an invariant set for the closed-loop system (2). We consider the sampled-data implementation of the controller under the following assumptions:

H.2 The control is piecewise constant, calculated based on the sampled-data version of the state

$$u(t) = K\big(x(t_k)\big), \quad \forall t \in [t_k, t_{k+1}), \ \forall k \in \mathbb{N}. \tag{3}$$

H.3 The set of sampling instants $\{t_k\}_{k\in\mathbb{N}}$ satisfies

$$0 < t_{k+1} - t_k \leq \bar{h}, \quad \forall k \in \mathbb{N},$$

for a given MSI $\bar{h}$, and $\lim_{k\to\infty} t_k = +\infty$.

H.4 For any initial condition $x(t_0) = x_0 \in \mathcal{D}$, the system

$$\dot{x}(t) = f(x(t)) + g(x(t))K(x_0), \tag{4}$$

admits a unique solution $x(t)$ originating from x_0 which is defined on the interval $[t_0, t_0 + \bar{h})$ and $x(t) \in \mathcal{D}$.

We obtain the closed-loop sampled-data system (see also Fig. 1):

$$\dot{x}(t) = f\big(x(t)\big) + g\big(x(t)\big)K\big(x(t_k)\big),$$
$$\forall t \in [t_k, t_{k+1}), \ \forall k \in \mathbb{N}. \tag{5}$$

If Assumptions H.1–H.4 hold, then the system solution $x(t)$ is constructed in an iterative manner by integrating (5) over the