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Delay-dependent stability analysis by using delay-independent integral evaluation[☆]

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ABSTRACT

For the stability analysis of time-delay systems, the available methods usually require the exact evaluation of some quantities. The definite integral stability method, originated from the Argument Principle or the Cauchy Theorem, is effective because it only requires a rough estimation of the testing integral over a finite interval to judge stability. However, no general rule is given in the literature for properly choosing the upper limit of the testing integral. In this paper, two simple algorithms are presented for finding the parameter-dependent critical upper limit and a parameter-independent upper limit without any restriction on the number of time delays. These results improve and complete the definite integral stability method. As illustrated by the numerical examples, the proposed algorithms work effectively and accurately.

1. Introduction

Time-delay systems are described by delay differential equations (DDEs) (Erneux, 2009; Michiels & Niculescu, 2007; Niculescu, 2001; Stepan, 1989). The present study is restricted to autonomous Retarded DDEs (RDDEs) and Neutral DDEs (NDDEs) only. Time delays come usually from controllers, filters, actuators, or the contact problems of pure mechanical systems. In control applications, we mention here the car following problem modeled by RDDEs, where the delay is the human driver's response time or the signal communication and processing time in autonomous vehicles (Brackstone & McDonald, 1999; Campbell, Egerstedt, How, & Murray, 2010; Orosz, Wilson, & Stepan, 2010); NDDE is used in modeling and reducing the sway of container cranes with a delayed controller (Masoud & Nayfeh, 2003; Nayfeh, Masoud, & Nayfeh, 2011; Zhang, Wang, & Hu, 2012). As an example for a pure mechanical system with delay originated in the contact of tool and workpiece, RDDE is used to model the machine tool vibration of the cutting process (Long & Balachandran, 2007; Stepan, 2001).

The possible negative effect of the delay on stability and performance has been a key issue in engineering applications. Lots of methods, criteria, or algorithms are available for the stability analysis of DDEs (Erneux, 2009; Kuang, 1993; Michiels & Niculescu, 2007; Niculescu, 2001; Olgac & Sipahi, 2002; Stepan, 1989). Lyapunov functionals and Lyapunov–Krasovskii functionals have been used for stability analysis of DDEs applicable also for global stability analysis (Gu, 2010; Gu & Liu, 2009; Niculescu, 2001). By rewriting the discrete-delay DDEs into distributed-delay DDEs, Lyapunov–Krasovskii functionals provide sufficient stability conditions of DDEs (Gu & Niculescu, 2000; Ivanescu, Niculescu, Dugard, Dion, & Verriest, 2003; Niculescu, 1999).

In order to obtain more dedicated results, methods based on the analysis of characteristic functions are preferred. Among the first ones is the Pontryagin method that goes back to 1942 (Pontryagin, 1942). The method of stability switch is effective in finding the stable intervals for a given parameter (Kuang, 1993). A different version of the stability switch method is based on introducing Rekasius substitution, where the critical conditions can be studied with polynomials (Olgac, 2004; Olgac & Sipahi, 2002; Sipahi &

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Olgac, 2006). Although the Nyquist criterion works effectively in some applications (Fu, Olbrot, & Polis, 1989, 1991), the Nyquist plot may not be good in judging stability because it may intersect itself in an intricate way, especially in case of oscillatory systems with multiple delays (Abdallah, Dorato, Benites-Read, & Byrne, 1993).

The Argument Principle based stability criteria proposed and developed in Hassard (1997), Kolmanovskii and Myshkis (1999), Stepan (1989) and Xu and Wang (2014) enable one to calculate analytically the number of characteristic roots in the right-half complex plane (unstable roots, for short) for RDDEs and most of NDDEs. To do this, Hassard (1997) and Stepan (1989) suggest the calculation of all the finite number of roots with positive real parts of a transcendental real function associated with the complex characteristic function, instead of calculating the corresponding improper integral. The definite integral method works effectively for RDDEs (Kolmanovskii & Myshkis, 1999) by changing the corresponding improper integral into a proper one, and it is proved to be applicable also for most of NDDEs (Xu & Wang, 2014). A special and useful feature of the definite integral method is that it only requires a rough estimation of the testing integral. In this way, it is of less complexity and its computational cost is low with a properly chosen upper limit of the integral. However, no general rules are given for estimating this upper limit.

This paper aims at proposing two general algorithms for choosing such an upper limit. With these, the number of unstable roots is calculated easily, so that the stability can be judged. The rest of the paper is organized as follows. In Section 2, a brief introduction of the stability criteria in definite integral form is given for DDEs. Then in Section 3, the algorithm for finding the critical upper limit of the definite integral is presented and illustrated with an example. In Section 4, the proposed algorithm is generalized to parameter-independent stability tests, and it is used to plot stability charts of NDDEs in an example with multiple delays. Finally in Section 5, some concluding remarks are drawn.

2. Problem statement

Let us consider a linear time-delay system described by

$$\dot{x}(t) + \sum_{i=1}^{m} N_i \dot{x}(t-\tau_i) = Ax(t) + \sum_{i=1}^{m} B_i x(t-\tau_i) \tag{1}$$

where $x \in \mathbb{R}^n$, $A, B_i, N_i \in \mathbb{R}^{n\times n}$. The characteristic function of Eq. (1) is given in the form

$$f(\lambda) = \lambda^n + \sum_{i=0}^{n} \alpha_i(\mathrm{e}^{-\lambda\tau_1}, \dots, \mathrm{e}^{-\lambda\tau_m})\lambda^{n-i} \tag{2}$$

where $\alpha_i(z_1, \dots, z_m)$, $i = 0, 1, \dots, n$ are real polynomials with respect to $z_1 = \mathrm{e}^{-\lambda\tau_1}, \dots, z_m = \mathrm{e}^{-\lambda\tau_m}$. Eq. (1) is a RDDE when $N_k = 0$ for all $k = 1, 2, \dots, m$, that is, $\alpha_0(z_1, \dots, z_m) \equiv 0$, while it is a NDDE when at least one $N_k \neq 0$ for some $k = 1, 2, \dots, m$, that is, $\alpha_0(z_1, \dots, z_m) \neq 0$. A trivial solution of a RDDE is asymptotically stable in Lyapunov sense if and only if all the characteristic roots stay in the open left half of the complex plane (Kuang, 1993). For a NDDE, further condition is needed that these roots are uniformly bounded away from the imaginary axis, because they may have accumulation points on the imaginary axis (Kuang, 1993).

Assume that $f(\lambda)$ has no roots on the imaginary axis. Let $\mathcal{N}$ be the number of all the unstable characteristic roots of Eq. (2) located in the right half complex plane, and let Δ_C denote the argument change over the contour C shown in Fig. 1, which consists of $\{C_1 : \lambda = R\mathrm{e}^{\theta\,\mathrm{i}} \mid \theta \in (-\pi/2, \pi/2)\}$ and $\{C_2 : \lambda = \omega\,\mathrm{i} \mid \omega \in (-R, R)\}$. Then the Argument Principle or Cauchy Theorem gives

$$\mathcal{N} = \lim_{R\to+\infty} \frac{\Delta_C \arg(f(\lambda))}{2\pi} = \lim_{R\to+\infty} \frac{1}{2\pi\mathrm{i}} \oint_C \frac{f'(\lambda)}{f(\lambda)}\,d\lambda. \tag{3}$$

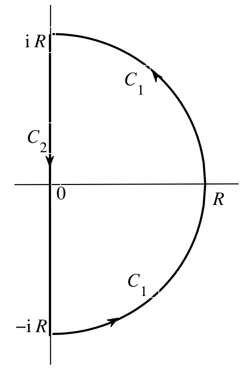

Fig. 1. The contour C of the line integral in Eq. (3).

Let $\Re(z)$ denote the real part of complex number z, and define a testing integral $F(T_1, T_2)$ as

$$F(T_1, T_2) \overset{\text{def}}{=} \int_{T_1}^{T_2} \Re\left(\frac{f'(\omega\,\mathrm{i})}{f(\omega\,\mathrm{i})}\right) \mathrm{d}\omega. \tag{4}$$

For a RDDE, Eq. (3) leads to Kolmanovskii and Myshkis (1999)

$$\mathcal{N} = \frac{n}{2} - \frac{1}{\pi} \lim_{T\to+\infty} F(0, T). \tag{5}$$

Hence simplified from Eq. (5) using a definite integral, $\mathcal{N} = 0$ if and only if there is a sufficient large $T > 0$ such that $F(0, T) > (n-1)\pi/2$, see Kolmanovskii and Myshkis (1999).

For a NDDE, the limit in (5) does not converge. However, under the following assumption (Hale & Lunel, 2002)

$$\sup_{\Re(\lambda)>0,\ |\lambda|\to\infty} \left|\alpha_0(\mathrm{e}^{-\lambda\tau_1}, \dots, \mathrm{e}^{-\lambda\tau_m})\right| < 1, \tag{6}$$

a lemma is derived in Xu and Wang (2014) from Eq. (3):

Lemma 1. *Assume that $f(\lambda)$ has no roots on the imaginary axis, and condition (6) holds, then there exists a sufficiently large $T_0 > 0$, such that for all $T \geq T_0$*

$$\mathcal{N} \in \left(-\frac{F(0,T)}{\pi} + \frac{n-1}{2}, -\frac{F(0,T)}{\pi} + \frac{n+1}{2}\right). \tag{7}$$

Because the length of the interval given in (7) is less than 1, the exact number $\mathcal{N}$ can be located in the interval (7) using an arbitrary T that is slightly larger than T_0. With the use of standard mathematical softwares, $\mathcal{N}$ can be calculated simply by using the round off command

$$\mathcal{N} = \text{round}\left(\frac{n}{2} - \frac{F(0,T)}{\pi}\right). \tag{8}$$

When $\mathcal{N} = 0$, system (1) is asymptotically stable. Thus, under assumption (6), the key issue for the stability analysis is to develop some effective algorithms for finding an upper limit T in the testing integral $F(0, T)$.

3. Parameter-dependent critical upper limit

This section is devoted to finding a critical upper limit T_0 of the testing integral, with which Eq. (8) gives the correct integer $\mathcal{N}$ for any $T > T_0$. The key idea is to introduce two real functions by using the real and imaginary parts of $\mathrm{i}^{-n} f(\omega\,\mathrm{i})$, given by

$$R(\omega) + S(\omega)\,\mathrm{i} = \mathrm{i}^{-n} f(\omega\,\mathrm{i}). \tag{9}$$