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Reset strategy for consensus in networks of clusters☆

Marcos Cesar Bragagnolo, Irinel-Constantin Morărescu, Jamal Daafouz, Pierre Riedinger

Université de Lorraine, CRAN, UMR 7039, 2 Avenue de la Forêt de Haye, Vandœuvre-lès-Nancy, France
CNRS, CRAN, UMR 7039, 2 Avenue de la Forêt de Haye, Vandœuvre-lès-Nancy, France

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ABSTRACT

This paper addresses the problem of consensus in networks structured in several clusters. The clusters are represented by fixed, directed and strongly connected graphs. They are composed by a number of agents which are able to interact only with other agents belonging to the same cluster. To every agent we associate a scalar real value representing its state. The states continuously evolve following a linear consensus protocol and approach local agreements specific to each cluster. In order to enforce a global agreement over the whole network, we consider that each cluster contains an agent that can be exogenously controlled. The state of this agent, called leader, will be quasi-periodically reseted by a local master controller that receives information from some neighboring leaders. In order to control the consensus value we have to firstly characterize it. Precisely we show that it depends only on the initial condition and the interaction topologies. Secondly, we provide sufficient Linear Matrix Inequality (LMI) conditions for the global uniform exponential stability of the consensus in presence of a quasi-periodic reset rule. The study of the network behavior is completed by a decay rate analysis. Finally we design the interaction network of the leaders which allows to reach a prescribed consensus value. Numerical implementation strategy is provided before illustrating the results by some simulations.

1. Introduction

Networks appear naturally in diverse areas of science and engineering as biology (Pavlopoulos et al., 2011; Ratmann, Wiuf, & Pinney, 2009; Reynolds, 2001), physics (Gfeller & Rios, 2008) and sociology (Hegselmann & Krause, 2002; Lorenz, 2005) as well as robotics (Bullo, Cortés, & Martinez, 2009) and communication (Pastor-Satorras & Vespignani, 2004). Studies concerning real networks revealed that the topology of interactions in communication, social or biological systems presents a cluster/community structure (Boccaletti, Latora, Moreno, Chavez, & Hwang, 2006; Hanski, 1998; Pastor-Satorras & Vespignani, 2004). In order to detect these communities, different algorithms are available in the literature (Lambiotte, Delvenne, & Barahona, 2009; Morărescu & Girard, 2011; Newman & Girvan, 2004). A consequence of the presence of decoupled clusters in the network is that consensus/synchronization cannot be reached and different local agreements are obtained (Morărescu & Girard, 2011; Touri & Nedic, 2012). To overcome this problem, we propose a quasi-periodic discrete controller meant to force the consensus in this type of clustered networks.

As in many works in the literature, in this paper we call agents the constitutive elements of the network and their number will define the network dimension. The consensus of the agents attracted a lot of interest in the last decade and it was studied in different frameworks: directed or undirected interactions, fixed or time-varying interaction graph, delayed or un-delayed, synchronized or desynchronized interactions, linear or nonlinear, continuous or discrete agent dynamics (Jadbabaie, Lin, & Morse, 2003; Morărescu, Niculescu, & Girard, 2012; Moreau, 2005; Olfati-Saber & Murray, 2004; Ren & Beard, 2005). The agreement speed in various frameworks has also been quantified (see for instance Olshevsky & Tsitsiklis, 2009; Xiao & Boyd, 2004). In order to guarantee the global coordination in networks with dynamic topologies, some works proposed controller designs that are able to maintain the network connectivity (Fiacchini & Morărescu, 2014; Zavlanos & Pappas, 2008).

A major concern in the last decade has been the control over networks with communication constraints (Anta & Tabuada, 2010;

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E-mail addresses: marcos-cesar.bragagnolo@univ-lorraine.fr (M.C. Bragagnolo), constantin.morarescu@univ-lorraine.fr (I.-C. Morărescu), jamal.daafouz@univ-lorraine.fr (J. Daafouz), pierre.riedinger@univ-lorraine.fr (P. Riedinger).

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Heemels, Johansson, & Tabuada, 2012; Postoyan, Tabuada, Nesic, & Anta, 2011). The focus was on the control of one system over a communication network by limiting the interactions between the controller and the plant. In this paper, only the exogenous control actions are constrained. We assume the network topology is known and the network is partitioned in several clusters. The agents continuously evolve by interacting with some other agents belonging to the same cluster. The global behavior of one cluster can be exogenously controlled by acting on the state of only one agent called leader. Due to communication constraint we assume that the exogenous actions take place at specific isolated instants that, for practical reasons, will be defined in the next section as a nearly/quasi periodic sequence. This strategy can be applied to control fleets of robots that are spatially clustered (Mahacek, Kitts, & Mas, 2011). The robots belonging to one cluster continuously interact but due to energy and communication constraint long distance interactions occurs discretely.

As mentioned in Bragagnolo, Morărescu, Daafouz, and Riedinger (2014), this model can be interpreted in terms of opinion dynamics. Each agent has an opinion that continuously evolves towards a local agreement representing the opinion of the community in which it lies. At specific instants, the leaders of the communities interact and they reset their opinion taking into account the ones of other leaders. The new opinions of the leaders will reset the values of the local agreements in each community. Iterating this process all the opinions will tend to a common value that depends only on the initial conditions and the network topology.

All the clusters are represented by fixed, strongly connected directed graphs. In order to enforce consensus the discrete control action of the leaders will be designed in a decentralized manner by taking into account only informations provided by some other leaders. In other words, we address the problem of consensus for agents subject to both continuous and discrete dynamics.

The aim of the paper is to control the consensus behavior in the network. A first contribution is related to the characterization of the consensus value in the framework under consideration. This is an important step that has to be done before imposing the consensus value. We note that the consensus value depends only on the initial conditions and the topologies of the involved networks (i.e the networks associated with the clusters and that associated with the leaders). It is noteworthy that the consensus value does not depend on the reset sequence used for the leaders' state. In order to study the stability of consensus we propose a LMI based condition that can be adapted for further goals of the paper encompassing the design of resets that allows reaching some network performances. The analysis of the network behavior finishes with the characterization of the convergence speed.

Another contribution of the paper is related to the design of the reset strategy of the leaders' state. In this part we design the interaction topology between leaders allowing to reach an a priori specified consensus value. In this part the network topology continue to be considered fixed and known for each cluster. The objective is to modify the consensus value of the whole network by changing the weights in the network of leaders. The set of consensus values that can be reached is contained in the interval defined by the minimum and maximum initial local agreements.

The paper is organized as follows. In Section 2 we formulate the problem under consideration. The agreement behavior and the possible consensus value are studied in Section 3. Sufficient conditions for the global uniform exponential stability of the consensus are provided in Section 4. These conditions are given in the form of a parametric LMI. Complementary results concerning the design of the network of interactions between the leaders allowing to reach a prescribed consensus value and the convergence speed are presented in Section 5. The problem of numerical implementation of the proposed developments is considered in Section 6. Precisely we show how the parametric LMI can be replaced by a finite number of LMIs. Section 7 is dedicated to numerical simulations which illustrate the results. Some conclusions and perspectives are presented at the end of the paper.

Notation. The following standard notation will be used throughout the paper. The sets of nonnegative integers, real and nonnegative real numbers are denoted by $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{R}_+$, respectively. For a vector x we denote by $\|x\|$ its Euclidean norm. The transpose of a matrix A is denoted by $A^\top$. Given a symmetric matrix $A \in \mathbb{R}^{n\times n}$, notation $A > 0$ ($A \geq 0$) means that A is positive (semi-)definite. By I_k we denote the $k \times k$ identity matrix. $\mathbb{1}_k$ and $\mathbf{0}_k$ are the column vectors of size k having all the components equal 1 and 0, respectively. We also use $x(t_k^-) = \lim_{t\to t_k, t\leq t_k} x(t)$. Throughout the paper we say that the LMI: $A > 0$ is satisfied on the subspace $\mathcal{K}$ if and only if $x^\top Ax > 0$ for all $x \in \mathcal{K}$.

2. Problem formulation

We consider a network of n agents described by the digraph (i.e. directed graph) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where the vertex set $\mathcal{V}$ represents the set of agents and the edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ represents the interactions.

Definition 1. A *path* in a given digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a union of directed edges $\bigcup_{k=1}^{p}(i_k, j_k)$ such that $i_{k+1} = j_k,\ \forall k \in \{1, \dots, p-1\}$.

Two nodes i, j are *connected* in a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ if there exists at least a path in $\mathcal{G}$ joining i and j (i.e. $i_1 = i$ and $j_p = j$).

A *strongly connected digraph* is such that any two distinct nodes are connected. A *strongly connected component* of a digraph is a maximal subset of the vertex set such that any of its two distinct elements are connected.

In the sequel, we consider that the agent set $\mathcal{V}$ is partitioned in m strongly connected clusters/communities $\mathcal{C}_1, \dots, \mathcal{C}_m$ and no link between agents belonging to different communities exists. Each community possesses one particular agent called leader and denoted in the following by $l_i \in \mathcal{C}_i,\ \forall i \in \{1, \dots, m\}$. The set of leaders will be referred to as $\mathcal{L} = \{l_1, \dots, l_m\}$. At specific time instants $t_k,\ k \geq 1$, called reset times, the leaders interact between them following a predefined interaction map $\mathcal{E}_l \subset \mathcal{L} \times \mathcal{L}$. We also suppose that $\mathcal{G}_l = (\mathcal{L}, \mathcal{E}_l)$ is strongly connected. The rest of the agents will be called followers and denoted by f_j. For the sake of clarity we consider that the leader is the first element of its community:

$$\mathcal{C}_i = \{l_i, f_{m_{i-1}+2}, \dots, f_{m_i}\}, \quad \forall i \in \{1, \dots, m\} \tag{1}$$

where $m_0 = 0,\ m_m = n$ and the cardinality of $\mathcal{C}_i$ is given by

$$|\mathcal{C}_i| \triangleq n_i = m_i - m_{i-1}, \quad \forall i \geq 1.$$

Example 1. To illustrate the notation (1) we consider a simple network of 6 agents partitioned in 2 clusters having 3 elements. Then $\mathcal{C}_1 = \{l_1, f_2, f_3\}$ and $\mathcal{C}_2 = \{l_2, f_5, f_6\}$.

In order to keep the presentation simple and making an abuse of notation, each agent will have a scalar state denoted also by l_i for the leader l_i and f_j for the follower f_j. We also introduce the vectors $x = (l_1, f_2, \dots, f_{m_1}, \dots, l_m, \dots, f_{m_m} = f_n)^\top \in \mathbb{R}^n$ and $x_l = (l_1, l_2, \dots, l_m)^\top \in \mathbb{R}^m$ collecting all the states of the agents and all the leaders' states, respectively.

We are ready now to introduce the linear reset system describing the overall network dynamics:

$$\begin{cases} \dot{x}(t) = -Lx(t), & \forall t \in \mathbb{R}_+ \setminus \mathcal{T} \\ x_l(t_k) = P_l x_l(t_k^-) & \forall t_k \in \mathcal{T} \\ x(0) = x_0 \end{cases} \tag{2}$$