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Direct learning of LPV controllers from data[☆]

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ABSTRACT

In many control applications, it is attractive to describe nonlinear (NL) and time-varying (TV) plants by linear parameter-varying (LPV) models and design controllers based on such representations to regulate the behavior of the system. The LPV system class offers the representation of NL and TV phenomena as a linear dynamic relationship between input and output signals, which is dependent on some measurable signals, *e.g.*, operating conditions, often called as scheduling variables. For such models, powerful control synthesis tools are available, but the way how to systematically convert available first principles models to LPV descriptions of the plant, to efficiently identify LPV models for control from data and to understand how modeling errors affect the control performance are still subject of undergoing research. Therefore, it is attractive to synthesize the controller directly from data without the need of modeling the plant and addressing the underlying difficulties. Hence, in this paper, a novel data-driven synthesis scheme is proposed in a stochastic framework to provide a practically applicable solution for synthesizing LPV controllers directly from data. Both the cases of fixed order controller tuning and controller structure learning are discussed and two different design approaches are provided. The effectiveness of the proposed methods is also illustrated by means of an academic example and a real application based simulation case study.

1. Introduction

The concept of *linear parameter-varying* (LPV) systems, introduced in Shamma and Athans (1990), offers a promising framework for modeling and control of a large class of *nonlinear* (NL) and *time-varying* (TV) systems. LPV systems can be seen as an extension of *linear time-invariant* (LTI) systems, with a linear dynamic relation between the input and the output signals. Unlike in the LTI case, these signal relations can change over the time and depend on a measurable time-varying signal, the so-called *scheduling variable*. Scheduling variables can be external signals like space coordinates or parameters used to describe changing operating conditions. In this way, the nonlinear and time-varying behavior of the system can be embedded in the solution set of a linear dynamic input–output relationship which varies with the scheduling variable (Tóth, Willems, Heuberger, & Van den Hof, 2011). The LPV modeling paradigm has evolved rapidly in the last two decades and has been applied in many applications like aircrafts (Lu, Wu, & Kim, 2006), automotive systems (Cerone, Piga, & Regruto, 2011; Corno, Tanelli, Savaresi, & Fabbri, 2011; Novara, Ruiz, & Milanese, 2011), robotic manipulators (Hashemi, Abbas, & Werner, 2012) and induction motors (Prempain, Postlethwaite, & Benchaib, 2002).

Accurate and low complexity models of LPV systems can be efficiently derived from data using *input–output* (IO) representation based model structures (Bamieh & Giarre, 2002; Butcher, Karimi, & Longchamp, 2008; Laurain, Gilson, Tóth, & Garnier, 2010; Piga, Cox, Tóth, & Laurain, 2015; Tóth, 2010), while state-space approaches appear to be affected by the curse of dimensionality and other approach-specific problems (Felici, van Wingerden, & Verhaegen, 2007; van Wingerden & Verhaegen, 2009; Verdult, Lovera, & Verhaegen, 2004; Verdult & Verhaegen, 2002, 2005). However, most of the control synthesis approaches are based on a state-

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space representation of the system dynamics (except a few recent works (Ali, Abbas, & Werner, 2010; Cerone, Piga, Regruto, & Tóth, 2012; Wollnack, Abbas, Werner, & Tóth, 2013)) and state-space realization of complex IO models is theoretically solved, but difficult to accomplish in practice (Tóth, 2010). This transformation results in a state-minimal representation, which can have rational dependency on time-shifted versions of the scheduling signals. Alternative approaches can reduce the complexity of the scheduling-variable dependency, but at the price of a non state-minimal representation, for which efficient model reduction is largely an open issue (Tóth, Abbas, & Werner, 2012). Moreover, the way the modeling error affects the control performance is unknown for most of the design methods and little work has been done on including information about the control objectives into the identification setting.

In this paper, a *method is proposed to design fixed-order LPV controllers in an IO form using directly the experimental data. In fact, this corresponds to designing controllers without deriving a model of the system*. This approach permits to avoid the critical (and time-consuming) approximation steps related to modeling, identification and state-space realization and it results in an automatic procedure in which only the desired closed-loop behavior has to be specified by the user. The proposed approach is developed for the case when the parametric structure of the controller is assumed to be given and also when the structure is needed to be selected (learnt) from data directly. The recent results in data-driven LPV model structure selection (Laurain, Tóth, Zheng, & Gilson, 2012; Tóth, Laurain, Zheng, & Poolla, 2011) employing *Least-Squares Support Vector Machines* (LS-SVM) (Suykens, Van Gestel, De Brabanter, De Moor, & Vandewalle, 2002) are exploited. In both cases, the controller synthesis problem is formulated as an optimization problem and *instrumental-variable* (IV) based identification techniques are used to efficiently cope with the noise affecting the signal measurements. The advantages of using an IV based approach are twofold: (i) it allows the design of the controller through convex optimization based on measured data from the system; (ii) the bias in the designed controller with respect to the optimal solution, due to the noise affecting the output measurements, is guaranteed to asymptotically converge to zero as the number of data samples increases.

Direct controller tuning approaches using a single set of IO data, also known as non-iterative data-driven control, have been already studied in the *linear time-invariant* (LTI) framework (Bazanella, Campestrini, & Eckhard, 2011). Well established approaches, like *Virtual Reference Feedback Tuning* (VRFT) (Campi, Lecchini, & Savaresi, 2002; Formentin, Savaresi, & Del Re, 2012) and *Non-iterative Correlation-based Tuning* (CbT) (van Heusden, Karimi, & Bonvin, 2011), have been widely discussed in the literature, see, *e.g.*, the recent results in Formentin, Cologni, Previdi, and Savaresi (2014), Formentin, Corno, Savaresi, and Del Re (2012), Formentin, De Filippi, Corno, Tanelli, and Savaresi (2013), Formentin and Karimi (2014), Formentin, Karimi, and Savaresi (2013) and Formentin, van Heusden, and Karimi (2013). Other recently introduced approaches are, *e.g.*, Formentin and Karimi (2013) and Radac, Precup, Petriu, Preitl, and Dragos (2013).

The first attempt to extend a data-driven method, namely the VRFT method, to LPV systems has been presented in Formentin and Savaresi (2011), where data-driven gain-scheduled controller design has been proposed to realize a user-defined LTI closed-loop behavior. Although satisfactory performance has been shown for slowly varying scheduling trajectories, this methodology is far from being generally applicable to LPV systems. As a matter of fact, in the method presented in Formentin and Savaresi (2011), the controller must be linearly parameterized and the reference behavior must be LTI. The latter requirement represents a strict limitation, since an LTI behavior might be difficult to realize in practice, as it may require too demanding input signals and dynamic dependence of the controller on the scheduling signal. On the other hand, the LPV extension of Non-iterative CbT has been found to be unfeasible, as the derivation of this approach is based on the commutation of the plant and the controller in the tuning scheme (Karimi, Van Heusden & Bonvin, 2007). Unfortunately, such a commutation does not generally hold for parameter-varying transfer operators (Tóth et al., 2011). The recent work in Novara (2013) also deals with LPV direct data-driven control, but the framework is completely different from the one proposed herein. Specifically, in Novara (2013), the system is given in state-space form with measurable state vector, the optimal controller is assumed to be Lipschitz continuous and the whole method is developed in a deterministic set-membership setting. A direct data-driven LPV solution has been presented in a stochastic framework for feed-forward precompensator tuning in Butcher and Karimi (2009). However, also in this case, no dynamic dependence is accounted for and the final objective is constrained to be LTI.

In summary, the main contributions of the paper are as follows: (i) a novel direct data-driven method is introduced for optimization of LPV controller parameters without the need of a model of the system to be controlled; the method is inspired by the VRFT concept, but it is entirely different from the straightforward LPV extension of VRFT in Formentin and Savaresi (2011); (ii) this is the first data-driven control method where learning the controller structure from data is achieved by the use of LS-SVM; (iii) we also show how inversion of a state-space reference model can be extended to the LPV case and how inversion up to a kth-order delay can be achieved in case of no direct feedthrough. Finally, we compare the proposed approach in detail with other existing direct data-driven techniques. In this paper, we will focus on the SISO setting only. A non-straightforward MIMO extension of the results is possible by the use of so-called kernel representations, but it is beyond the scope of this work. A preliminary version of this work has been presented in Formentin, Piga et al. (2013).

The paper is organized as follows. The formulation of the design problem is provided in Section 2, whereas Section 3 outlines the main idea behind the proposed methodology. Then, Section 4 illustrates the technical derivation of the method in case the structure of the controller is *a priori* fixed (parametric design) and Section 5 deals with the case where also the controller parameterization has to be determined from data (nonparametric design). Section 6 discusses the proposed approach in comparison with the existing techniques. The effectiveness of the developed control design methodologies is shown by means of a numerical example in Section 7 and a real application based simulation study inspired by Kulcsar, Dong, van Wingerden, and Verhaegen (2009) in Section 8. The paper is ended by some concluding remarks.

2. Problem formulation

Let $\mathcal{G}_{\mathrm{p}}$ denote an unknown *single-input single-output* (SISO) LPV system described by the difference equation

$$A(p,t,q^{-1})y_{\mathrm{o}}(t) = B(p,t,q^{-1})u(t), \tag{1}$$

where $u(t) \in \mathbb{R}$ is the input signal, $y_{\mathrm{o}}(t) \in \mathbb{R}$ is the noise-free output and $p(t) \in \mathbb{P} \subseteq \mathbb{R}^{n_{\mathrm{p}}}$ is a set of n_{p} (exogenous) measurable scheduling variables. Without the loss of generality, in the sequel, the case of $n_{\mathrm{p}} = 1$ will be considered in order to keep the notation simple. In (1), $A(p,t,q^{-1})$ and $B(p,t,q^{-1})$ are polynomials in the backward time-shift operator q^{-1} of finite degree n_{a} and n_{b}, respectively, *i.e.*,

$$A(p,t,q^{-1}) = 1 + \sum_{i=1}^{n_{\mathrm{a}}} a_i(p,t)q^{-i}, \tag{2a}$$

$$B(p,t,q^{-1}) \quad = \sum_{i=0}^{n_{\mathrm{b}}} b_i(p,t)q^{-i}, \tag{2b}$$