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Ground-state stabilization of quantum finite-level systems by dissipation ☆

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ABSTRACT

Control by dissipation, or environment engineering, constitutes an important methodology within quantum coherent control which was proposed to improve the robustness and scalability of quantum control systems. The system–environment coupling, often considered to be detrimental to quantum coherence, also provides the means to steer the system to desired states. This paper aims to develop the theory for engineering of the dissipation, based on a ground-state Lyapunov stability analysis of open quantum systems via a Heisenberg-picture approach. In particular, Lyapunov stability conditions expressed as operator inequalities allow a purely algebraic treatment of the environment engineering problem, which facilitates the integration of quantum components into a large-scale quantum system and draws an explicit connection to the classical theory of vector Lyapunov functions and decomposition–aggregation methods for control of complex systems. This leads to tractable algebraic conditions concerning the ground-state stability and scalability of quantum systems. The implications of the results in relation to dissipative quantum computing and state engineering are also discussed in this paper.

1. Introduction

Control of quantum systems lies at the core of the quantum technology (Altafini & Ticozzi, 2012; Dong & Petersen, 2010; Wiseman & Milburn, 2009), while stability analysis provides the appropriate tool for the systematic development of quantum control theory. The stability analysis has been used in several quantum control synthesis problems (James, Nurdin, & Petersen, 2008; Maalouf & Petersen, 2011; Mirrahimi & Van Handel, 2007; Petersen, Ugrinovskii, & James, 2012; Qi, Pan, & Guo, 2013; Zhang, Liu, Wu, Jacobs, & Nori, 2014). The applications include measurement-based feedback control and coherent control for the generation of quantum states as well as the regulation of system performance. Among all the methods for stability analysis, the Lyapunov stability approach is the most fundamental, as the energy of a quantum system is well-defined for most of the physical systems and a Lyapunov function can be easily constructed (Amini et al., 2013; Kuang & Cong, 2008; Sayrin et al., 2012; Ticozzi, Lucchese, Cappellaro, & Viola, 2012; Wang & Schirmer, 2010). In particular, as we will demonstrate in this paper, the Lyapunov method provides a means for engineering the dissipation to be used as coherent control.

Quantum computing often involves the execution of a sequence of unitary operations on quantum systems. However, the severe decoherence associated with the quantum systems presents a major obstacle to the scalability of this approach. For this reason, methods for robust realization of unitary operations are currently under discussion. The possible plans include topological quantum computing, adiabatic quantum computing and dissipative quantum computing. Among these schemes, the adiabatic quantum computing and dissipative quantum computing have direct relevance to the stability of quantum systems. For example, in dissipative quantum computing and state engineering, dissipation is introduced as a resource to coherently control the system (Verstraete, Wolf, & Ignacio Cirac, 2009). The idea is to consider open quantum systems, and stabilize their quantum states by engineering the system–environment interaction. If designed judiciously,

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the dissipation will drive the system to a target steady state regardless of the initial state and external perturbations. This method can be used to generate highly entangled quantum states, and perform quantum computation by encoding the computation result to the steady state of the system. Since dissipation of energy is the key physical principle behind this method, this kind of coherent control approach can be referred to as control by dissipation. Our goal in this paper is to formulate the method of control by dissipation within the framework of ground-state stability, and then propose approaches for the synthesis of system–environment dissipative interactions that rely on Lyapunov methods for stability analysis.

Stability of quantum states has been the focus of many theoretical studies. Many of them have successfully derived sufficient conditions for convergence of quantum Markov systems to a steady state (Frigerio, 1978; Koga & Yamamoto, 2012; Kraus et al., 2008; Pan et al., 2014; Sauer, Gneiting, & Buchleitner, 2013; Schirmer & Wang, 2010; Spohn, 1976). In particular, the stability of quantum states in a dissipative setting has been considered in Altafini and Ticozzi (2012), Schirmer and Wang (2010) and Ticozzi and Viola (2009, 2012). In these studies, the target state is often explicitly given and follows a Schrödinger-picture master equation. The dissipative couplings, compensated by Hamiltonian control, can generate a Markov process that converges to the target states (Ticozzi & Viola, 2009, 2012). The implementation of the system–environment couplings with the practical resources has been investigated experimentally. Dissipative engineering of several types of quantum systems has been demonstrated in recent years (Barreiro et al., 2011; Kastoryano, Reiter, & Sørensen, 2011; Krauter et al., 2011; Lin et al., 2013; Schindler et al., 2013; Shankar et al., 2013).

In this paper, we adopt an alternative path to approach the stability theory within the Heisenberg picture, where instead of designating target states explicitly, they are characterized as ground states a Lyapunov operator, and the stability problem is transformed to the problem of stabilization of the ground states of the Lyapunov operator. The formalism of Lyapunov stability can thus be conveniently introduced to engineer the desired system dissipation within this framework. This allows to derive tractable sufficient conditions expressed in terms of operator inequalities, which can be used for the synthesis of the desired system–environment coupling. Such conditions is the main contribution of this paper compared to our previous work (Pan et al., 2014). The general results regarding stability of Lyapunov operators obtained in (Pan et al., 2014) do not readily apply to the problem of control by dissipation.

An important advantage of the Heisenberg-picture approach developed here is that the target state does not need to be given in advance. In addition to the entangled-state engineering applications in which the Lyapunov operator is chosen based on the knowledge of the target state, there exists a large class of applications where the control goals are posed as minimization of the expectation of an operator while the target state with respect to which the expectation is taken is not known. For example, the problems of sequential quantum computation and the quantum satisfiability problem (SAT) (Bravyi, 2006; Nielsen & Chuang, 2004) involve operators which play the role of cost functions. In these problems, the target states which minimize the expectation of the operators are unknown and result from computation and/or control. Moreover, the target state in these applications may be not unique. This complicates the analysis based on the conventional Schrödinger-picture approach. Therefore, the Heisenberg-picture approach extends the applicability of the control by dissipation.

One of the main contributions of this paper is concerned with the scalability of the control by dissipation, when this control method is applied to large quantum systems comprised of multiple interacting subsystems coupled with the environment. The Heisenberg-picture Lyapunov approach has an advantage in that the problem can be treated in a way that resembles the decomposition–aggregation engineering (Bellman, 1962; Šiljak, 1978) for complex classical systems. Namely, a large-scale quantum system is decomposed into subsystems and an individual Lyapunov operator is associated with a subsystem. This allows us to establish conditions, expressed in terms of the subsystems' Lyapunov operators, under which the quantum system is guaranteed to converge to its ground state. Here we note a similarity with the classical connective stability conditions (Šiljak, 1978), which have proved to be useful in the synthesis of decentralized controllers for large-scale systems.

A typical methodology for the synthesis of dissipations involves two problems, the calculation of the stabilizing system–environment couplings and the implementation of these couplings using the available physical resources. For example, it is possible to construct a coherent optical network to realize a linear coupling (Nurdin, James, & Doherty, 2009). Therefore, in this paper we focus on the first problem of calculation of coupling operators that render the states of the ground energy asymptotically stable. Particularly, we can apply this method to check the feasibility of the solutions proposed in Verstraete et al. (2009). It is worth mentioning that the constraints on the system–environment couplings could be greatly relaxed if Hamiltonian control is available (Ticozzi & Viola, 2009, 2012).

The preliminary version of this paper has been accepted for presentation at the American Control Conference (Pan, Ugrinovskii, & James, 2015). Compared to the preliminary conference version, this paper has been substantially revised and expanded. It includes a detailed exposition of the background on open quantum systems, the new material on the scalability of the Lyapunov methods, synthesis of dissipation, the examples and an application to stabilization of quantum states associated with quantum toric codes. The paper also includes detailed proofs of all the results and gives detailed discussions of these results, which were not included in Pan et al. (2015).

The paper is organized as follows. In Section 2, we introduce the notations and the model considered in this paper. In Section 3 we present the ground-state stability analysis of Lyapunov operators. Section 4 discusses the scalability problem, where a large quantum system may be governed by more than one Lyapunov operators. Section 5 concerns with the synthesis of the dissipation. More explicitly, this section concerns with the calculation of the correct coherent couplings for the ground-state stabilization when the Lyapunov operator is given. Conclusion is given in Section 6. The proofs of the results are given in the Appendix.

2. Notations and preliminaries

2.1. Open quantum systems

Consider a Hilbert space $\mathcal{H}$ and define $\mathcal{B}(\mathcal{H})$ as the space of bounded operators on $\mathcal{H}$. We only consider finite-dimensional quantum systems throughout this paper. In other words, $\mathcal{H}$ is assumed to be finite-dimensional throughout the paper. Hence all bounded operators in our case are representable as complex matrices. Let $X \in \mathcal{B}(\mathcal{H})$. X^T denotes the transpose of X and $X^\dagger$ is the adjoint of X. An operator X is called an observable if $X^\dagger = X$. The notation $X \geq 0$ ($X \leq 0$) means the operator X is a positive (negative) semidefinite operator. We write $X > 0$ if X is positive definite. Also, we will use the notation $X \succeq 0$ for positive semidefinite operators X whose smallest eigenvalue is equal to 0.

Given a bounded observable $X \in \mathcal{B}(\mathcal{H})$ and a trace class operator ρ on $\mathcal{H}$, $\langle X \rangle_\rho$ denotes the trace of $X\rho$, $\langle X \rangle_\rho = \mathrm{Tr} X\rho$. When ρ is a density state, i.e., a matrix whose trace is equal to 1, then $\langle X \rangle_\rho$ is the mean value of X evaluated at the density state ρ.