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Brief paper

Enhanced nonlinear damping for a class of singularly perturbed interconnected nonlinear systems☆

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ABSTRACT

In this paper, we propose a method of enhanced nonlinear damping control for a class of singularly perturbed interconnected nonlinear systems (SPINSs). Instead of simply canceling out the interconnection between slow and fast subsystems, the proposed method transforms SPINS into a feedback connection of two output strictly passive subsystems. Then, enhanced nonlinear damping is implemented to improve the transient behavior of slow subsystem. The proposed method provides weighting factors for each tracking error which enable the energy function to be designed to shape the tracking performance for each state. We also design an augmented state observer to estimate unknown disturbances and unmeasurable states. Using a composite Lyapunov function, we prove that the origin of the tracking error dynamics is globally exponentially stable. It is also analyzed that the tracking errors are globally uniformly ultimately bounded for a time-varying bounded disturbance. Experiments were performed to evaluate the tracking performance of the proposed method and our results validate the effectiveness of the proposed method.

1. Introduction

Direct current motors and permanent magnet synchronous motors have been widely used for various servo systems. These systems can be classified as the singular perturbed interconnected nonlinear systems (SPINSs), which can be defined as

$$\begin{aligned}\dot{x} &= A_x x + B_x g_0(y)^T z - B_x w(t)\\ \varepsilon\dot{z} &= A_z z + g_1(y) B_x^T x + B_z u\\ y &= \begin{bmatrix} C_x & 0\\ 0 & I_m\end{bmatrix}\begin{bmatrix} x\\ z\end{bmatrix}\end{aligned} \tag{1}$$

where

$$A_x = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \ddots & \vdots\\ 0 & \ddots & \ddots & \ddots & 0\\ \vdots & \ddots & \ddots & 0 & 1\\ 0 & \cdots & 0 & 0 & b_d\end{bmatrix} \in \mathbb{R}^{r\times r}$$

$$B_x = \begin{bmatrix} 0 & \cdots & 0 & 1\end{bmatrix}^T \in \mathbb{R}^{r\times 1}, \qquad C_x = \begin{bmatrix} C_1^T & C_2^T\end{bmatrix}^T \in \mathbb{R}^{l\times r}$$
$$C_1 = \begin{bmatrix} 1 & 0 & \cdots & 0\end{bmatrix} \in \mathbb{R}^{1\times r}, \qquad C_2 \in \mathbb{R}^{l-1\times r}.$$

Here, $x = [x_1 \cdots x_r]^T \in \mathbb{R}^r$ is the state of the slow subsystem, $z = [z_1 \cdots z_m]^T \in \mathbb{R}^m$ is the state of the fast subsystem, $y \in \mathbb{R}^{l+m}$ is the measurable output, $u \in \mathbb{R}^m$ is the input of the system, w is an unknown disturbance bounded as

$$\sup_t |w(t)| \le w_{\max}, \qquad \sup_t |\dot{w}(t)| \le \dot{w}_{\max},$$

and the time scale factor is $0 < \varepsilon \ll 1$. $g_0(y) \in \mathbb{R}^m$ and $g_1(y) \in \mathbb{R}^m$ are sufficiently continuously differentiable nonlinear functions

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that satisfy

$$\gamma_1 g_0(y) + g_1(y) = 0 \tag{2}$$

$$0 < \|g_0(y)\|_2 \le \gamma_2 \tag{3}$$

where $\gamma_1 > 0$ and $\gamma_2 > 0$. $A_z \in \mathbb{R}^{m\times m}$ is the system matrix of the fast subsystem, $I_m \in \mathbb{R}^{m\times m}$ is the identity matrix, and $B_z \in \mathbb{R}^{m\times m}$ is a nonsingular matrix. $n = r + m$ is the dimension of the given dynamics and $b_d \le 0$ is a real number. The matrix A_x represents the series of integrators with viscous damping b_d of the mechanical system. For motor dynamics in the form of SPINS, the states of fast dynamics z and the input u are the phase currents and voltages, respectively. Therefore, we assume that z is measurable and B_z is nonsingular without loss of generality. Equality (2) represents Newton's third law of motion (action and reaction force pairs) and inequality (3) is required to hold the interconnection between the slow and fast subsystems for any y. Note that various systems, such as multi-link robot manipulators actuated by electric motors or electro-hydraulic actuators, can be transformed into a SPINS (1) by feedback linearization (Seo, Venugopal, & Kenné, 2007).

To improve the tracking performance of various systems in the form of SPINSs, various feedback control methods have been proposed (Bodson, Chiasson, Novotnak, & Rekowski, 1993; Fekih, 2008; Kim, Shin, & Chung, 2012, 2013; Kim, Won, Shin, & Chung, 2012; Liu & Li, 2012; Wang, Book, & Huggins, 2012; Yan & Wang, 2013). In these literature, the control input was designed such that the given SPINSs become cascade interconnected systems of tracking errors with asymptotically stable origins (Chaillet, Angeli, & Ito, 2014). This implies that the fast dynamics is perfectly decoupled from the slow dynamics but still affects the responses of the slow dynamics. To neglect the transient response of the slow dynamics affected by the fast dynamics, the previous methods only make the tracking error of the fast dynamics e_z quickly converge to zero. From the viewpoint of singular perturbation theory, however, this cannot be used to generate the motion of e_z to improve the stabilizing performance of the tracking error of the slow dynamics e_x because $e_z = 0$ is an asymptotically stable equilibrium point regardless of the motion of e_x. This implies that the equilibria of e_z as a function of e_x must be designed to improve the stabilizing performance of e_x in the transient period.

Through interconnection and damping assignment passivity based controllers, it is possible to design the equilibria of e_z to help improve the stabilizing performance of e_x in the transient period (Acosta, Ortega, Astolfi, & Mahindrakar, 2005; Gómez-Estern & Van der Schaft, 2004; Ortega, Spong, Gómez-Estern, & Blankenstein, 2002; Ortega, van der Schaft, Maschke, & Escobar, 2002). In the literature (Acosta et al., 2005; Gómez-Estern & Van der Schaft, 2004; Ortega, Spong et al., 2002; Ortega, van der Schaft et al., 2002), an analysis determined that the total energy function asymptotically converges to zero. However, it has not addressed how to design the total energy function to enable individually adjustment of the stabilization performance for each state. The effect of the interconnection between the dynamics of e_x and e_z on the convergence ratio of e_x has also yet to be analyzed. Furthermore, disturbances that may affect the tracking performance were also not considered.

In this paper, we propose a method of enhanced nonlinear damping control for a class of singularly perturbed interconnected nonlinear systems (SPINSs). The tracking errors, including the virtual inputs, are defined in order to transform SPINSs into tracking error dynamics that has individual inputs for each state. The virtual inputs are designed to make the system matrix of the tracking error dynamics of a slow subsystem become a special form, which includes weighting factors for each state. The control input is designed to ensure that the tracking error dynamics of SPINSs becomes a feedback connection of two output strictly passive subsystems. The proposed method enables the tracking performance of each state to be adjusted by the weighting factors. We analyze the interconnection between fast and slow subsystems that generate the nonlinear damping effect in the feedback connection of two output strictly passive subsystems. From this analysis, we show that a high convergence ratio for fast subsystems does not always improve the tracking performance of the slow subsystem especially in a slow manifold, since the nonlinear damping effect decreases as the convergence ratio of e_z increases. We also design an augmented state observer that estimates the unknown disturbances and unmeasurable states. Using a composite Lyapunov function, we prove that the origin of the tracking error dynamics is globally exponentially stable under an unknown constant disturbance. It is also analyzed that the tracking errors are globally uniformly ultimately bounded for an unknown time varying bounded disturbance. Experiments utilizing a permanent magnet stepper motor, which is a type of SPINS, are performed to evaluate the tracking performance of the proposed method. The experimental results validate the effectiveness of the proposed method.

2. Controller design

In this section, the controller design is presented. First, the tracking error dynamics, including the virtual inputs, are derived. Then, we design the virtual inputs such that the system matrix of the tracking error dynamics of x has a special form. The control input u is designed to make the tracking error dynamics of SPINS become output strictly passive.

2.1. Virtual input design

In this subsection, we derive the tracking error dynamics, which includes virtual inputs, to simplify the controller design. The virtual inputs x_i^* and z^* are designed such that the system matrix of the tracking error dynamics of x has weighting factors for each state. In the design process of the virtual inputs, we assume that all of the states and the disturbance x_i, z and $w(t)$ are measurable. Let us define the tracking errors as

$$\begin{aligned} e_x &= \begin{bmatrix} e_{x_1} & \cdots & e_{x_r}\end{bmatrix}^T \\ e_z &= z^* - z,\; e_{x_i} = x_i^* - x_i \quad i = 1, \ldots, r \end{aligned} \tag{4}$$

where the virtual inputs x_i^* and z^* will be designed. With (1) and (4), the tracking error dynamics can be rearranged as

$$\begin{aligned} \dot{e}_x &= A_x e_x + B_x g_0(y)^T e_z + B_x w(t) + I_r u_x \\ \varepsilon \dot{e}_z &= \varepsilon \dot{z}^* - A_z z - g_1(y) B_x^T x - B_z u \end{aligned} \tag{5}$$

where $I_r \in \mathbb{R}^{r\times r}$ is the identity matrix and

$$u_x = \begin{bmatrix} u_{x_1} & u_{x_2} & \cdots & u_{x_r}\end{bmatrix}^T$$

$$u_{x_i} = \begin{cases} \dot{x}_i^* - x_{i+1}^* & i < r \\ \dot{x}_r^* - b_d x_r^* - g_0(y)^T z^* & i = r. \end{cases}$$

The controllers for each state can be individually designed using the state feedback. To decouple the states x_i and z from the tracking errors e_{x_i} and e_z, each x_i^* should be designed with $x_1, \ldots, x_{i-1}$ and z^* should be a function of x. We choose the virtual inputs x_i^* and z^* as

$$x_i^* = \begin{cases} x_1^d & i = 1 \\ \dot{x}_1^d + \rho_1 e_{x_1} & i = 2 \\ \dot{x}_{i-1}^* + \kappa_{i-1} e_{x_{i-2}} + \rho_{i-1} e_{x_{i-1}} & \text{otherwise.} \end{cases} \tag{6}$$

$$z^* = \frac{g_0}{g_0^T g_0}\{\dot{x}_r^* - b_d x_r + \kappa_{r-1} e_{x_{r-1}} + \rho_r e_{x_r} + w(t)\}$$