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Technical Note

An image despeckling approach using quantum-inspired statistics in dual-tree complex wavelet domain

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ABSTRACT

This paper presents a novel despeckling algorithm to enhance image quality of medical ultrasound images. The proposed approach exploits coefficients in the *dual-tree complex wavelet transform* (DTCWT) domain to develop a new quantum-inspired thresholding function. The inter-scale correlation among the coefficients of different subbands and the intra-scale variance between the coefficient and its neighborhood in the same subband are utilized to develop a new thresholding function, which is further incorporated into a Bayesian framework to perform adaptive image despeckling. Experiments are conducted using both artificially generated and real-world medical images to demonstrate that the proposed approach outperforms the conventional image despeckling approaches.

1. Introduction

Ultrasonography is one of the most important modalities of low-cost and non-invasive medical imaging system [1,2]. However, they usually suffer from speckle noise, which significantly degrades the image quality and makes it difficult to discriminate fine details of the images in diagnostic examinations.

Many image despeckling algorithms have been developed in the literature to enhance the quality of medical ultrasound images; such as the Kuan filter [3], Frost filter [4], Lee filter [5], non-log transformed *generalized likelihood* (GenLik) ratio based wavelet-domain denoising method [6], and partial differential equation based Perona–Malik [7] method and *speckle reducing anisotropic diffusion* (SRAD) [8]. Although these filters perform well for removing speckle from the noisy image, they cannot preserve sharp features of the original image very well. The *block-matching and 3D filtering* (BM3D) method [9] is the state-of-the-art denoising method specifically designed for additive white Gaussian noise, which cannot effectively remove speckle noise from the images. *Discrete wavelet transform* (DWT) based approaches also have been utilized in despeckling [10]. However, it cannot provide phase information, which makes DWT less suitable for handling the medical ultrasound images [11]. Shearlet transform is also a directional representation system extending the wavelet transform and can better sparsely approximate the multivariate signals such as images [12]. *Dual-tree complex wavelet transform* (DTCWT) is used for the follow-up work to avoid the shortcoming introduced by DWT. In our early study, a quantum-inspired despeckling method [13] has been proposed to obtain fairly good despeckling performance. However, the method developed in [13] lacks the adaptability to determine the parameters of the statistical models for wavelet coefficients of different images.

To tackle the above challenges, a new approach is proposed in this paper by exploiting a quantum-inspired thresholding function based on characteristics of wavelet coefficients. More specifically, the proposed approach exploits two statistics of the coefficients in the DTCWT domain: (i) the inter-scale correlation among the parent and child subbands; and (ii) the intra-scale variance between the coefficient and its neighborhood in each individual subband. This differs from the conventional despeckling methods that smooth the image at all pixels to the same degree.

The rest of paper is organized as follows. Section 2 presents the proposed approach. Section 3 presents the experimental results using both artificially generated speckled images and real-world speckled images. Finally, Section 4 concludes this paper.

2. Image despeckling approach

In order to study the image despeckling problem, a mathematical problem formulation is presented in this paragraph. A speckled medical ultrasound image $\tilde{Y}$ is modeled as $\tilde{Y} = \tilde{N}\tilde{X}$, where $\tilde{X}$ is the ground truth and $\tilde{N}$ is the independent speck noise random field.

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In the DTCWT domain, the log-transformed ultrasound image can be modeled as $Y = X + N$, where $Y = Y_r + iY_i$, $X = X_r + iX_i$ and $N = N_r + iN_i$ are the complex wavelet coefficients of $\tilde{Y}$, $\tilde{X}$ and $\tilde{N}$, respectively, r and i indicate real and imaginary parts, respectively. With such formulation, the objective of image despeckling is to remove the speck noise from the observed noisy image.

2.1. Conventional thresholding

The Bayesian inference framework has been justified to be effective in image despeckling [10]. First, the distribution of log-transformed speckle noise is close to a Gaussian *probability density function* (PDF) as

$$p_n(N) = \frac{1}{2\pi\sigma_n^2} exp\left(-\frac{N_r^2 + N_i^2}{2\sigma_n^2}\right) \tag{1}$$

in which σ_n is the standard deviation of the PDF. On the other hand, a non-Gaussian statistical model of the complex wavelet coefficients is used for original image wavelet coefficients as

$$p_x(X) = \frac{3}{2\pi\sigma^2} exp\left(-\frac{\sqrt{3}}{\sigma}\sqrt{X_r^2 + X_i^2 exp(K)}\right) \tag{2}$$

where σ is the standard deviation and K is a smoothing factor which controls the degree of shrinkage. The maximum likelihood estimate of X is

$$\hat{X}(y) = \underset{x}{argmax}\, p_{x|y}(x|y) \tag{3}$$

According to Bayes' theorem, (3) can be rewritten as

$$\hat{X}(y) = \underset{x}{argmax}[p_n(y - x)p_x(x)] \tag{4}$$

the maximum likelihood estimate of the despeckled image X can be obtained based on (1) and (2). After applying log-transformation on (4), we can obtain the estimator that makes the estimated wavelet coefficients' derivative equals zero, which is

$$\hat{X} = \left(\sqrt{Y_r^2 + Y_i^2} - \sqrt{3} exp(K)\frac{\sigma_n^2}{\sigma}\right)_+ \times Y/\sqrt{Y_r^2 + Y_i^2} \tag{5}$$

where the algebra operation $(f)_+$ is defined as $max(f, 0)$. The estimator defined in (5) is the traditional soft thresholding function, where the parameters σ and K are defined in (2), and σ_n is estimated as

$$\sigma_n = \text{median}\left(\left|Y_r^{45^\circ}\right|\right)/0.6745 \tag{6}$$

where $\left|Y_r^{45^\circ}\right|$ denotes the magnitudes of the wavelet coefficients in 45° direction subband at the first scale.

2.2. Proposed thresholding using quantum-inspired statistics

The conventional shrinking function (5) shrinks all the wavelet coefficients to the same degree. In view of this, an extra threshold ΔT, which exploits the statistics of coefficients in the DTCWT domain, is added to the traditional threshold to avoid modifying the coefficients of high frequency signal of images. The proposed estimator of X is defined as

$$\hat{X} = \left(\sqrt{Y_r^2 + Y_i^2} - \left(\sqrt{3} exp(K)\frac{\sigma_n^2}{\sigma} + \Delta T\right)\right)_+ \times Y/\sqrt{Y_r^2 + Y_i^2} \tag{7}$$

The image despeckling problem boils down to the shrinkage function defined in (7), where the key challenge is the choice of the adaptive threshold ΔT. For that, a modified quantum-inspired shrinkage function is proposed as follows.

The definition of the proposed threshold ΔT exploits both the inter-scale and intra-scale statistics of coefficients in the DTCWT domain, together with the principle of the quantum signal processing, as follows.

- The first motivation of the proposed threshold ΔT relies on the *inter-scale* correlation among the coefficients in the DTCWT domain. The magnitudes of high frequency signal coefficients have stronger correlation than those of noise coefficients across scales. To be more specific, if a parent coefficient of signal yields large modulus, then its children are much more likely to be large too. Products of coefficients and their parents are exploited to distinguish between signal and noise in the high frequency subbands as

$$C^{dir} = \left|Y_1^{dir}\right| \times \left|Y_2^{dir}\right| \tag{8}$$

where dir is the orientation of the subbands, $\left|Y_1^{dir}\right|$ and $\left|Y_2^{dir}\right|$ denote the modulus of DTCWT coefficients at first and second scale, respectively. According to the principle of quantum signal processing [14], the state of C^{dir} is expressed as

$$\left|C^{dir}\right\rangle = p_0\left|0\right\rangle + p_1\left|1\right\rangle \tag{9}$$

where p_0 and p_1 are probability amplitudes of ground states $\left|0\right\rangle$ and $\left|1\right\rangle$, and $|p_0|^2$, $|p_1|^2$ are the measurement probability of noise state $\left|0\right\rangle$ and signal state $\left|1\right\rangle$, respectively, $|p_0|^2 + |p_1|^2 = 1$. p_0 and p_1 are defined as

$$p_0 = \cos\left(NC^{dir} \times \frac{\pi}{2}\right) \tag{10}$$

$$p_1 = \sin\left(NC^{dir} \times \frac{\pi}{2}\right) \tag{11}$$

Then the matrix C^{dir} is normalized to obtain NC^{dir}, which reflects the power of high frequency signal. In the quantum computing, the quantum interference is presented by the unitary transformation U of quantum gate, which is formulated as

$$Input\_state = U \times Output\_state \tag{12}$$

In which, quantum rotation gate $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ is used as the unitary transformation U. Eq. (9) can be rewritten as

$$\left|C^{dir}(m, n)\right\rangle = \cos\left(NC^{dir} \times \frac{\pi}{2}\right)\left|0\right\rangle + \sin\left(NC^{dir} \times \frac{\pi}{2}\right)\left|1\right\rangle \tag{13}$$

- The second motivation of the proposed threshold ΔT is determined by the *intra-scale* variance of the coefficient and its neighborhood. This is also inspired by the principle of quantum signal processing. According to observer effect in quantum mechanics, the measurement (statistics of the coefficients studied in the DTCWT domain) of the system (noisy image) affects the end state of the system (the denoised image). If the system measurement (the observed noisy coefficient in the image) is less similar to the environment (neighboring coefficients in the DTCWT domain), the observer is more conscious (a stronger shrinkage of the wavelet coefficients needs to be imposed). Motivated by this, the entropy and coefficients of variation of the wavelet coefficients are used to determine the degree of the shrinkage in the proposed approach.

To summarize, based on the aforementioned motivations, the proposed adaptive threshold in (7), which exploits both the inter-scale correlation and the intra-scale variance among the DTCWT coefficients, is defined as

$$\Delta T = \cos^2\left(NC^{dir} \times \frac{\pi}{2}\right) \times \left|e_1^{dir} - e_2^{dir}\right| \times ncv \times num^{dir} \tag{14}$$

The first two terms of (14) exploit the inter-scale statistics in the DTCWT domain, and the first one is derived from the inter-scale DTCWT statistics using (13), the e_1 and e_2 are the entropy for