Automatica 65 (2016) 132–139

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Transverse function control of a motorboat[☆]

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ARTICLE INFO

Article history:
Received 16 February 2015
Received in revised form
1 August 2015
Accepted 9 November 2015
Available online 8 December 2015

Keywords:
Transverse function control approach
Marine vehicles
Nonlinear feedback control

ABSTRACT

The Transverse Function (TF) control approach is applied to the control of a generic motorboat endowed with a surge force along the stern–bow direction and a torque actuation to modify the boat's orientation. With respect to more conventional methods this approach allows for uniform practical stabilization of any smooth reference pose (i.e. position+orientation) trajectory. This includes fixed-poses in the absence of a sea-current, whose asymptotic stabilization cannot be achieved by classical feedback controllers, and non-feasible pose trajectories, i.e. trajectories that are not solutions to the system's motion equations and thus cannot be stabilized asymptotically.

1. Introduction

A typical motorboat is an underactuated dynamic system with three main degrees of freedom (surge, sway and yaw) and only two control inputs (a surge force and a control torque for yaw modification). Associated motion control problems are commonly classified into three sub-categories, namely *path following*, *position tracking*, and *pose (position + orientation) tracking* including set-point stabilization. Path following refers to the problem of using orientation control in order to zero the lateral distance between the vehicle and a pre-specified geometric path, given a forward (non-zero) velocity. This problem, first addressed in the case of nonholonomic mobile robots (Samson, 1992), was subsequently extended to underactuated vehicles (Encarnaçao & Pascoal, 2000; Skjetne, Fossen, & Kokotović, 2004). Its adaptation to marine vehicles led to solutions including the hierarchical controllers derived in Fossen and Pettersen (2014) and the cascaded-systems approach in the presence of unknown ocean currents developed in Aicardi et al. (2001) and Moe, Caharija, Pettersen, and Schjolberg (2014). The present paper addresses position and pose tracking. Position tracking refers to the problem of stabilizing asymptotically a time-parametrized reference trajectory for a point located on the vehicle's main body. Due to the system's dynamics nonlinearities the determination, in the form of a time-parametrized function, of the vehicle's orientation along such a trajectory is not possible except for very specific trajectories (along straight lines and circles, for instance) commonly referred to as *trimming* trajectories. Along these trajectories a linear approximation of the system's dynamics can be explicited. Provided that the forward reference velocity does not vanish, the obtained linearized system is controllable and the tracking problem can be addressed with classical linear control techniques. To enlarge the stability domain and address more general reference position trajectories nonlinear control solutions have been studied. A way to passively stabilize the boat's orientation along a reference position trajectory with non-zero surge velocity consists in controlling the position of a *Virtual Reference Point* (VRP) located at the bow or ahead of the ship, as proposed in Berge, Ohtsu, and Fossen (1999). Other contributions take the option of complementing the reference position with a reference orientation that complies with the boat's motion equations so as to asymptotically stabilize the resulting *feasible* reference pose trajectory (Jiang, 2002; Lefeber, Pettersen, & Nijmeijer, 2003; Pettersen & Nijmeijer, 1998). A remaining difficulty with this approach is the calculation of the reference orientation, a difficulty amplified by the imprecise knowledge of the environmental forces exerted on the boat. In this latter respect more or less simplified modelling equations are used. For instance, in the above cited references, controllers are derived from a model that does not account for squared velocity dependent hydrodynamic forces and torques. In Silvestre, Pascoal, and Kaminer (2002) a more sophisticated model is considered and a gain scheduling control approach is applied. In Aguiar and Hespanha (2007) baskstepping and Lyapunov-based control solutions, derived for a class of underactuated systems,

[☆] This work is supported by FP7-ICT-2013-10 of the EU 7th Framework Programme, through the project DRONIC. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Jun-ichi Imura under the direction of Editor Toshiharu Sugie.

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http://dx.doi.org/10.1016/j.automatica.2015.11.040
0005-1098/

achieve ultimate boundedness of the position tracking errors without assuming non-zero surge velocity. The proposed control design is also applied to path following and is complemented with a supervisory adaptation policy to handle model uncertainties. As for set-point stabilization, the problem consists in stabilizing either asymptotically or practically a fixed desired pose in the absence of environmental perturbations (sea-current or wind). It is more challenging than the position tracking problem in the sense that classical control design methodologies do not provide solutions. The first reason is that the linearization of the system dynamics at the desired pose is not controllable. An even more troublesome fact is that the system does not satisfy Brockett's necessary condition, Brockett (1983) for the existence of a time-invariant feedback asymptotic stabilizer, either linear or nonlinear. Solutions circumventing this difficulty were first developed for nonholonomic mobile robots and consisted in working out nonlinear time-varying stabilizers (Samson, 1992). Adaptations of this type of control to the practical stabilization of a motorboat are presented in Lefeber et al. (2003) and Pettersen and Fossen (2000). More recently, this type of problem has given rise to the development of the so-called *Transverse Function control approach* that allows for the practical stabilization of *any* pose trajectory, either feasible or non-feasible (Morin & Samson, 2003). This approach thus applies directly to position tracking and set-point stabilization, with the complementary asset of providing solutions to a fourth control problem seldom addressed in the control literature, namely the practical stabilization of any pose trajectory, either feasible (i.e. complying with the system's motion equations) or non-feasible. In the case of a motorboat, a simple example of a non-feasible pose trajectory is a fixed desired pose whose orientation is not aligned with the direction of the sea-current. The present paper may be viewed as an extension of the hovercraft control solution example pointed out in Morin and Samson (2005). More precisely, it explains in more detail how the TF approach applies to the design of motorboat controllers, with environmental forces taken into account explicitly, and it provides proofs of complementary stability and convergence results of interest for practical purposes. The attentive reader will also find out that the proposed control design is much related to those described in Behal, Dawson, Dixon, and Fang (2002) and Do (2010). The first of these references aims at achieving the uniform practical stabilization of feasible pose trajectories via the introduction of a dynamic oscillator in the control law. The TF approach provides an important clarification as for the existence of this dynamic oscillator that we relate to the existence of specific transverse functions associated with the control vector fields involved in the system's dynamic equations and the property of controllability that they infer. The motorboat model that we consider is also more realistic because it includes added mass effects, as well as sea-currents. As for the second reference, which evokes the TF function approach in the introduction and explicitly addresses the case of non-feasible trajectories, it does not clearly account for the role of this approach in the control design, nor for the origin and determination of the transverse function that is used. In addition, the result of uniform ultimate boundedness of the tracking errors reported in both references is here complemented with a study of the asymptotic behaviour of the controlled system, with exponential stability of equilibria proven in two nominal operational conditions, namely position tracking along a straight path and stabilization of a fixed-pose whose orientation is given by the *a priori* unknown direction of the sea-current.

The paper is organized as follows. Section 2 provides some notation, describes the system modelling and explains how transverse functions that can be used to control the system are derived. Control design is presented in Section 3 with complementary explanations, along with three propositions that specify the properties of the proposed controllers in terms of stability and convergence. In Section 4, simulation results illustrate the performance of the proposed controllers in the presence of an unknown sea-current. Concluding remarks are reported in Section 5.

2. Modelling equations

The generic motorboat here considered is sketched in Fig. 1. The following notation is used.

- C is the boat's centre of mass (CoM) and $\mathcal{I} = \{O; \boldsymbol{i}_0, \boldsymbol{j}_0\}$ is an inertial frame.
- The boat's orientation with respect to (w.r.t.) the inertial frame is given by the angle θ.
- $\mathcal{B} = \{C; \boldsymbol{i}, \boldsymbol{j}\}$ is a body-fixed frame, with unit vectors parallel to the boat's principal axes.
- The intensities of the propulsion force and of the actuation torque used to control the boat's position and orientation are denoted respectively as $f = (f_1 + f_2)$ and $\tau = 0.5(f_1 - f_2)d$, with d the distance between the two propulsors.
- The boat's proper mass and angular inertia about the vertical axis are denoted as m and I_z respectively. The total mass matrix including added masses Δm_i, Δm_j, and ΔI_z along the boat's principal axes, as well as off-diagonal coupling yaw/sway coefficients m_{c1} and m_{c2}, is denoted as

$$M_a = \begin{bmatrix} \begin{matrix} & M & \end{matrix} & \begin{matrix} 0 \\ m_{c1} \end{matrix} \\ \begin{matrix} 0 & m_{c2} \end{matrix} & I \end{bmatrix}$$

  with $M = \mathrm{diag}\{m_i, m_j\}$, $m_i = m + \Delta m_i$, $m_j = m + \Delta m_j$, and $I = I_z + \Delta I_z$. Due to the profiled shape of a standard ship one can safely assume that $m_i < m_j$.
- $\dot{x}_c$ and v_c are the vector of coordinates of the sea-current velocity in $\mathcal{I}$ and $\mathcal{B}$ respectively. In this work $\dot{x}_c$ is assumed constant.
- $v = (v_i, v_j)^T$ (resp. $v_h = v - v_c$) is the vector of coordinates in $\mathcal{B}$ of the CoM's velocity w.r.t. $\mathcal{I}$ (resp. w.r.t. the water surface).
- $\bar{C}$, of coordinates $x = (x_1, x_2)^T$ in $\mathcal{I}$, is the point on the boat's longitudinal axis located at the (typically small) distance $\frac{m_{c1}}{m_j}$ from the CoM, i.e. $C\bar{C} = \frac{m_{c1}}{m_j}\boldsymbol{i}$.
- $\bar{v} = (\bar{v}_i, \bar{v}_j)^T$ (resp. $\bar{v}_h = \bar{v} - v_c$) is the vector of coordinates in $\mathcal{B}$ of the velocity of $\bar{C}$ w.r.t $\mathcal{I}$ (resp. w.r.t. the water surface). It is related to the CoM's velocity according to $\bar{v}_i = v_i$ and $\bar{v}_j = v_j + \frac{m_{c1}}{m_j}\omega$ with $\omega = \dot{\theta}$ the boat's angular velocity.
- The boat evolves in a fluid (water) which exerts on it reaction *manoeuvring* forces that are responsible for longitudinal drag and lateral lift. The vector of coordinates of the resultant of linear viscous forces, expressed in $\mathcal{B}$, is classically modelled by $f_l = -C_l\bar{v}_h$ with $C_l = \mathrm{diag}\{c_{l,i}, c_{l,j}\}$, and $c_{l,i}$ and $c_{l,j}$ denoting positive coefficients. Similarly, and following a simplified expression based on the theory of slender bodies immersed in a fluid, the resultant of reaction forces involving quadratic terms is modelled by $f_q = -|\bar{v}_h|C_q\bar{v}_h$ with $C_q = \mathrm{diag}\{c_{q,i}, c_{q,j}\}$, and $c_{q,i}$ and $c_{q,j}$ denoting positive coefficients. These forces are *dissipative* in the sense that $\bar{v}_h^T f_l \leq 0$ and $\bar{v}_h^T f_q \leq 0$, so that they tend to reduce the boat's kinetic energy. Linear viscous effects are rapidly dominated by quadratic terms when $|\bar{v}_h|$ is not very small. Other environmental forces exerted on the boat (wind and waves forces, in particular) are neglected in this work.
- $R(\theta)$ is the rotation matrix in the plane of angle θ, and $0_{m,n}$ is the null matrix with m lines and n columns.

The following model of the boat's motion equations, used here for simulation and control design purposes, relies on a model derived in Fossen (1994) for surface ships.

$$M_a\nu + C(\nu)\nu = U + U_{hydro} \tag{1}$$

with $\nu = (v_h^T, \omega)^T$, $U = (f, 0, \tau)^T$ the vector regrouping the thrust force and the actuation torque that control the boat's motion, $U_{hydro} = ((f_l + f_q)^T, \tau_l + \tau_q)^T$ with τ_l and τ_q denoting linear and