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Brief paper

Output feedback control for uncertain nonlinear systems with input quantization ☆

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ABSTRACT

In this paper, we propose a new adaptive output-feedback tracking control scheme for a class of uncertain nonlinear systems with input quantized by a newly-proposed quantizer. This quantizer is a combination of a logarithmic (or a hysteresis) quantizer and a uniform quantizer, and it has the advantages of both logarithmic and uniform quantizers in ensuring reducible communication expenses and acceptable quantization errors for better system performances. Compared with existing results in adaptive control, the proposed scheme provides a way to relax certain restrictive conditions, in addition to solving the problem of adaptive output-feedback control with input quantization. It is shown that the designed adaptive controller ensures global boundedness of all the signals in the closed-loop system and enables the tracking error to exponentially converge towards a compact set which is adjustable.

1. Introduction

Quantization can be seen as a map from continuous signals to discrete finite sets. Recently, a great deal of attention has been paid to the study of quantization problems because of its theoretical and practical importance in modern engineering such as hybrid systems, discrete-event systems, digital control systems and control with information constraints, see Ishii and Francis (2002), Lunze, Nixdorf, and Schroder (1999), Tatikonda and Mitter (2004) and Wong and Brockett (1999). For systems with information quantization, a continuous control input or state is quantized by a quantizer which results in an inevitable quantization error. Thus the effects of the quantization error to the performances of the closed system, especially system stability, need to be carefully and clearly studied.

Up to now, quantized control for systems with uncertainties has been mainly considered based on robust approaches. Elia and Mitter (2001) studied the coarsest quantizer for single-input–single-output (SISO) linear systems and proved that the coarsest quantizer should follow a logarithmic law. This result was extended to nonlinear systems in Liu and Elia (2004) and the ideas of using robust control Lyapunov function to get robust quantized controllers were developed. In Fu and Xie (2005), the coarsest quantizer was taken as a sector bound uncertainty to study quantized control with quadratic stability and H_2 and H_∞ performance criteria were developed. The quantized control problem with input and output quantization was also considered in Coutinho, Fu, and De Souza (2010). In Phat, Jiang, Savkin, and Petersen (2004), robust stabilization problem for linear discrete-time systems via a limited communication channel was addressed. More results on robust quantized control could be found in De Persis (2009), De Persis and Mazenc (2010), Liberzon and Hespanha (2005), Liu, Jiang, and Hill (2012a), Nair and Evans (2004), Xing, Wen, Su, Liu, and Cai (2015) and Yang, Hong, Jiang, and Wang (2009).

Besides robust control, adaptive control is another important approach to deal with system uncertainties as it can provide on-line estimation of unknown parameters. In Hayakawa, Ishii, and Ysumura (2009b) and Sun, Hovakimyan, and Basar (2010), adaptive control with quantized input signals for linear systems is reported. In Hayakawa, Ishii, and Tsumura (2009a), adaptive quantized control for nonlinear systems is considered. It is noted that a sector bounded property on quantization errors is used to establish the results. However, the resulting stability conditions depend on control signals, which is hardly checkable in advance as control signals are unavailable before the designed controller is

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put in operation. In Zhou, Wen, and Yang (2014), a backstepping-based adaptive control scheme is presented for a class of strict-feedback uncertain systems with input quantization. Although the proposed design method can avoid stability conditions depending on control input and make the stabilization error arbitrarily small, the nonlinearities of the system to be controlled should satisfy global Lipschitz conditions with known Lipschitz constants and their partial derivatives to be bounded. Recently in Xing, Wen, Su, Cai, and Wang (2015), such a restriction was relaxed by using the sector bound property of the logarithmic quantizers. However, the unknown parameters are only contained in the last nonlinear function of the system. In addition, to the best of our knowledge, all the existing results in adaptive quantized control are based on state feedback. Therefore, it is also important to investigate output-feedback quantized control, besides relaxing the conditions mentioned above.

It should be noted that all the mentioned results above consider either logarithmic quantizer or the extended hysteresis quantizer. The main reason is that in network control, the logarithmic quantizer (or hysteresis quantizer) can largely reduce communication burden when the amplitude of the input signal is large and decrease quantization error when the amplitude is small, due to its varying quantization levels compared to a uniform quantizer. However, such quantizers have certain disadvantages. Their structure determines that the quantization level becomes coarser as the magnitude of the signal gets bigger (away from the origin). Inevitably, excessively large amplitude signals will result in very large quantization errors, which may be unbounded. Actually this is unnecessary just for decreasing communication cost, since such large quantization errors may degrade the performance of the system or even result in instability. For example in tracking control, if the reference signal requires the control signal to be big, the resulting large quantization errors would result in large tracking errors.

In this paper, we solve the problems mentioned above by proposing a new adaptive output-feedback control scheme for a class of uncertain nonlinear systems with input quantization. Firstly, we propose a new quantizer which is a combination of a logarithmic (or hysteresis) quantizer and a uniform quantizer. More specifically, when the magnitude of the control signal is smaller than a threshold specified by designer, the proposed quantizer is a logarithmic (or hysteresis) quantizer; on the other hand, when the magnitude of the control signal is larger than the threshold, the quantizer becomes a uniform quantizer. In this way, the coarseness of the quantization level remains unchanged after the logarithmic quantizer is replaced by a uniform quantizer. Clearly such a quantizer has the advantages of both logarithmic and uniform quantizers in ensuring reducible communication expenses and acceptable quantization errors for better system performances. This is illustrated in the example in Section 2. Then based on this quantizer, a method to design the state observers is given. Compared with the currently available design schemes without input quantization, the state estimation error can only be ensured to converge to a bounded set independent of the control signal u, instead of zero. With our proposed adaptive controller, we successfully compress the effects of the quantization error on the final tracking error into a bounded set that can be decreased by choosing suitable design parameters. With our newly proposed scheme and quantizer, we are able to remove the assumptions imposed in Zhou et al. (2014) that the nonlinearities of the system to be controlled should satisfy global Lipschitz conditions with known Lipschitz constants and their partial derivatives to be bounded. In addition, in contrast to Hayakawa et al. (2009b,a), the established stability conditions do not depend on control signals, either. It is shown that the designed adaptive controller in this paper ensures global boundedness of all the signals in the closed-loop system and enables the tracking error to exponentially converge towards a compact set which is adjustable.

The remaining part of this paper is organized as follows. Section 2 first describes the existing sector-bounded quantizers, then a new quantizer is proposed to overcome the disadvantage of the existing logarithmic (or hysteresis) quantizer. An example is given to show the effectiveness of the newly-proposed quantizer. Section 3 describes the system to be controlled and also gives the control objective. Section 4 presents the adaptive controller designed based on backstepping technique and analyses the stability and tracking performance of the closed-loop system. A simulation example is given in Section 5 to illustrate the control scheme and verify the established theoretical results. Finally Section 6 concludes the paper.

2. Quantizer description

2.1. Sector-bounded quantizer

Let $\Delta_q = q(u) - u$, denoting the quantization error. A sector-bounded quantizer is a quantizer with its quantization error satisfying the following sector bound property introduced in Liu, Jiang, and Hill (2012b).

$$|\Delta_q| \le \delta|u| + (1-\delta)d, \tag{1}$$

where $0 \le \delta < 1$ and d are known parameters of quantizers to be described below. Based on Fu and Xie (2005) and Liu et al. (2012b), most practical quantizers belong to such a class with logarithmic quantizer, hysteresis quantizer, and a uniform quantizer being typical examples.

2.1.1. Logarithmic quantizer

In this paper, the logarithmic quantizer described in Yang et al. (2009) is considered. It is modelled as

$$q_l(u) = \begin{cases} u_i & \dfrac{u_i}{1+\delta} < u \le \dfrac{u_i}{1-\delta} \\ 0 & 0 \le u < \dfrac{d}{1+\delta} \\ -q(-u) & u < 0 \end{cases} \tag{2}$$

where $u_i = \varrho^{(1-i)}d$ with $i = 1, 2 \dots$. The parameters $d > 0$ and $0 < \varrho < 1$, $\delta = \frac{1-\varrho}{1+\varrho}$ determine the quantization density of $q(u)$. $q(u)$ is in the set $U = \{0, \pm u_i\}$. $u_{min} = \frac{d}{1+\delta}$ determines the size of the deadzone for $q(u)$. For this quantizer, some remarks about its range, number of quantization levels and quantization density are given in De Persis and Mazenc (2010).

2.1.2. Hysteresis quantizer

In this paper, the hysteresis quantizer employed is described in the following form, similar to Zhou et al. (2014).

$$q_h(u(t)) = \begin{cases} u_i \mathrm{sgn}(u) & \dfrac{u_i}{1+\delta} < |u| \le u_i,\ \dot{u} < 0, \text{ or} \\ & u_i < |u| \le \dfrac{u_i}{1-\delta},\ \dot{u} > 0 \\ u_i(1+\delta)\mathrm{sgn}(u) & u_i < |u| \le \dfrac{u_i}{1-\delta},\ \dot{u} < 0, \text{ or} \\ & \dfrac{u_i}{1-\delta} < |u| \le \dfrac{u_i(1+\delta)}{1-\delta},\ \dot{u} > 0 \\ 0 & 0 \le |u| < \dfrac{d}{1+\delta},\ \dot{u} < 0, \text{ or} \\ & \dfrac{d}{1+\delta} \le |u| \le d,\ \dot{u} > 0 \\ q(u(t^-)) & \dot{u} = 0 \end{cases} \tag{3}$$