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Simple homogeneous sliding-mode controller[☆]

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ABSTRACT

High-order sliding mode (HOSM) control is known to provide for finite-time-exact output regulation of uncertain systems with known relative degrees. Yet the corresponding universal HOSM controllers are typically constructed by special recursive procedures and have complicated form. We propose two new families of homogeneous HOSM controllers of a very simple form. Lyapunov functions are provided for a significant part of the first-family controllers. The second family consists of quasi-continuous controllers, which can be done arbitrarily smooth everywhere outside of the HOSM manifold. A regularization procedure ensures high-accuracy output regulation by means of control with required smoothness level. Output-feedback controllers are constructed. Controllers of the orders 3–5 are demonstrated.

1. Introduction

Control of uncertain nonlinear systems is a hot topic of the modern control theory, and sliding mode (SM) control (SMC) remains one of the most effective tools to handle such uncertain systems (Edwards & Spurgeon, 1998; Sabanovic, 2011; Utkin, 1992).

The SMC idea is to keep properly chosen functions (so-called sliding variables) at zero, effectively reducing the system uncertainty. The control is chosen discontinuous in order to dominate the uncertainties. The corresponding motion is said to be in SM and features a high, theoretically infinite control switching frequency. Unfortunately, the resulting system vibration can be destructive (the chattering effect (Fridman, 2003; Sabanovic, 2011; Utkin, 1992)). Another restriction is that the control usually needs to appear explicitly already in the first total time derivative of sliding variables (Edwards & Spurgeon, 1998; Utkin, 1992). High-order SMs (HOSMs) have been introduced to cope with these obstacles.

Let a dynamic system be understood in the Filippov sense (Filippov, 1988) and $s_1, \dots, s_m$ be its scalar outputs. Suppose that the system is closed by some possibly-dynamical discontinuous feedback, so that the successive total time derivatives $s_i, \dot{s}_i, \dots, s_i^{(n_i-1)}$, $i = 1, \dots, m$, are continuous functions of the closed-system state-space variables; and the n-sliding set $s_i = \cdots = s_i^{(n_i-1)} = 0$, $i = 1, \dots, m$, is a non-empty integral set, $n = (n_1, \dots, n_m)$. Then the motion on the set is said to be in the n-sliding (nth-order sliding) mode (n-SM). The vector $n = (n_1, \dots, n_m)$ is called the sliding order (Levant, 1993, 2003). The standard sliding modes (Edwards & Spurgeon, 1998; Utkin, 1992) are of the first order (s_i are continuous, and $\dot{s}_i$ are discontinuous, $n = (1, \dots, 1)$).

The relative degree of the sliding variable (i.e. the minimal order of its total derivative explicitly containing control (Isidori, 1989)) has become the main parameter of the HOSM application. HOSMs (Bartolini, Pisano, Punta, & Usai, 2003; Levant, 1993, 2003, 2005a, 2010; Plestan, Glumineau, & Laghrouche, 2008; Shtessel & Shkolnikov, 2003) are applicable for any relative degrees. They hide the switching in the higher derivatives of the sliding variables, while preserving the finite-time transient to the SM and improving the SM accuracy in the presence of switching imperfections, noises and disturbances.

Introducing integrators in the control channel, one artificially increases the relative degree, produces arbitrarily smooth control and simultaneously removes the dangerous high-energy chattering (Bartolini, Ferrara, & Usai, 1998; Bartolini et al., 2003; Levant, 2010). Such controllers directly solve the control problem, if

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the sliding variable is a tracking error. Another important application of SMs is robust finite-time-exact differentiation and observation (Bartolini, Pisano, & Usai, 2000; Bejarano & Fridman, 2010; Levant, 2003; Shtessel & Shkolnikov, 2003; Utkin, 1992; Yu & Xu, 1996). These differentiators are to provide the necessary information for the output-feedback application of the HOSM controllers. HOSM control (HOSMC) has been successfully applied to numerous real control systems, such as wheel slip control (Amodeo, Ferrara, Terzaghi, & Vecchio, 2010), mobile robot (Ferrara & Rubagotti, 2008), aircraft control (Shtessel & Shkolnikov, 2003), etc.

Most of the aforementioned HOSM controllers are usually obtained by the homogeneity analysis and design (Bernuau, Efimov, Perruquetti, & Polyakov, 2014; Levant, 2005a; Orlov, 2005). The controllers are mostly provided by long complicated formulas (Harmouche, Laghrouche, & Chitour, 2012; Levant, 2003, 2005b). A 5-SM controller formula at least takes a few lines. This is also often true in the particular case of the finite-time integrator-chain stabilization (Hong, 2002). For example, a 3-SM controller in Harmouche et al. (2012) is based on Hong (2002) and takes 3 lines. A simple homogeneous HOSM controller family still lacks.

Being constructive, the HOSM convergence proofs involve the recursive choice of sufficiently-large control parameters (Levant, 2003, 2005a,b). The Lyapunov analysis of HOSMs has been recently performed in Cruz-Zavala and Moreno (2014); Harmouche et al. (2012); Orlov, Aoustin, and Chevallereau (2011); Pico, Pico-Marco, Vignoni, and De Battista (2013); Polyakov (2012); Polyakov, Efimov, and Perruquetti (2015); Polyakov and Poznyak (2012). The Lyapunov method provides for explicit relations between the design parameters and allows the direct evaluation of the SM accuracy in the presence of various perturbations. Unfortunately, such estimations are mostly very conservative, and direct simulation often provides for much better results.

Two new HOSM controller families for uncertain systems of arbitrary relative degrees are developed in this paper. The main advantage of the new HOSM controllers is their ultimate simplicity. One does not need anymore to use complicated recursive procedures in order to develop the controller form for arbitrarily-high relative degrees.

The first-family controllers can be considered a generalization of terminal SM controllers (Feng, Yu, & Man, 2002; Man, Paplinski, & Wu, 1994) to high relative degrees, or generalization of continuous controllers (Bhat & Bernstein, 2007). We call them relay polynomial HOSM controllers. The derived controllers feature large freedom of fractional powers' choice. The proof is based on homogeneity, but also Lyapunov functions are presented in many cases.

The quasi-continuous versions of these controllers, continuous outside of the HOSM manifold, constitute the second family, called quasi-continuous polynomial HOSM controllers. They feature significantly reduced chattering. For the first time quasi-continuous controllers can be made arbitrarily smooth outside of the HOSM manifold. A regularization procedure is proposed to maintain approximate HOSMs by means of control with a prescribed smoothness level. The proofs are based on the homogeneity of controllers.

2. The problem statement and some new controllers

Consider a single-input single-output system of the form

$$\dot{x} = A(t,x) + B(t,x)\,u, \qquad s = s(t,x), \tag{1}$$

where $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}$ is the control, $s : \mathbb{R}^{n_x+1} \to \mathbb{R}$ and A, B are unknown smooth functions. The dimension value n_x is not used in the sequel. All differential equations are understood in the Filippov sense (Filippov, 1988) in order to allow discontinuous controls. The control task is to make s vanish in finite time and to keep it at zero afterwards.

The relative degree n of system (1) is assumed to be constant and known. It means (Isidori, 1989) that for the first time the control explicitly appears in the nth total time derivative of s, i.e.

$$s^{(n)} = h(t,x) + g(t,x)u, \tag{2}$$

where $h(t,x)$, $g(t,x)$ are some *unknown* smooth functions, $g \neq 0$. Note that no continuous feedback solves the problem because of the uncertainty of h, g, also such classic methods as back-stepping are not applicable.

According to the standard HOSM control approach (Levant, 2003), let

$$0 < K_m \leq g(t,x) \leq K_M, \qquad h(t,x)| \leq C, \tag{3}$$

for some $K_m, K_M, C > 0$. Also assume that solutions of (2) are infinitely extendible in time for any Lebesgue-measurable bounded control $u(t)$.

In practice the operational region of any plant is always bounded. In that case conditions (3) hold locally, and the results can be respectively reformulated (Levant, 2003).

Introduce the notation: $\forall x \neq 0\ \lfloor x \rceil^{\gamma} = |x|^{\gamma}\operatorname{sign} x$; $\forall \gamma > 0\ \lfloor 0 \rceil^{\gamma} = 0$; $\lfloor x \rceil^{0} = \operatorname{sign} x$.

The controls proposed in this paper have a very simple form. Choose any $a > 0$ and introduce the *relay polynomial* n-SM controller

$$u = -\alpha \operatorname{sign}\left(\lfloor s^{(n-1)} \rceil^{\frac{a}{1}} + \tilde{\beta}_{n-2}\lfloor s^{(n-2)} \rceil^{\frac{a}{2}} + \cdots + \tilde{\beta}_0 \lfloor s \rceil^{\frac{a}{n}}\right), \tag{4}$$

and the *quasi-continuous polynomial* n-SM controller

$$u = -\alpha \frac{\lfloor s^{(n-1)} \rceil^{\frac{a}{1}} + \tilde{\beta}_{n-2}\lfloor s^{(n-2)} \rceil^{\frac{a}{2}} + \cdots + \tilde{\beta}_0 \lfloor s \rceil^{\frac{a}{n}}}{|s^{(n-1)}|^{\frac{a}{1}} + \tilde{\beta}_{n-2}|s^{(n-2)}|^{\frac{a}{2}} + \cdots + \tilde{\beta}_0 |s|^{\frac{a}{n}}}. \tag{5}$$

Denote $\vec{s}_j = (s, \dot{s}, \ldots, s^{(j)})$ for any natural j. Note that the absolute value of the nominator of (5) does not exceed the denominator. Thus, the right-hand side of (5) is formally not defined at the n-sliding set $\vec{s}_{n-1} = 0$. Since $\vec{s}_{n-1} = 0$ is a set of the measure 0, the values of u on it do not affect the system behavior (Filippov, 1988), and in implementation some value from the range $[-\alpha, \alpha]$ is prescribed to u.

Provided the coefficients $\tilde{\beta}_j > 0$ are properly chosen, both controllers solve the stated problem with sufficiently large α. If $n = 2$, controller (4) becomes the terminal SMC (Levant, 1993; Man et al., 1994) for $a = 1$, and the nonsingular terminal SMC (Feng et al., 2002) for $a = 2$.

While the first controller is a "usual" discontinuous SM controller, the second one is *quasi-continuous* (Levant, 2005b), i.e. the control is only discontinuous, if the system is in the n-SM $\vec{s}_{n-1} = 0$. While in the SM the control reveals the typical SMC chattering. Nevertheless, while not in the n-SM, it becomes locally Lipschitz for $a \geq n$, and even k times continuously differentiable if $a > kn$, $k = 1, 2, \ldots$.

Quasi-continuous controllers feature much less chattering, since in practice the above n-SM equalities $\vec{s}_{n-1} = 0$ are never observed due to various switching imperfections, noises and disturbances. Thus the control remains continuous all the time. Note that the denominator of (5) actually measures the n-SM accuracy. The worse the SM accuracy the further the denominator from zero, which results in slower control changing (also see Sections 4.1 and 5).

In the sequel we prove the above statements and propose additional controllers, construct a Lyapunov function for controller (4) with $a \geq n$, provide numeric and analytic methods for coefficient adjustment, and propose a regularization procedure to solve the stated problem approximately by control featuring any needed smoothness level.

3. Homogeneity and sliding mode control

Obviously, (2) and (3) imply the differential inclusion

$$s^{(n)} \in [-C, C] + [K_m, K_M]u, \tag{6}$$