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A robustly stabilizing model predictive control strategy of stable and unstable processes☆

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ABSTRACT

This paper deals with the development of a robust model predictive control strategy with guarantee of stability, applicable to the stable and unstable processes. The model uncertainty is assumed to be described by a discrete set of linear models (multi-plant uncertainty), and the robustness is achieved by assembling cost-contracting constraints for all the possible models in the uncertainty domain. On the basis of a suitable state-space model description, an offset free control law is obtained by means of a one-step optimization formulation. The usefulness of the method proposed here is illustrated with control simulations of an unstable reactor system taken from the literature.

1. Introduction

Open-loop unstable processes, such as heat and mass exchange networks, recycle and exothermic reaction, are commonly found in the process industry. In particular, when an unstable reactor system (e.g. a continuous stirred tank reactor, CSTR) is required to be controlled, a greatly desired property of the controller is the guarantee of closed-loop system stability. Furthermore, within an economic standpoint, it is well known that CSTR schemes often must operate at open-loop unstable operating regions in order to maximize its profitability (Biagiola & Figueroa, 2004). On this matter, more sophisticated control strategies, as model predictive control (MPC), should then be designed to drive in a stable manner such reactor systems from one unstable and profitable steady-state to another. Particularly, strategies as robust model predictive control (RMPC) seem fairly appealing and justifiable to successfully pursue both operational and economic goals of unstable reactor systems.

The search for robustly stabilizing controllers has received plentiful attention in the control literature, so that the several approaches that have been developed so far can be grouped into four main categories, and their essential features are summarized in Table 1.

The first approach aims to achieve robust stability by minimizing the control objective function for the worst possible realization of the uncertainty description. Among the available studies on this topic, the main research work that makes use of such an approach is the one that was reported by Lee and Yu (1997). They proposed robustly stabilizing infinite-horizon MPC (IHMPC) algorithms for open-loop stable systems with norm-bounded time-invariant or time-varying parametric uncertainties. In their method, it is also shown that the control cost is a Lyapunov function to the dynamic system when the objective is calculated considering all possible combinations of system models along the control horizon. Following the same line, Lee and Cooley (2000) extended the method to systems with integrating poles. There, in order to turn the infinite-horizon control cost bounded, the integrating states of the system at the end of the control horizon are forced to be minimized through a new optimization problem that considers the worst-case cost. Then, the solution of this proposed optimization problem is properly transferred to the original min–max control optimization problem as an equality constraint, which guarantees the convergence properties of the controller. With the aim of producing a highly structured convex optimization problem for the Lee and Cooley's control formulation, Ralhan and Badgwell (2000) added cost contracting constraints associated with the uncertainty description to the original control problem, allowing for more efficient numerical solutions. A major limitation behind the

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Table 1
General characteristics of the available RMPC methods.

Category	Model uncertainty	Prediction horizon	Pursuit economic targets	System dynamics
Min–Max control formulations	Polytopic	Infinite	No	Stable and integrating
Stabilizing state-feedback schemes	Polytopic, bounded disturbance	Finite and infinite	Yes	Applicable to any dynamic
Control Lyapunov functions	Bounded disturbance	Finite	No	Nonlinear without time delays
Cost-contracting constraints	Polytopic and multi-plant	Infinite	Yes	Stable, integrating, time delays

aforementioned approaches is the assumption that the plant steady-state is always known (set-point at the system origin), i.e. the min–max predictive controllers are simply regulators, and consequently as soon as a disturbance enters the process or the set-point changes to values away from the origin, the control law cannot eliminate the resulting offset. This shortcoming is overcome in the work by Odloak (2004) that proposed a min–max robust MPC to the general case where the system equilibrium point is unknown (output tracking case).

Even so the min–max control theories suffer from serious limitations, namely: the main barrier is the fact that the application of these strategies results in a very conservative control performance under the conditions in which there are guarantee of stability; other disadvantage of the min–max predictive control formulations is its computational intensity since their resulting optimization problems are very expensive to solve on-line. Therefore, from the point of view of the process industry such a control approach can become infeasible for practical implementation purposes (Ralhan & Badgwell, 2000).

One of the most heavily studied robust stability methods is the one based on the state-feedback scheme. The core of these control strategies consists of incorporating the state feedback control law into the RMPC problem formulation. The state-feedback RMPC schemes involve either adding contracting constraints to all possible system states along the infinite horizon, or applying the dual-mode control structure, which is composed of two distinct control modes: in the first mode, the RMPC drives the uncertain system over a finite horizon from the initial state towards a point inside a given robust positively invariant (RPI) set, while in the second control mode a local controller having the form of a state-feedback takes over and holds the state within this set for all admissible uncertainties.

Following the state-contracting based approach, the first RMPC appeared in the work by Kothare, Balakrishnan, and Morari (1996), where the authors proposed a robust MPC strategy for polytopic uncertain models, which is based on an infinite-horizon linear quadratic regulator (LQR). The controller was extended to the constrained case, through the inclusion of conservative linear matrix inequality (LMI) constraints on the inputs and outputs. Even though, with this method, stability can be achieved for stable, unstable or integrating systems with some classes of model uncertainty, it yields a quite conservative control law due to the fact that the control actions are obtained by a fixed state-feedback gain throughout the infinite prediction horizon. Besides, it may cause feasibility problems because of the hard state contraction constraints. There have been attempts in the literature to improve the feasibility issues of the method, by applying a parameter-dependent Lyapunov function (Cuzzola, Geromel, & Morari, 2002; Mao, 2003), by introducing relaxation matrices in the robust control optimization problem (Lee, Won, & Park, 2008), by providing LMIs as approximations to the state contracting constraints (Jia, Krogh, & Stursberg, 2005), by re-formulating the control problem as a quasi-worst-case optimization problem (Lu & Arkun, 2000), or by inserting a norm-bounded disturbance in the system model and so the stability is successfully achieved by quadratic boundedness (Ding & Zou, 2014).

With regard to the dual-mode prediction methods, its major idea consists of adding a set of free control inputs along a finite horizon, which in turn would be considered as decision variables in the optimization problem, and subsequently one applies the state-feedback control law inside the terminal robustly stabilizable set. Within this approach, an intensive research to synthesize RMPC controllers is focused on uncertain systems described by polytopic uncertainty, e.g. Casavola, Giannelli, and Mosca (2000), Ding (2010), Ding, Xi, and Li (2004), Pluymers, Suykens, and De Moor (2005), Schuurmans and Rossiter (2000) and Wan and Kothare (2003), even though there are some works based upon system models described by bounded disturbance type uncertainty (Mayne, Raković, Findeisen, & Allgöwer, 2006; Mayne, Seron, & Raković, 2005), and more recently there is a promising research line aimed at RMPC designs concerning both uncertainty descriptions (Tahir & Jaimoukha, 2013). The polytopic uncertainty-based RMPC formulations proposed so far can differ considerably from each other due mainly to the state-feedback control law adopted inside the terminal constraint set. In particular, these RMPC methods can then be classified into four groups, namely, open-loop (Casavola et al., 2000; Ding et al., 2004), partial feedback (Schuurmans & Rossiter, 2000), feedback (Pluymers et al., 2005; Wan & Kothare, 2003) and parameter-dependent open-loop (Ding, 2010), whose works related to the two last groups have sought to overcome the limitations from the two first ones with respect to the enlarging of the attraction domain and improving the optimality of the controllers.

On the other hand, the state-feedback RMPC schemes for the polytopic uncertain model cited above have the drawback of huge on-line computational burdens stemming from the combinatorial nature of the resulting optimization problem. Furthermore, the rigorous stability and feasibility of these methods are guaranteed only if the desired reference values for the system inputs and states are fixed equilibrium points (typically the origin). Within the dual-mode prediction approach the finite (control) horizon must be large enough such that the states at the end of the horizon lie in the RPI set and, in addition, the off-line calculation concerning the parameters determination of the RPI set is not trivial. A possible way, proposed by Limon, Alvarado, Alamo, and Camacho (2010), remedies in part the aforementioned drawbacks by pointing out a robust tube based dual-mode MPC formulation. In their research work, the reference tracking case is solved by considering the steady-state conditions of the system as decision variables (artificial reference) of a single RMPC optimization problem, whose recursive feasibility and enlarged domain of attraction of the controller are achieved because the terminal constraint (state-feedback law) that is a RPI set accommodates any equilibrium point (non-zero set-points). Since the resulting optimization problem is posed as a quadratic programming, its computational burden is drastically reduced if compared to the LMI problem formulations. However, such improvements are efficiently attained by virtue of the model uncertainty being restricted to a bounded additive disturbance, which describes in a simpler fashion the system uncertainty.

Alternatively, the Control Lyapunov Function (CLF)-based MPC schemes are motivated by the fact that is possible to explicitly characterize the stability region, guaranteed feasibility and closed-loop stability of the controller without the need of imposing an infinite prediction horizon (Mhaskar, El-farra, & Christofides, 2005, 2006); please see the work of Christofides, Scattolini, Muñoz de la