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Predictive inverse model allocation for constrained over-actuated linear systems[☆]

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ABSTRACT

The paper presents a model predictive allocation scheme for constrained over-actuated linear systems, for which input redundancy entails the existence of multiple trajectories in the state space yielding a given reference output. The method relies upon the concept of inverse model allocation, where dynamic allocation of reference state and input trajectories is accomplished within the framework of output regulation while maintaining invariance of the error-zeroing subspace. The study focuses on the design of model predictive allocator to achieve constraint satisfaction and asymptotic evolution of the trajectories to a pre-computed steady-state target. In particular, the objective of this study is the analysis of the stability and feasibility properties of the proposed schemes and the characterization of suitable sufficient conditions for stability in geometric terms. In support of the theoretical findings, this study presents an example where the proposed methodology is applied to solve a constrained tracking control problem for the linearized model of the air path system of a turbocharged Diesel engine.

1. Introduction

Control allocation is a standard tool in the design of control systems for plant models possessing a redundant set of control inputs (see Bodson, 2002; Johansen & Fossen, 2013 and references therein.) A typical formulation involves squaring the plant model by factoring out the null space of the input map, computing the control policy for the ensuing virtual square system, and lifting the control policy to the original higher dimensional input space by means of optimization techniques. Recent contributions have started to address more challenging cases of input-redundant systems that cannot be rendered square by projection. The work of Zaccarian (2009) proposes a taxonomy of input redundant systems, in which the above situation was explicitly recognized. The case of injective input maps in over-actuated systems was termed *weak input redundancy* to distinguish this scenario from the standard case of *strong input redundancy*, which can be addressed solely by projection. Geometric characterizations of weak input redundancy have been recently proposed in the context of full-information output regulation (Galeani, Serrani, Varano, & Zaccarian, 2015; Serrani, 2012), and for more general tracking problems in Cristofaro and Galeani (2014). In these works, it was shown that weakly input redundant systems possess a non-unique inverse model, where redundancy yields the existence of a family of *state and input trajectories* that are compatible with a given output trajectory. This allows for the definition of control allocation policies aimed at shaping the behavior of the regulated system by providing a suitable on-line selection of the steady-state behavior of the closed-loop system. This selection can be accomplished with performance criteria in mind, which possibly include avoidance of constraint violation in both the input and state space, while preserving invariance of the error-zeroing subspace. In particular, in Serrani (2012) a redundant inverse model is parameterized as a controllable system with assigned dynamics, where a remarkable feature is that the allocation problem is decoupled from error regulation, and can be formulated independently as a dynamic optimization problem. Building upon this work, a model-predictive approach was proposed in Zhou, Fiorentini, Canova, and Serrani (2013) for the design of the allocation module, in the specific context of control of over-actuated turbocharged internal combustion engines for automotive applications.

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For systems that are subject to constraints, Model Predictive Control (MPC) is one of the few methodologies that are able to address state/input constraints in a systematic fashion (Mayne, Rawlings, Rao, & Scokaert, 2000). Considerable study has been devoted to application of MPC to the tracking problem, and solutions have been developed by resorting to appropriate formulations in the MPC setting of the *internal model principle* (Limon, Alvarado, Alamo, & Camacho, 2008; Maeder, Borrelli, & Morari, 2009; Maeder & Morari, 2010; Pannocchia & Rawlings, 2003). For systems with redundant inputs, the use of constrained optimization (Bodson & Frost, 2011) and model predictive control has also been explored in the context of control allocation in applications such as aerospace systems (Luo, Serrani, Yurkovich, Oppenheimer, & Doman, 2007), thermal management (Vermillion, Sun, & Butts, 2011), marine and underwater vehicles (Johansen, Fossen, & Tøndel, 2005), automotive systems (Zhou et al., 2013), and fault tolerant control (Cristofaro & Johansen, 2014). The concept of dynamic reference management has also been proposed within the framework of reference governors (Bemporad, Casavola, & Mosca, 1997), where the manipulation of external reference signals is accomplished via predictive optimization.

This paper presents a model predictive allocation scheme for *constrained weakly input redundant* system, based on the concept of inverse model allocation (Serrani, 2012; Zhou et al., 2013). Differently from reference or command governors, an inverse model allocator aims at optimizing the state/input trajectories while maintaining invariance of the tracking error, that is, without modifying the output reference to be tracked. Moreover, with a compensator pre-designed for stabilizing purposes, the model predictive allocator performs as an *add-on module*, which mainly focuses on selecting optimal reference trajectories from the redundant family of states/inputs and addressing the constraint requirements. In particular, the cost function to be optimized does not incorporate the tracking error, as opposed to conventional MPC tracking control (Pannocchia & Rawlings, 2003). By expanding on the preliminary results in Zhou et al. (2013), the paper offers a detailed analysis of the stability and feasibility of the proposed approach, and a characterization of sufficient geometric conditions for stability. The investigation focuses on optimality and stability of the allocation of the steady-state trajectories, with an eye to incorporating performance requirements on the transient behavior as well. Finally, a case study is presented to illustrate how the proposed methodology may be applied to a constrained tracking control problem for linearized models of turbocharged Diesel engines characterized by redundant actuation in the air path system.

The paper is organized as follows: Section 2 presents the problem statement. In Section 3, the concept of inverse model allocation is recalled, together with the salient aspects of the synthesis of the extended reference model of Serrani (2012). The design of the dynamic allocation module is described in Section 4, whereas stability and feasibility analysis under the assumption of perfect tracking are presented in Section 5. The case study on the control of the air path system of a linearized over-actuated Diesel engine model is presented in Section 6. Conclusions are offered in Section 7, whereas notation and technical details are found in the Appendix.

2. Problem statement

Consider the following output regulation problem for discrete-time system

$$\begin{aligned} w(t+1) &= A_e w(t) & x(t+1) &= Ax(t)+Bu(t)+B_e w(t)\\ y(t) &= Cx(t) & z &= Dx(t)\\ r(t) &= C_e w(t) & e &= Cx(t)-C_e w(t) \end{aligned} \tag{1}$$

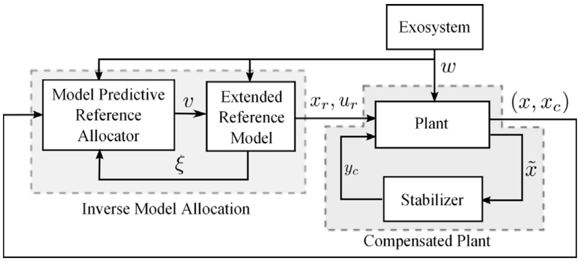

Fig. 1. Architecture of the proposed regulator.

with time $t \in \mathbb{N}_+ := \{0, 1, \ldots\}$, plant state $x \in \mathscr{X} \simeq \mathbb{R}^n$, exosystem state $w \in \mathscr{W} \simeq \mathbb{R}^q$, control input $u \in \mathscr{U} \simeq \mathbb{R}^m$, regulated output $y \in \mathscr{Y} \simeq \mathbb{R}^p$, reference $r \in \mathbb{R}^p$ and auxiliary performance output $z \in \mathscr{Z} \simeq \mathbb{R}^s$. State and input trajectories of the plant model are subject to polyhedron constraints of the form

$$\mathcal{P} \triangleq \{u \in \mathcal{U},\ x \in \mathcal{X},\ w \in \mathcal{W}\} \tag{2}$$

where $\mathcal{U} \subset \mathscr{U}$, $\mathcal{X} \subset \mathscr{X}$ and $\mathcal{W} \subset \mathscr{W}$ are given compact sets. It is required that $\mathcal{W}$ be a forward invariant set for the exosystem. The following assumptions characterize the class of plant and exosystem models:

Assumption 1. (1) The pair (A, B) is stabilizable.
(2) The matrix A_e is semi-simple with $\operatorname{spec} A_e \cap \mathbb{C}^+ = \emptyset$, where $\mathbb{C}^+ := \{\lambda \in \mathbb{C} : |\lambda| > 1\}$.
(3) The set of invariant zeros of the triple (C, A, B) and the spectrum of A_e are disjoint.
(4) The triplet (C, A, B) is right-invertible.

The linear model (1) is assumed to be *over-actuated*, that is, $\dim y < \dim u$. It is further assumed that the matrix B has full column rank and the matrix C has full row rank. This scenario has been termed *weak input redundancy* (Zaccarian, 2009). Finally, it assumed that a dynamic state-feedback stabilizer

$$\begin{aligned} x_c(t+1) &= A_c x_c(t) + B_c u_c(t)\\ y_c(t) &= C_c x_c(t) + D_c u_c(t) \end{aligned} \tag{3}$$

with state $x_c \in \mathscr{X}_c$, input $u_c \in \mathscr{U}_c$ and output $y_c \in \mathscr{Y}_c$ has been designed such that the closed-loop matrix

$$A_a := \begin{bmatrix} A + BD_c & BC_c \\ B_c & A_c \end{bmatrix}$$

resulting by setting $u_c = x$, $u = y_c$ satisfies $\operatorname{spec} A_a \subset \mathbb{C}^-$, where $\mathbb{C}^- := \{\lambda \in \mathbb{C} : |\lambda| < 1\}$.

The control goal is to find a full-information *plug-in regulator* with interconnection structure shown in Fig. 1, and the largest possible set of initial conditions such that all trajectories of the closed-loop system remain confined within the constraint set and satisfy $\lim_{t\to\infty} e(t) = 0$, while minimizing a given cost functional $J(z, u)$ of performance output and input trajectories. It is worth noting that the cost function is not required to incorporate penalties on the tracking error. In this study, the cost function is chosen as a quadratic functional of the form

$$J(z, u) = \sum_{t=0}^{\infty} z(t)^T W_z z(t) + u(t)^T W_u u(t) \tag{4}$$

where W_z, W_u are positive definite matrices. This is done for convenience, as efficient algorithms are available to calculate the solution (Bemporad, Morari, Dua, & Pistikopoulos, 2002).