Digital Signal Processing 82 (2018) 1–10

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

Hilbert spectrum analysis of piecewise stationary signals and its application to texture classification

Zhihua Yang [a], Qian Zhang [b], Feng Zhou [a], Lihua Yang [c,*]

[a] Information Science School, Guangdong University of Finance and Economics, Guangzhou 510320, China
[b] College of Mathematics and Statistics, Shenzhen University, Shenzhen 518060, China
[c] Guangdong Province Key Laboratory of Computational Science, School of Mathematics, Sun Yat-sen University, Guangzhou 510275, China

ARTICLE INFO

Article history:
Available online 2 August 2018

Keywords:
Piecewise stationary signals
Empirical Mode Decomposition (EMD)
Hilbert–Huang Transform (HHT)
Texture classification

ABSTRACT

The piecewise stationary signal is a special kind of non-stationary signals, which exist widely in the real world. Many time-frequency techniques are developed to process non-stationary signals. However, almost all the classical time-frequency methods depends strongly on the choice of the 'basis', which makes them not match adaptively the real time-frequency structure of signals. This paper presents new insights on the Hilbert–Huang transform. It is shown that the Hilbert spectra can capture fine time-frequency structures of piecewise stationary signals by generating the 'bases' adaptively. Based on that, the harmonic components of high energy can be utilized to generate feature vector for texture image classification. This feature vector is shown to be robust to rotation, uneven illumination and noise. Experimental results on three commonly used texture datasets give challenging recognition rates.

1. Introduction

According to the Fourier Analysis, a periodic signal can be written as a sum of harmonic waves of constant frequency and amplitude if only it satisfies to the Dirichlet condition:

$$x(t) = \sum_k A_k \cos(2\pi\omega_k t + \varphi_k). \tag{1}$$

Most signals in the real world can be approximately modeled by this model. The basic idea of the classical spectrum analysis methods for signal processing is to extract the useful harmonic components with well-designed filters [1–3]. The Fourier spectrum analysis has become the classic tool for stationary signal processing. However, if the signal is not stationary then these harmonic waves may not coincide with its physical oscillations, which are called pseudo ones. There are many non-stationary signals in the real world, such as the physiological rhythm [4], textural images [5], and speeches [6]. To analyze and process non-stationary signals, some theory and techniques, including the short-time Fourier transform, Wavelet Transform, and Hilbert–Huang transform, have been developed in the past half century.

The Hilbert–Huang Transform (HHT) provides a powerful tool for nonlinear and non-stationary signal processing [7]. It decomposes a signal adaptively into finite (often small) number of the so-called intrinsic mode functions (IMFs) by an algorithm, called empirical mode decomposition (EMD). It is shown that the EMD essentially acts as an adaptive filter bank [8]. The adaptivity of the decomposition to nonlinear and non-stationary signals have attracted a lot of interests and many developments have been proposed in the last twenty years, such as [9–15]. After EMD, the IMFs can be viewed as adaptive versions of the harmonic waves. Different from the Fourier and wavelet decompositions, the IMFs are generated adaptively during the decomposition, rather than being specified in advance.

The IMF is an empirical mode for monocomponent signals. A detailed discussion on the coincidence between the monocomponent signals and IMFs is given in [16–18]. Once the IMFs are extracted, the instantaneous frequency and amplitude of each IMF can be found by using the Hilbert transform [19,20] or other demodulation algorithms [21–23], which form an energy distribution with explicit physically significant [7]. The energy distributions of all the IMFs are combined to generate the so-called Hilbert spectrum. It has been shown that the Hilbert spectrum characterizes the signal locally on both the time and frequency domains. As a powerful tool for adaptive time-frequency analysis, the Hilbert–Huang transform has been extensively applied in many applications [24–28].

* Corresponding author.
E-mail addresses: yangzh@gdufe.edu.cn (Z. Yang), mazhangq@szu.edu.cn (Q. Zhang), fengzhou@gdufe.edu.cn (F. Zhou), mcsylh@mail.sysu.edu.cn (L. Yang).

https://doi.org/10.1016/j.dsp.2018.07.020
1051-2004/

The piecewise stationary signal is a special kind of non-stationary signals. It may be not stationary on the whole time axis but stationary piecewise, that is, it can be formulated by Eq. (1) segment by segment on the time (space) domain. This locality makes the HHT the ideal candidate for processing of such signals. Based on the Hilbert spectrum, this paper shows that the harmonic components of the highest energies in the Hilbert marginal spectrum are stable and dominant features of the piecewise stationary signals.

Texture classification is very important to many applications in pattern recognition, such as the understanding of remote sensing images [29], medical images [30], content-based image retrieval to text page segmentation [31]. Some texture classification methods have been proposed in the past few decades [32–41]. Texture image are non-stationary signals. For such signals, people generally establish suitable time-frequency (TF) and time-scale (TS) representations to extract classification feature [42]. In this study, we treat texture image as piecewise stationary signals and use the harmonic components with the highest energies to generate a feature vector for texture classification. It is shown that the proposed features are robust to rotation, uneven illumination and noise. Experiments on three commonly used texture data sets from the Brodatz database are implemented and compared with the existing methods. The proposed features are supported by the encouraging experimental results.

The rest of this manuscript is organized as follows. Section 2 presents a brief introduction to the Hilbert spectrum and its marginal spectrum. Some useful traits of HHT are highlighted and the model of piecewise stationary signals is also given. In Section 3, a new type of features is established for texture classification based on the Hilbert marginal spectrum. The robustness to rotation, illumination and noise are analyzed theoretically. Finally, the experiments and comparison with existing methods are given in Section 4. Section 5 is a short conclusion and discussion.

2. Hilbert marginal spectrum for piecewise local stationary signals

The Hilbert–Huang Transform is proposed by Huang et al. in [7]. When processed with the HHT, an arbitrary discrete signal $x(t)$ is adaptively decomposed by the Empirical Mode Decomposition (EMD) algorithm into a finite and often small number of Intrinsic Mode Functions (IMFs), denoted by $x_i(t),\ \ i = 1, \cdots, n$, and a residue $r(t)$:

$$x(t) = \sum_{i=1}^{n} x_i(t) + r(t). \tag{2}$$

Since each IMF $x_i(t)$ is an approximation of monocomponent signal, its instantaneous phase $\theta_i(t)$ and amplitude $a_i(t)$ can be calculated as

$$a_i(t) = \sqrt{x_i^2(t) + (Hx_i)^2(t)}, \quad \theta_i(t) = \arctan\frac{Hx_i(t)}{x_i(t)}, \tag{3}$$

where $Hx_i(t)$ is the Hilbert transform of $x_i(t)$ defined by

$$Hx_i(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{x_i(\tau)}{t-\tau} d\tau \tag{4}$$

with P indicating the Cauchy principal value integral. Accordingly, the instantaneous frequency of $x_i(t)$ is defined as

$$\omega_i(t) = \frac{d\theta_i(t)}{dt}. \tag{5}$$

With the instantaneous frequency and amplitudes, the Hilbert spectrum of $x(t)$ is defined as

$$H(\omega, t) = \sum_{i=1}^{n} a_i(t)\delta(\omega, \omega_i(t)), \tag{6}$$

where $\delta(x, y)$ is the Kronecker delta function which is 1 if the variables are equal and 0 otherwise. We integrate $H(\omega, t)$ over the time axis to calculate the following *Hilbert marginal spectrum*:

$$h(\omega) = \int_{0}^{T} H(\omega, t) \mathrm{d}t. \tag{7}$$

Three key traits of the Hilbert marginal spectrum need to be highlighted.

- Different from the Fourier decomposition and wavelet decomposition, EMD has no specified 'basis'. Its 'bases' are adaptively produced during the decomposition process, which avoids the possible pseudo components as in the Fourier expansion. Hence, IMFs are more intrinsic and natural than the harmonic waves used in the Fourier transform and the mother wavelet in the wavelet transform.
- The Hilbert transform $Hx_i(t)$ is defined as the convolution of $x_i(t)$ and $1/t$ by Eq. (4), it emphasizes the local properties of $x_i(t)$ even though the transform is global. This makes the energy distribution of each IMF possess very fine local properties in the time (space) domain. Meanwhile, EMD is similar to sifting; it separates the local modes of the data from the finest-scale component to the mean trend. The first IMF includes the finest scale component, namely, the highest-frequency component, and the residue is the mean trend, which is the lowest-frequency component or the substitute of the DC term in the Fourier expansion. The oscillations of the same scale would never occur in two different IMFs at the same location. If $\omega_i(t)$ is the frequency of the ith IMF at time t then $\omega_1(t) > \omega_2(t) > \cdots$. Thus, with the Hilbert spectrum, frequencies of a signal are distinguishable at any time.
- The frequency in the marginal spectrum has an entirely different meaning from that in Fourier spectral analysis. In the Fourier representation, the existence of energy at a frequency means a component of a sine or cosine wave has persisted through the time (space) span of the data. However, in the Hilbert marginal spectrum, the existence of energy at the frequency denotes a high likelihood for such a wave to have appeared locally. This means, the frequency components of local oscillations can be captured by the Hilbert marginal spectrum successfully.

A signal $x(t)$ defined on the interval $[0, T]$ is called piecewise stationary if it is stationary segment by segment, that is, the whole time interval $[0, T]$ can be divided into n segments:

$$0 = t_0 < t_1 < \cdots < t_n = T,$$

such that $x(t)$ is stationary on each $[t_{i-1}, t_i]$ for $i = 1, \cdots, n$. That is, $x(t)$ can be written as

$$x(t) = \begin{cases} \sum_{k\in N_0} A_k^0 cos(2\pi\omega_k^0 t + \varphi_k^0) & t \in [t_0, t_1) \\ \cdots & \cdots \\ \sum_{k\in N_i} A_k^i cos(2\pi\omega_k^i t + \varphi_k^i) & t \in [t_i, t_{i+1}) \\ \cdots & \cdots \\ \sum_{k\in N_{n-1}} A_k^{n-1} cos(2\pi\omega_k^{n-1} t + \varphi_k^{n-1}) & t \in [t_{n-1}, t_n] \end{cases} \tag{8}$$

The traits of the signal suggest that the HHT may be an ideal tool to analyze piecewise local stationary signals. First, adaptivity