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Learning by doing with spillovers: Strategic complementarity versus strategic substitutability[☆]

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ABSTRACT

"Learning by doing", in which unit production costs decrease with cumulative production experience, is extensively observed in economies of scale. It has been shown that, in the case of mature technologies, learning curves exhibit a linear behavior in cumulative production. In this paper we consider two competing firms that produce fully substitutable products and whose experience levels have a linear effect on their unit production costs. We assume that production experience is affected by random causes, and that the learning process may involve spillovers of experience from the competing firm. As in the Cournot competition, in this differential game the firms compete by choosing the quantities of products that they will produce. In contrast to the Cournot assumption, according to which firms maximize their profits by taking as given the quantity produced by the competitor, we assume that a firm may not be able to determine its competitor's reaction to a change in its output and instead may conjecture the competitor's response. We find that, in contrast to the case of a steady state, an open-loop equilibrium over a finite planning horizon may result in greater output (more competitive behavior) compared with a subgame-perfect equilibrium. We show that this result is due to the fact that strategic behavior in feedback Nash equilibrium depends on the relationship between the level of proprietary learning by doing (which encourages strategic complementarity) and the level of spillovers (which involves strategic substitutability).

1. Introduction

"Learning by doing", a phenomenon in which a firm's unit production cost decreases with cumulative production, has long been observed in economic activities (Thorndike, 1927). Though its economic implications have been extensively documented (see the reviews in Arrow, 1962; Dutton & Thomas, 1984; McDonald & Schrattenholzer, 2001; Yelle, 1979), the consequences of learning by doing for conjecture-based competitive behaviors remain under-investigated. This paper seeks to fill this gap by analyzing how a firm's conjecture-based competitive behaviors are affected by the extent to which it accumulates experience.

In practice, a process of learning by doing can be characterized along three dimensions: *intensity*, *regularity* and *proprietary nature*. In terms of intensity, two alternative representations of learning by doing have been suggested (Newell, Mayer-Kress, & Liu, 2001): *power law and exponential functions*. In a power law, learning rates are decreasing and do not follow a single time scale (Schroeder, 1991). Accordingly, a learning curve is evaluated when cumulative production doubles. In contrast, exponential functions correspond to a constant learning rate and to a fixed time-scale, such that the inverse of the learning rate defines the intrinsic time-scale of the system (Cohn & Tesauro, 1992). In the case of mature technologies, such learning curves exhibit a linear behavior in cumulative production quantity (e.g., Australian Business Council for Sustainable Energy, 2003). Notably, the linearity assumption is widely used in the literature (see, for example, Fudenberg & Tirole, 1983; Jørgensen & Zaccour, 2000). In terms of regularity, learning by doing is an empirical process of knowledge accumulation that is often exposed to exogenous random factors, and is therefore inherently noisy (Cross, 1973; Flood, 1952; Rakhlin, 2006). In this regard, the process in which firms accumulate production experience can be assumed to be stochastic. Finally, regarding

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Table 1
Observability of experience levels and firms' production strategies.

		Experience levels are	
		Unobservable	Observable
A firm's production's strategy is based on	The competitor's current production strategy	Section 4	Section 5
	The competitor's conjectured production strategy	Section 4	Section 5

the proprietary nature of the process, learning by doing in a competitive context may not be perfectly proprietary but may also involve spillovers; that is, a firm might learn not only from its own experience but also from the experience of its competitors, as in R&D processes (Cellini & Lambertini, 2009; D'Aspremont & Jacquemin, 1988; El Ouardighi, Shnaiderman, & Pasin, 2014; Griliches, 1992).

To investigate how a firm's accumulation of experience affects its conjecture-based competitive behaviors, we consider a mature duopolistic industry involving fully substitutable products, in which two firms, competing on quantities (Cournot competition), undergo an imperfectly proprietary, noisy process of learning by doing. This process is characterized by a linear effect of production experience on unit production costs. As is typical to a Cournot game, the choice of quantities determines the market prices of the homogeneous products and thereby the firms' profits. In this setup, we assume that each firm's production strategy is based either on the current production strategy of its competitor (Cournot, 1838), or on the firm's beliefs (conjectures) about its competitor's responses to its own behavior (Bowley, 1924).[2] In the latter case, we consider consistent conjectures, meaning that they indeed coincide with the competitor's actual optimal policy, e.g., beliefs about the slope of the competitor's reaction function coincide with the actual slope (Bresnahan, 1981).

Because of its cumulative nature, production experience is interpreted herein as a stock variable, which requires a dynamic approach. In dynamic Cournot games, regardless of whether each firm's production strategy is based on the competitor's current or conjectured production strategy, the mutual observability of the firms' experience levels over time plays an essential role (Dockner, Jørgensen, Long, & Sorger, 2000). In the observable case, in which each firm's current stock of experience (state) is known to both parties, a firm's production strategy is *contingent* on the current values of its own stock of experience and that of its competitor. This case is referred to as a closed-loop, or feedback scenario. In the unobservable case, in which neither firm has precise knowledge of either its own stock of experience or that of its competitor (e.g., due to noisy experience accumulation), each firm precommits to its production strategy at the beginning of the time horizon, and its strategy at each time period depends on time only. This case is referred to as an open-loop scenario. Overall, four scenarios are possible, as shown in Table 1. In the following sections we evaluate each scenario and compare the four scenarios in terms of the product's overall market price and each party's payoffs.

In the next section, we briefly review the relevant literature. Section 3 develops a finite-time-horizon differential game model in which two firms compete on quantities, and benefit from learning by doing with experience spillovers. Sections 4 and 5 study the four scenarios outlined in Table 1. Section 6 uses analytical and numerical means to compare the outcomes of the four scenarios. Section 7 concludes the paper.

[2] For a recent introduction on conjectural variations, see Figuières, Jean-Marie, Quérou, and Tidball (2004).

2. Literature review

Our research is related to a broad area of differential games in economics and management science (e.g., Dockner et al., 2000; Jørgensen & Zaccour, 2004; Long, 2010).

Jarmin (1993) develops and estimates an empirical multi-period model to study the intertemporal nature of learning by doing and spillovers. He finds evidence of both proprietary and spillover learning and shows that a firm's ability to learn from its own experience differs from its ability to learn from its rival's experience. Early analytical models accounting for the effect of experience typically consider quantity-based competition over two periods, with proprietary learning-by-doing and/or spillovers, especially when analyzing contingent equilibria (see, for example, Fudenberg & Tirole, 1983; Jørgensen & Zaccour, 2000; Spence, 1981). They find higher output in contingent equilibrium than in precommitment equilibrium, unless one of the firms enters the competition only in the second period. In particular, Jørgensen and Zaccour (2000) use a two-stage model that includes an incumbent firm, which enters in the first period, and an entrant firm, which enters in the second period. In the first period, the incumbent gains experience, which reduces its unit cost in the next period. Moreover, a fraction of the incumbent's experience spills over to the entrant (who enters in period two), affecting the latter's unit production cost as well. The authors find that, depending on the level of spillover, the entrant's equilibrium output under precommitment can be either higher or lower than that under a contingent equilibrium scenario. The authors suggest that future models should consider additional periods in order to study possible spillover from the entrant back to the incumbent, and the parties' mutual interactions over time due to cost reductions. Notably, discounting of future cash flows has a critical role in the results obtained in the two-period models cited above. That is, if the discount factor is set to zero, the strategic outcome is canceled out and the outcome obtained under contingent equilibrium becomes identical to that associated with precommitment equilibrium.

In a more general model, Stokey (1986) develops a differential game with production dynamics and complete spillovers, i.e., the unit cost for any firm depends only upon cumulative (infant) industry production. Stokey finds persistent free riding due to complete spillovers and shows that the contingent equilibrium output in this scenario is higher than that of the static Cournot–Nash equilibrium with no learning. To overcome tractability issues and determine explicit steady-state equilibrium in a similar problem, Miravete (2003) assumes no spillovers and only fixed cost reduction due to accumulated output, while the unit cost remains constant.

In this paper we present a novel contribution with the goal of comparing firms' output levels (also referred to as "competitiveness", where higher output corresponds to higher competitiveness) across different Cournot game scenarios, in which the firms' marginal costs are affected both by proprietary learning and experience spillovers. To the best of our knowledge, the duopoly with spillovers and conjectures has not been studied. Furthermore, we show that a firm's strategic behavior determines the relationship between those output levels that can be altered, depending on the level of learning by doing (which encourages strategic complementarity), and the level of spillovers (which