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Reconstructing signal from quantized signal based on singular spectral analysis

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ABSTRACT

Singular spectral analysis (SSA) is a nonparametric spectral estimation method for performing the time series analysis. It represents a signal as the sum of its components. In this manuscript, a nearly cyclostationary signal is considered. The signal is quantized and the SSA is employed to reconstruct the original signal based on the quantized signal. First, the reconstructed signal is modeled as the weighted sum of the SSA components. In order to estimate the weights, each quantization level is considered as a class. Different signal values are associated with different probabilities of the corresponding classes via the sigmoid functions defined based on the distances between the signal values and the corresponding quantized levels. Therefore, our proposed method provides the optimal estimate of a given signal in the minimum cross entropy sense. Computer numerical simulation results show that our proposed method can reduce the quantization error and reconstruct the original signal more accurately compared to some existing algorithms.

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1. Introduction

Quantization is widely employed in many signal processing systems such as in the data compression [1] and the analog to digital conversion [2,27] systems. However, it is an inherently nonlinear process. More precisely, it is a many to few mapping because it involves an irreversible quantization error. That means, the same output value is obtained even though the input values are different [3]. Therefore, in general it is impossible to exactly recover the original signal value if only the quantized value is given.

To reduce the quantization error, some preprocessing methods where the encoder structures are changed were proposed for reducing the quantization error. The most common preprocessing method is the dithering approach [4,5]. A uniformly distributed noise is added and subtracted before and after the quantizer, respectively, in order to widen the spectra of the input and the output of the quantizer so as to achieve the statistical independence between the input signal and the quantization error. However, implementing the uniformly distributed noise generator is challenging from a practical situation viewpoint. On the other hand, the sigma delta modulation is also widely used in reducing the quantization error [6–8]. A loop filter is added before the quantizer while the output of the quantizer is negatively fedback to the input of the loop filter. If the input signal is oversampled, then the frequency band of the input signal will be localized in a very narrow band. By properly designing the loop filter, the quantization error will be shaped away from the signal band. Therefore, this method can achieve a very high signal to noise ratio (SNR). However, some high order sigma delta modulators suffer from the instability issue [7].

On the other hand, some promising postprocessing methods were proposed to improve the signal quality without the need of changing the encoder structure [9–14]. The most common and classical method is to process the quantized signal by a lowpass filter [9]. The lowpass filtering can smooth the blocking artifacts and extract some useful information from the quantized signal. There are many existing works on designing the filters including the filter design using the discrete cosine transform [28], the filter design based on the optimization approaches [29–33] and so on. However, the filtering method is not effective for the quantizer with a very small number of bits. Besides, the wavelet based thresholding method is employed to reduce the quantization error. It can significantly improve the signal quality for some signals [10, 13,14]. However, this method does not perform well for all practical signals. Also, there is no general rule for predefining the basis function and the threshold value. On the other hand, an adaptive Wiener filtering approach was proposed to reduce the quantization error [12]. The mean squares error between the ideal spectrum of

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the unquantized signal and the estimated spectrum of the quantized signal is minimized. However, this approach requires a full statistic knowledge on the unquantized signal.

In this manuscript, a nonparametric spectral estimation method based on the singular spectral analysis (SSA) [15] is proposed. This is a postprocessing method to reconstruct the original signal from the quantized signal. Here, the input signal is assumed to be nearly cyclostationary. Also, a referenced signal is assumed to be priori known for performing the training. The quantized signal is first decomposed into several SSA components. Then, the reconstructed signal is represented as a weighted sum of the SSA components. Here, it is required to determine the weights. To address this issue, every quantization level is considered as a class. The signal values are associated with the probabilities of the classes via the sigmoid functions defined based on the distances between the signal values and the quantization levels. In other words, if the signal value is close to a quantization level, then the probability of having the signal value belonging to this class is relatively high. On the other hand, if the signal value is far away from the quantization level, then the probability is low. The probability function is modeled by the sigmoid function which is widely used in the machine learning community [16]. Therefore, the weights can be optimally estimated based on minimizing the cross entropy between a referenced signal and its approximation.

A greedy algorithm called the matched sign pursuit (MSP) is employed to reconstruct the signal from the quantized measurements in the compressive sensing (CS) applications [25,26]. However, it is different from our proposed method. First, the signal is sparse and compressible in the CS applications [25,26]. That is, the signal can be represented in the domain where most of the coefficients in this domain are zero or with small values. Only a few coefficients are with large values. Therefore, the quantized measurements in the CS applications are obtained through a transformed domain. However, the nearly cyclostationary signal is considered in this manuscript. The nearly cyclostationary signal is quantized directly by a uniform quantizer. As a result, the quantized measurements are obtained in time domain. Second, the objective function of the reconstruction problem in the CS applications is to minimize the l_1 norm based on the sparsity prior [25,26]. However, the objective function of the reconstruction problem in this manuscript is to minimize the cross entropy based on a referenced signal. Further, the reconstruction problem in [25,26] is nonconvex since its constraints are nonconvex. However, the optimization problem in this manuscript is proved to be convex. Third, the MSP algorithm is employed to solve the reconstruction problem in the CS applications [25,26]. However, the solution is not guaranteed to be global by using this greedy algorithm. The interior point method is applied to solve the optimization problem in this manuscript. The solution is guaranteed to be global since the optimization problem is convex.

The main contributions of this manuscript include two aspects. First, the SSA method is proposed to use for reducing the quantization error. It is different from the conventional filtering methods since the dictionaries used for reducing the quantization error are formed in different ways. In particular, the filtering method is based on a dictionary which is formed by a set of orthonormal basis functions. More precisely, the sine functions and the cosine functions are used in the Fourier transform. The basis functions are predefined and they do not change as the input quantized signal changes. On the other hand, the SSA method can decompose a signal into several components automatically. Therefore, the SSA components change adaptively as the input quantized signal changes. As a result, the dictionary which is formed by the SSA components changes automatically according to the input quantized signal. The SSA method has the advantage of adaptiveness over the filtering methods. The experimental results are presented. They show that the SSA method is potentially useful for reconstructing the original signal from the quantized signal. Second, this manuscript proposes to consider every quantization level as a class. The signal values are associated with the probabilities of the classes via the sigmoid functions. Further, the weights are optimally estimated in the minimum cross entropy sense. The minimum cross entropy criterion has the advantage over the conventional minimum mean squares error criterion. This is because the last several eigenvalues may be nearly equal to zero for some signals in the SSA procedures particularly when the windows length is large. The corresponding vectors of the SSA components will be close to the zero vectors. Therefore, the conventional mean squares error criterion suffers from the ill posed issue which results to several abnormal weights. However, since the singular values are restricted to the range between 0 and 1 due to the probability constraint, the minimum cross entropy criterion will work properly.

The outline of this manuscript is as follows. Section 2 briefly reviews the procedures for performing the SSA. Section 3 presents our proposed method for reconstructing the signal from the quantized signal. Some computer numerical simulation results are presented in Section 4. Finally, a conclusion is drawn in Section 5.

2. Brief review on the SSA

The SSA is a nonparametric time series analysis method. It is widely used in different areas such as in the spectral estimation, the biomedical signal denoising [17], the climatic time series forecasting [18] and the hyperspectral data analysis [19].

Let the length of a one dimensional discrete time signal be N and the vector of the signal be $\mathbf{x} = [x_1 \ \cdots \ x_N]^T$. Here, the superscript T denotes the transposition operator. The procedures for performing the SSA usually consist of the following steps:

Step 1. Forming the Hankel matrix

The signal is segmented into the overlapped pieces. Define the lengths of these overlapped pieces or the window length as L where $1 < L \le \frac{N}{2}$. Define K as the total number of these overlapped pieces. That is, $K = N - L + 1$. Let these overlapped pieces of the signal be $\mathbf{x}_k = [x_k \cdots x_{k+L-1}]^T$ for $k = 1, \ldots, K$. Define the trajectory matrix as $\mathbf{X} = [\mathbf{x}_1 \cdots \mathbf{x}_K]$. That is,

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & \cdots & x_{N-L+1} \\ x_2 & x_3 & \cdots & x_{N-L+2} \\ \vdots & \vdots & \ddots & \vdots \\ x_L & x_{L+1} & \cdots & x_N \end{bmatrix}. \tag{1}$$

Note that $\mathbf{X}$ is a Hankel matrix.

Step 2. Performing the singular value decomposition (SVD)

The SVD is applied to the matrix $\mathbf{X}\mathbf{X}^T$. Denote λ_j for $j = 1, \ldots, L$ as the singular values of the obtained diagonal matrix. Denote $\boldsymbol{\Lambda} = diag(\lambda_1, \ldots, \lambda_L)$. Here, $diag(\lambda_1, \ldots, \lambda_L)$ denotes the diagonal matrix with its diagonal elements being equal to λ_j. Let the column vectors of the obtained singular matrices be $\mathbf{v}_j$ for $j = 1, \ldots, L$. Denote $\mathbf{V} = [\mathbf{v}_1 \cdots \mathbf{v}_L]$. If the SVD is applied to $\mathbf{X}$, then λ_j and $\mathbf{v}_j$ for $j = 1, \ldots, L$ are the squares of the singular values of the obtained diagonal matrix and the corresponding column vectors of the obtained singular matrices, respectively. That is,

$$\mathbf{X}\mathbf{X}^T = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^T. \tag{2}$$

Assume that λ_j for $j = 1, \ldots, L$ are sorted in the descending order. That is, $\lambda_1 \ge \cdots \ge \lambda_L \ge 0$. Define $\tilde{\mathbf{X}}_j = \mathbf{v}_j\mathbf{v}_j^T\mathbf{X}$ for $j = 1, \ldots, L$. Then, it can be shown that

$$\mathbf{X} = \sum_{j=1}^{L} \tilde{\mathbf{X}}_j. \tag{3}$$

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