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Brief paper

Input-to-error stable observer for nonlinear sampled-data systems with application to one-sided Lipschitz systems ☆

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ABSTRACT

In this paper the design of sampled-data state observers for nonlinear plants is investigated under the effect of system and measurement disturbance signals. We establish general design principles using the standard approaches of (i) direct discrete-time design via approximation and (ii) discretization of a continuous-time observer (emulation). By interpreting the disturbances as exogenous inputs affecting the error dynamics, sufficient conditions are derived which ensure the input-to-state stability (ISS) of the observer error system in a semiglobal practical sense for the unknown exact discrete-time model. Next, we focus on systems whose vector fields are one-sided Lipschitz to develop constructive design techniques via linear matrix inequalities (LMIs). Numerical simulations of an academic example and a chaotic attractor corroborate the effectiveness of the proposed sampled-data estimators.

1. Introduction

Observers are well accepted as one of the fundamental building blocks in system theory with extensive research results in the literature (see, e.g., Radke & Gao, 2006 and the references therein). Throughout this paper we study sampled-data nonlinear observers, understood as observers for continuous-time systems implemented using a digital computer via sample and hold devices. The main difficulty encountered in the sampled-data design is that most nonlinear differential equations of interest do not have a closed-form solution and therefore the designer is forced to rely on approximate models. Research on nonlinear sampled-data systems that takes explicit account of the lack of exact discrete-time models was pioneered by Nesić and Teel and has lead to a comprehensive body of literature (Laila, Nešić, & Astolfi, 2006; Nešić & Teel, 2004; Nešić, Teel, & Kokotović, 1999). See also Beikzadeh and Marquez (2013, 2014, 2015), Liu, Marquez, and Lin (2008), Polushin and Marquez (2004) for extensions to multirate systems.

Despite advances in nonlinear sampled-data control, sampled-data observers have received much less attention and there remain several challenging open issues. Significant results in this context include the Newton observers proposed by Moraal and Grizzle (1995) for nonlinear systems with sampled measurements, Biyik and Arcak (2006) that resolves the problem of unknown exact discrete-time models in Dabroom and Khalil (2001), Moraal and Grizzle (1995) that studies discretized high-gain observers, Arcak and Nešić (2004) that proposes a general framework for sampled-data observer design and Postoyan and Nešić (2012) that develops hybrid emulation-based techniques over communication networks. One important element in sampled-data observers design that requires further research is the effect of disturbances on the estimation error. Incorporating disturbance action in observers is nontrivial given that in the presence of external disturbances the reconstructed observer cannot converge to that of the true plant and therefore the classical Lyapunov tools cannot be employed. One way to tackle this problem is to consider the mapping from disturbance to observer error and employ the notion of input-to-state stability (ISS) (Sontag, 1989) to characterize the error dynamics. This concept was already applied to observer design of continuous-time plants with slope-restricted nonlinearities (Arcak & Kokotović, 2001) and Lipschitz systems (Alessandri, 2004). In this article we present two prescriptive sampled-data estimation procedures for general nonlinear systems based on (i) discrete-time design (DTD), and (ii) continuous-time design (CTD) or emulation (see also Laila et al., 2006, Nešić et al., 1999). We show that, given a continuous-time

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nonlinear plant model, then under some standard assumptions and Lyapunov-ISS conditions, the proposed observers converge to the true plant state in an input-to-state stable, semiglobal practical sense.

The second half of the paper is dedicated to systems satisfying a *one-sided Lipschitz* condition, in order to obtain constructive algorithms for a special class of systems. One-sided Lipschitz systems can be viewed as a generalization of the popular Lipschitz condition that relaxes the assumption of linear dominance and reduces the conservatism in the classical Lipschitz based results. Hu (2006, 2008) present a complete analysis of the observer convergence problem for one-sided Lipschitz systems. Existence conditions are discussed in Zhao, Tao, and Shi (2010) and feedback stabilization is formulated in Fu, Hou, and Duan (2013). These works focus on *stability* and make use of a modified one-sided Lipschitz condition in which the nonlinearity is scaled via a fixed symmetric matrix that makes the design problem tractable, but affects the value of the one-sided Lipschitz constant and brings additional constraints on the Lyapunov function. Observer design for the original one-sided Lipschitz condition remains relatively unexplored. The authors in Abbaszadeh and Marquez (2010) introduced an alternative approach which eliminates the need for scaling at the expense of an additional condition on the nonlinearity, known as *quadratically inner bounded*. This approach was further developed in Zhang, Su, Liang, and Han (2012) and in Benallouch, Boutayeb, and Zasadzinski (2012) for the discrete-time case. Using our proposed setups in the first half of the paper, we consider sampled-data observer design for one-sided Lipschitz systems in the presence of disturbance inputs. We present DTD and CTD-based schemes formulated in terms of LMIs that ensure *input-to-error stability*. We show that while the DTD observer necessitates the quadratically inner-bounded condition, the CTD observer does not. Instead, it employs a quasi-one-sided Lipschitz condition (Fu et al., 2013; Hu, 2008) on the plant nonlinearity to facilitate the design procedure.

2. Definitions and problem setting

Notation. For a given function $d:\mathbb{R}^+\to\mathbb{R}^q$, $d(k)$ indicates the value of $d(\cdot)$ sampled at $t=kT$, $k\in\mathbb{Z}^+$ and $\bar{d}=d[k]:=\{d(t):t\in[kT,(k+1)T]\}$ for $k\in\mathbb{Z}^+$ with the norm $\|d\|_\infty=\|d[k]\|_\infty=ess\sup_{\tau\in[kT,(k+1)T]}|d(\tau)|$. $\langle\cdot,\cdot\rangle$ is the natural inner product, i.e., given $x,y\in\mathbb{R}^n$, then $\langle x,y\rangle=x^\intercal y$, where $x^\intercal$ is the transpose of x.

We consider the following nonlinear system:

$$G:\begin{cases}\dot{x}(t)=f(x(t),u(t),d(t))\\ y(t)=g(x(t),u(t),d(t))\end{cases}\tag{1}$$

where $x\in\mathbb{R}^n$, $u\in\mathbb{R}^m$, $d\in\mathbb{R}^q$ and $y\in\mathbb{R}^p$ are respectively the state vector, control input, exogenous disturbance and measured output. The nonlinear function f is sufficiently regular such that there exists at least one solution for each fixed constant input and admissible disturbance. Assume that u is sampled and held via a (zero-order) hold device $\mathcal{H}$ at the same sampling period $T>0$ as y via an ideal sampler $\mathcal{S}$ in a sampled-data configuration. The exact discrete-time model of (1) is then given by

$$\begin{aligned}x(k+1)&=F_T^e(x(k),u(k),d[k])\\ y(k)&=g(x(k),u(k),d(k))\end{aligned}\tag{2}$$

where $F_T^e(x,u,\bar{d})$ is the solution of the differential equation in (1) over sampling interval $[kT,(k+1)T)$ with a constant input u starting at initial condition x. The need for a closed form solution of the differential equation (1) makes it impossible to obtain the model (2) in most practical cases. Therefore, consistent with the literature on nonlinear sampled-data systems, we refer to F_T^e as the exact discrete-time model of the system (1) and assume that it is unknown. Instead we employ a family of approximate discrete-time models $F_{T,h}^a(x(k),u(k),d[k])$, where h is a *modelling parameter* utilized to refine the approximate model for a given T. Clearly, our results also apply when the exact discretized model is available.

Definition 1. The approximate model $F_{T,h}^a$ is said to be one step consistent with F_T^e if there exist a class-$\mathcal{K}$ function $\rho(\cdot)$ and $T_1>0$ such that given any strictly positive numbers $(\delta_1,\delta_2,\delta_3)$ and each fixed $T\in(0,T_1]$, there exists $h_1\in(0,T]$ such that $|F_T^e(x,u,\bar{d})-F_{T,h}^a(x,u,\bar{d})|\le T\rho(h)$ for all $|x|\le\delta_1$, $|u|\le\delta_2$, $\|d\|_\infty\le\delta_3$ and $h\in(0,h_1]$.

We design a family of sampled-data observers of the form

$$\hat{x}(k+1)=F_{T,h}^a(\hat{x}(k),u(k),0)+\ell_{T,h}(\hat{x}(k),y(k),u(k)),\tag{3}$$

where $\hat{x}(k)$ denotes the state estimate, $F_{T,h}^a(\hat{x}(k),u(k),0)$ is the approximate model with zero disturbance and $\ell_{T,h}$ is the correction function zero at zero.

Our main question is under what conditions, and in what sense, an estimator like (3) guarantees approximate convergence to the true plant state when applied to the exact model (2). Note that it is well established that, even in the absence of disturbance, asymptotic convergence of an observer design based on the approximate model does not necessarily guarantee convergence of the true (exact) model (see Arcak & Nešić, 2004). We use the following definition to characterize the convergence behaviour.

Definition 2. The observer (3) is said to be input-to-error stable *semiglobal in T* and *practical in h*, if there exist $\beta\in\mathcal{KL}$ and $\gamma\in\mathcal{K}$ such that for any $\delta_1,\delta_2>0$ and compact sets $\mathcal{X}\subset\mathbb{R}^n$, $\mathcal{U}\subset\mathbb{R}^m$, we can find $T_1>0$ such that for any $T\in(0,T_1]$ and $\nu\in(0,\delta_1)$, there exists $h_1\in(0,T]$ such that $\forall h\in(0,h_1]$, $|x(0)-\hat{x}(0)|\le\delta_1$, $\|d\|_\infty\le\delta_2$ with x as the solution of the exact discrete-time model and $x(k)\in\mathcal{X}$, $u(k)\in\mathcal{U}$, $\forall k\in\mathbb{Z}^+$ implies

$$|x(k)-\hat{x}(k)|\le\beta(|x(0)-\hat{x}(0)|,kT)+\gamma(\|d\|_\infty)+\nu.\tag{4}$$

This definition is an extension of the notion of *semiglobal practical* convergence introduced by Arcak and Nešić (2004) when the plant is exposed to disturbance inputs. Note that for $d=0$, Definition 2 reduces to Arcak and Nešić (2004, Definition 2(b)). The effect of the sampling period as well as the refining parameter on the residual observer error is investigated in Section 6.

3. Observer design via approximation and input-to-state stability

In this section, we derive conditions based on the approximate model and DTD method that guarantee input-to-error stability of the sampled-data observer for the exact model in the sense of Definition 2. From (2) and (3), the observer error $e:=x-\hat{x}$ satisfies

$$\begin{aligned}e(k+1)&=E_{T,h}(e(k),x(k),u(k),d[k])\\ &+F_T^e(x(k),u(k),d[k])-F_{T,h}^a(x(k),u(k),d[k])\end{aligned}\tag{5}$$

where

$$E_{T,h}(e,x,u,\bar{d}):=F_{T,h}^a(x,u,\bar{d})-F_{T,h}^a(\hat{x},u,0)-\ell_{T,h}(\hat{x},y,u)\tag{6}$$

indicates the nominal estimation error dynamics for the approximate design, and $F_{T,h}^a-F_T^e$ is the mismatch between the approximate and exact plant models.