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Brief paper

Distributed finite-time velocity-free attitude coordination control for spacecraft formations[☆]

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ABSTRACT

In this paper, the finite-time velocity-free attitude coordination control for spacecraft formation flying under an undirected communication graph is addressed. A finite-time observer is introduced to obtain an accurate estimation of unmeasurable angular velocity and a decentralized finite-time observer is employed to estimate the angular acceleration of the virtual leader. With the application of the finite-time observer, the decentralized finite-time observer, and the homogeneous method, a continuous distributed finite-time attitude coordination control law is designed for a group of spacecraft without requiring angular velocity measurements. A rigorous proof shows that semi-global finite-time stability of the overall closed-loop system can be achieved and the proposed velocity-free control law guarantees a group of spacecraft to simultaneously track a common time-varying reference attitude in finite time even when the reference attitude is available only to a subset of the group members. The performance of the control scheme derived here is illustrated through numerical simulations.

1. Introduction

In recent years, attitude coordination control for spacecraft formation flying (SFF) has attracted significant attention. This is because SFF is an applicable technology for many space missions such as Earth monitoring, geodesy, deep space imaging and exploration, and in-orbit servicing and maintenance of spacecraft.

A class of decentralized coordination tracking control laws was developed in VanDyke and Hall (2006). Ren (2007) proposed control laws for a team of spacecraft through local information exchange. With consideration of external disturbances and time delays, Jin, Jiang, and Sun (2008) presented a decentralized variable structure controller for attitude coordination control of multiple spacecraft. Using a state-dependent Riccati equation technique, Changa, Park, and Choi (2009) proposed a decentralized attitude coordination control algorithm for satellite formation flying. Chung, Ahsun, and Slotine (2009) employed a Lagrangian approach and nonlinear contraction analysis to study the problem of cooperative tracking control for SFF. Cai and Huang (2014) studied the leader–follower attitude consensus problem for a multiple rigid spacecraft system. In these works, the control laws require full state measurements. Based on a bi-directional ring topology, Lawton and Beard (2002) proposed a passivity based formation control law for multi-spacecraft attitude alignment. Later, Ren (2010) extended the work of Lawton and Beard (2002) to the case of a general undirected connected communication topology. In Lawton and Beard (2002) and Ren (2010), the case when the final angular velocity is zero is considered, and the extension of the obtained results to the attitude consensus tracking is not straightforward. Abdessameud and Tayebi (2009) proposed a velocity-free attitude tracking and synchronization control scheme for a group of spacecraft. However, the common time-varying reference attitude was assumed to be available to each spacecraft in the group, which implies that there exists a central station or a leader which cannot only obtain the group reference but also communicate with each group member in the formation. The requirement of such a leader introduces an apparent limitation and the information relay will result in increased complexity especially when there are a large number of spacecraft. Therefore, in practical applications, it may be more realistic that a common time-varying reference attitude is available only to a subset of the group members. A velocity-free attitude coordination control scheme has been designed for such a group of spacecraft in Zou, Kumar, and Hou (2012).

The aforementioned attitude coordination laws achieve asymptotic stability with infinite convergence time. In the SFF attitude coordinated control, the finite-time control implies faster formation

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rearrangement capability, which leads to an enhanced application efficiency of SFF. Finite-time attitude control for a single spacecraft has been studied in Jin and Sun (2008), Zhu, Xia, and Fu (2011), Zou, Kumar, Hou, and Liu (2011), Du, Li, and Qian (2011), Du and Li (2012, 2013), Lu and Xia (2013) and Zou (2014). However, the extension of the finite-time attitude control algorithms from the single spacecraft case to the multiple spacecraft case is nontrivial especially for the case when there exists a dynamic (virtual) leader whose state is not accessible to all followers.

Several authors have investigated the finite-time attitude cooperative control problem, e.g., Meng, Ren, and You (2010), Du et al. (2011), Zou and Kumar (2012), Zhou, Hu, and Friswell (2013); Zhou, Xia, Wang, and Fu (2015). In Meng et al. (2010), based on a distributed sliding-mode estimator and a nonsingular sliding mode surface, a distributed finite-time control law was designed for a group of rigid bodies with a dynamic leader. However, the control law is discontinuous, and the discontinuity of the control input may cause chattering behavior and excite unmodeled high-frequency system dynamics. Du et al. (2011) proposed a distributed finite-time attitude control scheme for SFF under a communication graph which has a hierarchical structure. However, the control law is not applicable to finite-time attitude coordination control for SFF under an undirected communication graph. In Zhou et al. (2013), a quaternion-based finite-time attitude coordination control law was proposed for satellite formation flying. However, it is only shown that the vector part of the quaternion of each member in the group can track the desired trajectory, and it is not clear whether the attitude synchronization and tracking can be achieved in finite time. Based on the adaptive sliding mode control technique, decentralized finite-time attitude control laws were proposed for multiple rigid spacecraft in Zhou et al. (2015). However, each spacecraft in the formation has its own reference trajectory, and the control laws are not extendable to the case when there is a common time-varying reference attitude which is available to only a subset of the group members. Furthermore, the aforementioned cooperative finite-time attitude control laws rely on the availability of angular velocity measurements. However, in practical applications, due to either cost limitations or implementation considerations, angular velocity measurements may not be available. Therefore, it is highly desirable to design a velocity-free distributed attitude coordination control law that can provide finite-time control for SFF.

In this paper, we study finite-time velocity-free attitude coordination control for SFF under an undirected communication graph. We consider the case when the common time-varying reference attitude is available only to a subset of the team members. The attitude of each spacecraft in the formation is represented by modified Rodrigues parameters (MRPs). Using the finite-time observer introduced in Zou (2014), a decentralized finite-time observer and the homogeneous method, we propose a distributed semi-global velocity-free finite-time attitude coordination control law for SFF. In this paper, the term "semi-global stability" refers to the attitude system using MRPs-based representation. The proposed control law is useful and valuable during formation acquisition and deployment phases.

2. Background and preliminaries

2.1. Notation, definitions and lemmas

The notation $\|\cdot\|$ refers to the Euclidean norm of a vector or the induced norm of a matrix. I_n represents the $n \times n$ identity matrix. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the maximum and minimum eigenvalues of a matrix, respectively. The Kronecker product is denoted by $\otimes$. Given a vector $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ and $\alpha \in R$, define $x^\alpha = [x_1^\alpha, x_2^\alpha, \ldots, x_n^\alpha]^T$, $\mathrm{sig}^\alpha(x) = [\mathrm{sgn}(x_1)|x_1|^\alpha, \mathrm{sgn}(x_2)|x_2|^\alpha, \ldots, \mathrm{sgn}(x_n)|x_n|^\alpha]^T$, and $\mathrm{diag}(|x|^\alpha) = \mathrm{diag}(|x_1|^\alpha, |x_2|^\alpha, \ldots, |x_n|^\alpha)$, where $\mathrm{sgn}(\cdot)$ denotes the signum function defined by $\mathrm{sgn}(y) = 1$ if $y \geq 0$ and $\mathrm{sgn}(y) = -1$ if $y < 0$, $\forall y \in R$. For any $\lambda > 0$ and any set of real parameters $r_i > 0 (i = 1, 2, \ldots, n)$, a dilation operator $\delta_\lambda^r : R^n \longmapsto R^n$ is defined by $\delta_\lambda^r(x_1, x_2, \ldots, x_n) = (\lambda^{r_1}x_1, \lambda^{r_2}x_2, \ldots, \lambda^{r_n}x_n)$, where $r = [r_1, r_2, \ldots, r_n]^T$.

Definition 1 (*Nakamura, Yamashita, & Nishitani, 2004*)**.** A continuous function $f : R^n \longmapsto R$ is homogeneous of degree k with respect to the dilation δ_λ^r if $\forall \lambda > 0, f(\delta_\lambda^r(x)) = \lambda^k f(x)$, where $k > -\min\{r_i\}, i = 1, 2, \ldots, n$. A differential system $\dot{x} = f(x)$ (or a vector field f), with continuous $f : R^n \longmapsto R^n$, is homogeneous of degree k with respect to the dilation δ_λ^r if $\forall \lambda > 0, f_i(\delta_\lambda^r(x)) = \lambda^{k+r_i} f_i(x), i = 1, 2, \ldots, n$.

Definition 2 (*Hong, Wang, & Cheng, 2006*)**.** Consider the following system:

$$\dot{x} = f(x, t), f(0, t) = 0, \quad x \in U \subset R^n \tag{1}$$

where $f : U \times R^+ \to R^n$ is continuous on an open neighborhood U of the origin $x = 0$. The zero solution of (1) is (locally) finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U_0 \subseteq U$ of the origin. The "finite-time convergence" means: If, for any initial condition $x(t_0) = x_0 \in U_0$ at any given initial time t_0, there is a setting time $T > 0$, such that every solution $x(t; t_0, x_0)$ of system (1) is defined with $x(t; t_0, x_0) \in U_0 \setminus \{0\}$ for $t \in [t_0, T)$, $\lim_{t\to T} x(t; t_0, x_0) = 0$, and $x(t; t_0, x_0) = 0, \quad \forall t > T$. When $U = U_0 = R^n$, the zero solution is said to be globally finite-time stable.

Lemma 1 (*Hong et al., 2006*)**.** *Suppose that there is a Lyapunov function $V(x, t)$ defined on $U_1 \times R^+$, where $U_1 \subseteq U \in R^n$ is a neighborhood of the origin, and*

$$\dot{V}(x, t) \leq -lV^a(x, t), \quad \forall x \in U_1 \setminus \{0\} \tag{2}$$

where $l > 0$ and $0 < a < 1$. Then, the origin of system (1) is locally finite-time stable. The settling time satisfies $T \leq \frac{V^{1-a}(x(t_0), t_0)}{l(1-a)}$ for a given initial condition $x(t_0) \in U_1$.

Corollary 1. *Suppose that there is a Lyapunov function $V(x, t)$ defined on $U_1 \times R^+$, where $U_1 \subseteq U \in R^n$ is a neighborhood of the origin, and*

$$\dot{V}(x, t) \leq -lV^a(x, t) + kV(x, t), \quad \forall x \in U_1 \setminus \{0\} \tag{3}$$

where $l, k > 0$ and $0 < a < 1$. Then, for a given initial condition $x(t_0)$ at any initial time t_0, the origin of system (1) is locally finite-time stable if $x(t_0) \in \{U_1 \cap U_2\}$, where $U_2 = \{x | V^{1-a}(x, t) < l/k\}$ is a neighborhood of the origin and satisfies that $U_2 \subseteq U_1$ or $U_1 \subseteq U_2$. The settling time satisfies $T \leq \frac{V^{1-a}(x(t_0), t_0)}{(l - kV(x(t_0), t_0))(1-a)}$ for a given initial condition $x(t_0) \in \{U_1 \cap U_2\}$.

Proof. Note that if $x \in \{U_1 \cap U_2\}$, then we have

$$\begin{aligned} \dot{V}(x, t) &\leq -lV^a(x, t) + kV(x, t) \\ &= -(l - kV^{1-a}(x, t))V^a(x, t) \leq 0 \end{aligned} \tag{4}$$

which implies that $V(x, t) \leq V(x(t_0), t_0)$ for any initial condition $x(t_0) \in \{U_1 \cap U_2\}$. Thus, (4) becomes

$$\dot{V}(x, t) \leq -(l - kV^{1-a}(x(t_0), t_0))V^a(x, t). \tag{5}$$

The conclusion follows from Lemma 1. □

Lemma 2 (*Qian & Lin, 2001*)**.** *For any $x \in R, y \in R, c > 0, d > 0$, and $\gamma > 0$, $|x|^c|y|^d \leq c\gamma|x|^{c+d}/(c + d) + d|y|^{c+d}/[\gamma^{c/d}(c + d)]$.*