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Simplicity-based recovery of finite-alphabet signals for large-scale MIMO systems

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ABSTRACT

In this paper, we consider the problem of finite-alphabet source separation in both determined and underdetermined large-scale systems. First, we address the noiseless case and we propose a linear criterion based on ℓ_1-minimization combined with box constraints. We investigate also the system conditions that ensure successful recovery. Next, we apply the approach to the noisy massive MIMO transmission and we propose a quadratic criterion-based detector. Simulation results show the efficiency of the proposed detection methods for various QAM modulations and MIMO configurations. We mention that there is no change in the computational complexity when the constellation size increases. Moreover, the proposed method outperforms the classical Minimum Mean Square Error (MMSE)-based detection algorithms.

1. Introduction

Source separation problems in digital signal processing deal with the recovery of original source signals from the observed mixture signal [1,2]. In the overdetermined case, the number of observations exceeds the number of sources and the recovery is possible without making strong assumptions about the sources or the mixing parameters [3]. However, the separation problem becomes more difficult if the number of underlying sources is larger than the number of observations. Separation of such underdetermined mixtures requires the separation algorithm to exploit additional information about the source signals and the mixing parameters compared to the overdetermined case. Compressed sensing (CS) technique [4,5] has attracted considerable attention as it promises to surpass the traditional limits of sampling theory [6]. It is a signal processing technique to efficiently acquire and reconstruct signals, by finding solutions to underdetermined linear systems. It exploits the sparsity of the signal to recover it and thus, it uses far fewer samples than required by the sampling theorem [7,8]. A source is sparse in a given representation domain if most of its elements are close to zero. The CS technique requires the sparsity of the sources which restricts its application [9]. In [10], the authors proposed to apply the CS technique to solve underdetermined real-valued finite-alphabet source recovery problems. They introduced a suitable sparse decomposition to derive a sparse recovery problem solvable by CS techniques. In [11], Mangasarian et al. treated the binary alphabet case and showed that the source can be successfully recovered by resolving a linear program with higher detection probability as the number of observations exceeds half the number of sources. Mangasarian analysis remains true for all size-2 alphabets [10]. The recovery problem is especially important in data communications which is the field of interest of this paper and in image processing.

In this paper, we address the problem of finite-alphabet signal recovery for large-scale MIMO systems involving high-dimensional problems. We first consider the noiseless general case as in [10,11]. However, contrary to the aforementioned references, we study the case of complex-valued alphabets with any cardinality and complex-valued mixing matrices. We then propose a linear program for CS technique based on signal simplicity. Simplicity was first introduced by Donoho et al. in [12]. A signal is considered *simple* if most of its elements are equal to the extremes of the finite alphabet. We then prove that this proposed scheme provides the same efficient recovery performance as the schemes in [10,13,14] with lower computational complexity especially when the alphabet cardinality is high. We also show that the recovery scheme performs better when the dimensions of the mixing matrix increase.

In a second step, we propose to apply the principle to noisy massive MIMO transmission, which can be considered as a particular case of large-scale MIMO systems. Massive MIMO technology has been selected in the 5G standard definition as a solution to provide higher throughput under spectrum limitations [15]. It

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promises significant gains and offers the ability to serve more users at higher data rates with better reliability. Large number of antennas and/or users is involved, which makes receiver design critical from complexity point of view.

Research for high-performance receiver design that can lead to practical realization of massive MIMO systems is both nascent as well as promising. Sphere decoders [16], which are based on maximum likelihood (ML), require an exhaustive search within an hypersphere whose dimensions remain high in the massive MIMO case, yielding computationally-unsolvable detection. Usual linear equalizers such as minimum mean square error (MMSE) [17,18] and zero-forcing (ZF) [19] have lower computational complexity but degraded error rate performance compared to ML detectors, especially in the undetermined uncoded case. Successive interference cancellation (SIC) schemes were proposed such as MMSE-SIC in [20] to enhance the linear equalizer performance at the expense of higher complexity. Further error rate decrease was achieved by combining SIC and lattice reduction (LR) schemes as done for instance in the MMSE-SIC-LR studied in [21]. In this paper, we address the problem of detection in both determined and underdetermined systems. Underdetermined configuration is expected in future 5G system uplink, as the number of connected users times their transmit antenna number could be much higher than the base station receive antenna number. To carry out this study, we first extend the noiseless detection algorithm to the noisy case to design a low-complexity detector which exploits the simplicity of sources and we show its efficiency compared to existing detection techniques by investigating the error rate performance and the computational complexity.

In this paper, we deal with the problem of recovery of finite-alphabet signals in both determined and underdetermined large-scale systems by exploiting the simplicity property of the alphabet. Compared to previous work [10,11,13,14], the proposed criterion applies whatever the alphabet size or domain (real or complex-valued) and achieves the best performance with computational cost independent of the alphabet size. The efficiency of the proposed simplicity-based technique can be explained by the added constraints. These constraints ensure that some estimated output are highly reliable and will not contribute to the error propagation. Thus, the error propagation is reduced compared to iterative MMSE-based techniques.

Our contributions are: ***(i)*** a new criterion based on the simplicity property of finite-alphabet signals, ***(ii)*** the necessary condition of successful recovery in the noise-free case, ***(iii)*** the extension of the proposed criterion to the noisy case, ***(iv)*** the theoretical probability density function of the proposed algorithm output, ***(v)*** the theoretical symbol error probability in the case of M-QAM modulations.

This paper is organized as follows. Section 2 describes the system models considered in the following and provides an overview of state-of-the-art compressed sensing techniques. Section 3 deals with source separation problem in noise-free systems. Section 4 describes how the proposed source separation scheme is extended to be applied in massive MIMO systems. Finally, Section 5 concludes the paper.

Notations: Boldface upper case letters and boldface lower case letters denote matrices and vectors, respectively. For transpose, transpose conjugate and conjugate operations we use $(.)^T$, $(.)^H$ and $(.)^*$, respectively. $\otimes$ is the Kronecker product. $\boldsymbol{I}_k$ is the $k \times k$ identity matrix and $\mathbf{1}_k$ is the all-one size-k vector. Let $\boldsymbol{z} \in \mathbb{C}^k$ be a complex-valued vector of size k. We denote by $\underline{\boldsymbol{z}} \in \mathbb{R}^{2k}$ its real-valued transformed vector which can be defined by $\underline{\boldsymbol{z}} = \begin{pmatrix} \mathrm{Re}(\boldsymbol{z}) & \mathrm{Im}(\boldsymbol{z}) \end{pmatrix}^T$. Let also $\boldsymbol{H} \in \mathbb{C}^{n \times N}$ a complex-valued matrix with size $n \times N$, we denote by $\underline{\boldsymbol{H}} \in \mathbb{R}^{2n \times 2N}$ its real-valued matrix version, which is defined by $\underline{\boldsymbol{H}} = \begin{pmatrix} \mathrm{Re}(\boldsymbol{H}) & -\mathrm{Im}(\boldsymbol{H}) \\ \mathrm{Im}(\boldsymbol{H}) & \mathrm{Re}(\boldsymbol{H}) \end{pmatrix}$. $\mathrm{erfc}(\cdot)$ is the complementary error function. It is defined as $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\, dt$. $\delta(\cdot)$ is the Dirac delta function and $\mathbb{1}_{\mathcal{A}}(\cdot)$ is the indicator function of the subset $\mathcal{A}$.

2. System model and overview

2.1. Noise-free large-scale systems

We first consider the noise-free mixing model, which can be described by the following linear system:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x}, \tag{1}$$

where $\boldsymbol{x} \in \mathbb{C}^N$ is the $N \times 1$ complex-valued source vector, $\boldsymbol{y} \in \mathbb{C}^n$ is the $n \times 1$ complex-valued observation vector and $\boldsymbol{H} \in \mathbb{C}^{n \times N}$ is an $n \times N$ complex-valued random matrix. We assume that the components of $\boldsymbol{H}$ are independent and circularly symmetric Gaussian with zero mean and unit variance. The vector $\boldsymbol{x}$ belongs to a complex-valued finite alphabet. It can be decomposed from its real and imaginary parts as $\boldsymbol{x} = \boldsymbol{a} + j\boldsymbol{b}$ where $(\boldsymbol{a}, \boldsymbol{b}) \in \mathcal{F}^N \times \mathcal{F}^N$ and $\mathcal{F} = \{\alpha_1, \alpha_2, .., \alpha_p\}$. The equivalent real-valued linear system can then be written as:

$$\underline{\boldsymbol{y}} = \underline{\boldsymbol{H}\boldsymbol{x}}, \qquad \underline{\boldsymbol{x}} \in \mathcal{F}^{2N}. \tag{2}$$

We assume that the elements of $\mathcal{F}$ are equiprobable under the realization of $\underline{\boldsymbol{x}}$. Then, our problem is the recovery of $\underline{\boldsymbol{x}}$ from $\underline{\boldsymbol{y}}$ given $\underline{\boldsymbol{H}}$ and $\mathcal{F}$.

A special case was introduced by Mangasarian et al. in [11]. They considered the real-valued problem with $\boldsymbol{H}$ an $n \times N$ real-valued generic random matrix[1] and the vector $\boldsymbol{x}$ belonging to the real-valued finite alphabet $\{-1, 1\}$. In this case, $\boldsymbol{x}$ can be recovered by solving the ℓ_∞-norm minimization

$$(P_\infty) : \arg\min_x \|\boldsymbol{x}\|_\infty \quad \text{subject to} \quad \boldsymbol{y} = \boldsymbol{H}\boldsymbol{x}. \tag{3}$$

This optimization system was reformulated by a linear programming problem and the authors proved that the probability of successful recovery equals the probability that all of the columns of the generic random matrix lie in the same hemisphere. This probability is determined by the following theorem.

Theorem 2.1 *(Wendel [22]). Let $\boldsymbol{H}$ an $n \times N$ real-valued generic random matrix. The probability that all of its columns lie in the same hemisphere is precisely equal to*

$$P_{n,N} = 2^{-N+1} \sum_{i=0}^{n-1} \binom{N-1}{i}. \tag{4}$$

As an extension of this work, the authors in [10] generalized the problem to all size-2 constellations $[\alpha_1, \alpha_2]$ thanks to a simple translation.

In the complex case given by Eq. (1), we demonstrate in the Appendix A that given the properties of the complex-valued matrix $\boldsymbol{H}$, its real-valued matrix version $\underline{\boldsymbol{H}}$ is random generic. Then, the probability of successful recovery is equal to the probability that all of the columns of $\boldsymbol{H}$ lie in the first quadrant of the complex plane, that is to say the probability that all of the columns

[1] A matrix $\boldsymbol{H}$ is a generic random matrix if all sets of ℓ columns are linearly independent with probability 1 and each column is symmetrically distributed about the origin [11].

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