Automatica 67 (2016) 165–170

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Finite-time trajectory tracking control in a task space of robotic manipulators ☆

Mirosław Galicki

Faculty of Mechanical Engineering, University of Zielona Góra, Zielona Góra, Podgórna 50, Poland

ARTICLE INFO

Article history:
Received 14 April 2015
Received in revised form
17 November 2015
Accepted 27 November 2015
Available online 5 February 2016

Keywords:
Robotic manipulator
Task space trajectory tracking
Finite-time control
Lyapunov stability

ABSTRACT

This work addresses the problem of the accurate task space control subject to finite-time convergence. Dynamic equations of a rigid robotic manipulator are assumed to be uncertain. Moreover, globally unbounded disturbances are allowed to act on the manipulator when tracking the trajectory by the end-effector. Based on suitably defined task space non-singular terminal sliding vector variable and the Lyapunov stability theory, we derive a class of absolutely continuous Jacobian transpose robust controllers, which seem to be effective in counteracting uncertain dynamics, unbounded disturbances and (possible) kinematic and/or algorithmic singularities met on the end-effector trajectory. The numerical simulations carried out for a robotic manipulator of a SCARA type consisting of two revolute kinematic pairs and operating in a two-dimensional task space, illustrate performance of the proposed controllers.

1. Introduction

In recent years, interest has increased in applying robotic manipulators to useful practical tasks requiring extremely high precision and stability of the performance. In most situations met in practice, such tasks are specified in terms of a trajectory expressed in Cartesian coordinates to be tracked by the end-effector. In order to apply known joint space control techniques (see e.g. our recent work Galicki, 2015) for tracking such a trajectory, an inverse or pseudo-inverse kinematics algorithm has to be utilized. The process of kinematic inversion is both time consuming (there does not exist, in general, an analytic form of inverse mapping) and becomes very complicated when the Cartesian trajectory generates kinematic and/or algorithmic singularities (Balleieul, 1985). Thus, a controller to be designed should accurately track desired end-effector trajectory despite possible singularities met on this trajectory, uncertain dynamic equations, unknown payload to be transferred by the end-effector and external disturbances. Moreover, such controller has to generate at least absolutely continuous control signals (torques) to avoid undesirable chattering. Due to the challenging nature of the aforementioned control design problems, many researchers have proposed different types of controllers. In such a context, one can distinguish three major approaches of controlling the robotic manipulators in the task space. The control techniques offered in the first approach (Kelly & Moreno, 2005; Moreno-Valenzuela & Gonzales-Hernandez, 2011; Nakanishi, Cory, Mistry, Peters, & Schaal, 2008; Ott, Dietrich, & Schaffer, 2015; Siciliano, Sciavicco, Villani, & Oriolo, 2010) require the full knowledge of the dynamics neglecting the external disturbances. In the second approach, works (Braganza, Dixon, Dawson, & Xian, 2008; Cheah, Liu, & Slotine, 2006; Colbaugh & Glass, 1995; Galicki, 2014; Li & Cheah, 2013; Tatlicioglu, Braganza, Burg, & Dawson, 2008) propose adaptive control algorithms to compensate for parametric uncertainties in dynamic model including only the linearly parametrizable friction terms (viscous friction) and also neglecting the external (non-linearly parametrizable) disturbances (except of Colbaugh & Glass, 1995 which permits globally bounded disturbances). In the third approach, model based robust control scheme was proposed in work (Ozbay, Sahin, & Zergeroglu, 2008). In addition, all the aforementioned control schemes (except of Galicki, 2014) require explicit inverse or pseudo-inverse of a Jacobian matrix, which may result in numerical instabilities due to (possible) kinematic and/or algorithmic singularities (Balleieul, 1985). Furthermore, all the control schemes assume globally bounded disturbances when tracking the trajectory whereas e.g. a viscous friction term is globally unbounded. Finally, all the aforementioned controllers provide only at most asymptotic stability what may be insufficient for accomplishment of tasks requiring the extremely high precision (e.g. assembly of electronic components on the small surface of printed circuit boards). Consequently, all those algorithms are not able to generate continuous controls resulting

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Yoshihiko Miyasato under the direction of Editor Toshiharu Sugie.

E-mail address: M.Galicki@ibem.uz.zgora.pl.

http://dx.doi.org/10.1016/j.automatica.2016.01.025
0005-1098/

in finite-time stability of the equilibrium when (possible) singular configurations may appear on the trajectory, dynamic equations are uncertain and (unbounded) disturbances act on the robotic manipulator. In this study, a new task space non-singular terminal sliding manifold (TSM) is introduced to track the end-effector trajectory. The proposed TSM manifold makes it possible to simultaneously join the first order sliding mode approach possessing the finite-time control capabilities with the second order sliding mode techniques generating the (absolutely) continuous controls. The solution of the tracking control problem is based herein on introducing a new dynamic version of a static computed torque approach presented e.g. in works (Siciliano et al., 2010; Spong & Vidyasagar, 1989). By fulfilment of reasonable assumption regarding the Jacobian matrix, the proposed Jacobian transpose control scheme is shown to be finite-time stable. The remainder of the paper is organized as follows. Section 2 formulates the finite-time trajectory tracking task. Section 3 sets up a class of task space robust absolutely continuous controllers solving the trajectory tracking problem in a finite-time subject to uncertain robot dynamic equations and unbounded disturbances. Section 4 presents computer examples of the end-effector trajectory tracking by a robotic manipulator of a SCARA type, consisting of two revolute kinematic pairs and operating in two-dimensional task space. Finally, some concluding remarks are drawn in Section 5. Throughout this paper, $\lambda_{\max}(\cdot)$, $\lambda_{\min}(\cdot)$ denote the maximal and minimal, respectively, eigenvalues of the symmetric matrices $(\cdot)$.

2. Problem formulation

The robust control scheme designed in the next section is applicable to holonomic mechanical systems comprising robotic manipulators considered here which are described, in general, by the following dynamic equations, expressed in generalized (joint) coordinates $q \in \mathbb{R}^n$ (Spong & Vidyasagar, 1989):

$$M(q)\ddot{q} + H(q, \dot{q}) + G(q) + D(t, q, \dot{q}) = v, \tag{1}$$

where $\dot{q}$ and $\ddot{q}$ represent the velocity and acceleration, respectively. The $n \times n$ inertia matrix $M(q)$ is positive definite and symmetric. $H = B(q)(\dot{q}\cdot\dot{q}) + C(q)(\dot{q}^2)$, where B and C are the $n \times \frac{n(n-1)}{2}$ and $n \times n$ matrices of coefficients of the Coriolis and centrifugal forces, respectively. $(\dot{q}\cdot\dot{q}) = (\dot{q}_1\dot{q}_2, \ldots, \dot{q}_{n-1}\dot{q}_n)^T$ and $(\dot{q}^2) = (\dot{q}_1^2, \ldots, \dot{q}_n^2)^T$, respectively. $v = (v_1, \ldots, v_n)^T$ stands for the n-dimensional vector of controls (torques/forces). $G(q)$ is the n-dimensional vector of generalized gravity forces. $D(t, q, \dot{q})$ means the n-dimensional external disturbance signal which is (by assumption) at least absolutely continuous with $\dot{D}(t, q, \dot{q})$ as being a locally bounded Lebesgue measurable mapping. Moreover, $\|D\|$ and $\|\dot{D}\|$ are (by assumption) upper estimated as follows $\|D\| \le \alpha_0(t)$, $\|\dot{D}\| \le \alpha_1(t)$, where α_0, α_1 stand for the known, non-negative functions. The general kinematic and differential mappings between joint coordinates q and task ones $p \in \mathbb{R}^m$ can be written as

$$p = f(q), \qquad \dot{p} = J\dot{q}, \tag{2}$$

where $n \ge m$ is the dimension of the Cartesian space in which the end-effector operates; $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ and $J = \frac{\partial f}{\partial q}$ is the $m \times n$ Jacobian matrix. Due to the fact that kinematic redundancy is not significant in the design of our controller, m is assumed to be equal to n. A task accomplished by the end-effector consists in tracking a desired trajectory $p_d(t) \in \mathbb{R}^n$, $t \in [0, \infty)$ which is assumed to be at least triply continuously differentiable, i.e., $p_d(\cdot) \in C^3[0, \infty)$. By introducing the task tracking error $e = f(q) - p_d$, we may formally express the finite-time control problem by means of the following equations:

$$\lim_{t\to T} e(t) = \lim_{t\to T} \dot{e}(t) = \lim_{t\to T} \ddot{e}(t) = 0, \tag{3}$$

where $0 \le T$ denotes a finite-time of convergence of $f(q)$ to p_d. In further analysis, J is assumed to be of the full rank in the operation region, i.e.,

$$\operatorname{rank}(J(q)) = n. \tag{4}$$

Let us note that condition (4) may be made somewhat more weak. It suffices that for $0 \ne x \in \mathbb{R}^n$ and singular configuration q', the following condition holds true: $x \notin \ker(J^T(q'))$. In the sequel, useful properties of (1) are summarized which will be utilized while designing the controller. The following inequalities are satisfied (Spong & Vidyasagar, 1989):

$$0 < \|M^{-1}\|_F \le \Lambda_{\max}, \qquad \|B\|_F + \|C\|_F \le c_1, \qquad \|G\| \le c_2, \tag{5}$$

where $\|\ \|_F$ means the Frobenius (Euclidean) matrix norm; c_1, c_2, $\Lambda_{\max}$ are known positive scalar coefficients. Moreover, the following inequalities hold true for revolute kinematic pairs:

$$\|J(q)\|_F \le c_3, \qquad \left\|\frac{\partial J}{\partial q}\right\|_F \le c_3', \qquad \left\|\frac{\partial^2 J}{\partial q^2}\right\|_F \le c_3'', \tag{6}$$

where c_3, c_3', c_3'' are known scalar coefficients. Furthermore, from (4) and (5), one also obtains that

$$0 < \lambda\mathbb{I}_n \le JM^{-1}J^T \le \Lambda\mathbb{I}_n, \tag{7}$$

where λ, Λ denote estimations of minimal and maximal, respectively eigenvalues of matrix $JM^{-1}J^T$; $\mathbb{I}_n$ stands for the $n \times n$ identity matrix. In order to obtain at least absolutely continuous control v, let us differentiate (1) with respect to time thus obtaining $M(q)\frac{d^3q}{dt^3} + F(q, \dot{q}, \ddot{q}, t) = \dot{v}$, where $F = \dot{M}\ddot{q} + \dot{B}(\dot{q}\cdot\dot{q}) + \dot{C}(\dot{q}^2) + B\frac{d}{dt}(\dot{q}\cdot\dot{q}) + C\frac{d}{dt}(\dot{q}^2) + \dot{G} + \dot{D}$. Motivated in part by the static computed torque methodology (Siciliano et al., 2010; Spong & Vidyasagar, 1989), we propose a new dynamically computed torque vector $\dot{v}$ of the form

$$\dot{v} = J^T\hat{M}(q)u + \hat{F}(q, \dot{q}, \ddot{q}, t), \tag{8}$$

where $\hat{M}$ and $\hat{F}$ denote known estimates of the corresponding unknown terms M and F, respectively; $u \in \mathbb{R}^n$ is a new control to be found. If F is known mapping, we can take $\hat{F} = F$. Alternatively, $\hat{F} = 0$ if no model of F is available. Definition of $\hat{M}$ is given in the next section. In the sequel, we introduce the following auxiliary matrix $\mathcal{R} = JM^{-1}J^T\hat{M}$ which will play a crucial role by designing our controller. Let us triply differentiate e with respect to time thus obtaining $\frac{d^3e}{dt^3} = u + (\mathcal{R} - \mathbb{I}_n)u + Q - \frac{d^3p_d}{dt^3}$, where $Q = JM^{-1}(\hat{F} - F) + \ddot{J}\dot{q} + 2\dot{J}\ddot{q}$. Furthermore, based on definition of Q, an upper estimation on $\|Q\|$ takes the form

$$\|Q\| \le \mathcal{W}(t, q, \dot{q}, \ddot{q}), \tag{9}$$

where $\mathcal{W} = c_3\Lambda_{\max}\|\hat{F}\| + c_4\|\dot{q}\|\|\ddot{q}\| + c_5\|\dot{q}\|^3 + c_6\|\dot{q}\| + c_3\Lambda_{\max}\alpha_1$; c_4, c_5 and c_6 are (known by assumption) positive scalar coefficients for which the following inequalities hold true: $c_4 \ge \|JM^{-1}\|_F(\|\frac{\partial M}{\partial q}\|_F + \|B\|_F + \|C\|_F) + 3\|\frac{\partial J}{\partial q}\|_F$; $c_5 \ge \|JM^{-1}\|_F(\|\frac{\partial B}{\partial q}\|_F + \|\frac{\partial C}{\partial q}\|_F) + \|\frac{\partial^2 J}{\partial q^2}\|_F$ and $c_6 \ge \|JM^{-1}\|_F\|\frac{\partial G}{\partial q}\|_F$. Based on (8), the next section will present an approach to the solution of the control problem (1), (3) making use of the Lyapunov stability theory.

3. Control of the robotic manipulator

In the sequel, we start the analysis of a controller design by the assumption that joint positions, velocities and accelerations are available from measurements. Based on (7), we can make the following remark:

$$(\exists\hat{M} > 0)(\exists\rho > 0)(|\lambda_{\max}(\mathcal{R} - \mathbb{I}_n)| \le \rho < 1). \tag{10}$$