Digital Signal Processing ••• (••••) •••–•••

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

Nonlinear space–time varying parameter estimation using consensus-based in-network distributed strategy

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ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Distributed signal processing
DPS
ADMM
Fusion center
Least-square

ABSTRACT

In the present world, distributed signal processing plays a significant role in applications ranging from surveillance and tracking to exploration and monitoring. In this paper, an online distributed framework of spatio-temporal Wiener model is presented. A conventional Wiener model is extended to nonlinear distributed parameter systems (DPSs), which comprises of a linear time-invariant (LTI) system in series with a static nonlinear element. The standard Wiener model identification framework is reformulated as the minimization of multiple constrained optimization subtasks that get solved using alternating direction method of multipliers (ADMM) along with coordinate descent techniques. DPSs are significantly used in industrial processes e.g. thermal process, fluid process, etc. Almost all the real-time data contain non-linearity in them which is modeled using several methods: Wiener modeling is one of them. Adaptive as well as distributed implementation of such model is considered to take the advantages of both adaptive and distributed signal processing. The proposed method overcomes the limitations concerning fusion center (FC) and least-square (LS) based approaches. Unknown parameters of Wiener DPS are identified in an adaptive and distributed manner. To dignify the effectiveness of the proposed methodology, simulations on a catalytic rod (an example of a parabolic system) are illustrated.

1. Introduction

A recent in-vogue application of distributed signal processing is the decentralized estimation of unknown parameters of a system using the observations collected across a wireless sensor network (WSN). Distributed signal processing is involved in most of the important industrial applications ranging from surveillance and tracking to exploration and monitoring [1]. It deals with extracting the information from data gathered at different nodes. These nodes are spread over a wide geographical area under monitoring. The data collected by these sensor nodes are noisy, and the processing is done on these noisy data to extract required parameters. These nodes exploit the spatial and temporal diversity to improve the robustness of the processing tasks [2,3]. Fusion center (FC) based estimation (centralized approach) can also do the job [4] but has some limitations arises due to; i) the need of enormous amount of communication resources for transmitting each node information to FC that limit the self-sufficiency of the network [5] and ii) lack of robustness (as there is a point of failure of the whole system if fusion center gets corrupted). In addition, the FC-based approach is lacking the ability to respond in a real-time varying scenario which deteriorates the tracking performance [2,6]. To get rid of these detriments, an advanced way of online in-network distributed approach [7] is encountered to estimate the parameters of interest. An online in-network implementation allows to process on-the-fly information collected at each instant of time. The online implementation has been proved to recover the desired parameters of interest under ideal inter-sensor links [8]. The objective of the distributed estimation is to find out an estimate that is as close as possible to the one that would be obtained if every node had the information of the entire network.

The goal of this research is to develop, analyze and numerically test a methodology to estimate the unknown parameters of Wiener model in a decentralized manner. Previously, the LS-based approach has been employed to estimate the parameters of Wiener modeling [9], but this approach is an offline process where all the data need to be gathered at one place. However, this technique is prone to environmental changes as variables change with time.

Wiener modeling approach has been developed for lumped parameter systems (LPSs) and nonlinear DPSs [9]. To achieve the objective for modeling industrial processes in a distributed manner, the proposed article deals with the nonlinear DPS. Some of the widely used industrial examples of DPSs are thermal process, fluid process, etc. [9]. These industrial processes are basically described

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https://doi.org/10.1016/j.dsp.2018.05.007
1051-2004/

through partial differential equations (PDEs) [10–12] and possess complex coupling in space and time. Aforementioned processes are of infinite dimension and contain nonlinear dynamics. The spatio-temporal coupling, nonlinear dynamics and infinite dimensionality make the processes very difficult to analyze, model and control. Modeling is essential for any DPS to predict, analyze and control. Due to infinite-dimensionality, direct modeling of any PDE cannot be used for implementation. In fact, the dimensionality reduction based models are often required to approximate the systems with finite dimensions. Many approaches have been reviewed for model reduction and control problems when the PDEs of the systems are known or unknown [9,13–15]. Due to incomplete process knowledge, the PDEs of DPS are unknown. Hence, there are uncertainties like unknown parameters and undefined non-linearity. In order to overcome these uncertainties, data-based modeling is required, i.e. model has to be obtained considering input and output measured data [13,16–18]. In this article, an online distributed estimation of unknown uncertainties is presented using the spatio-temporal data collected at different sensor nodes. The underway distributed modeling methodology in this study can be termed as '*distributed Wiener modeling*' as the Wiener model parameters are identified in a distributed manner. Wiener model is chosen because of its ability to approximate a wide range of nonlinear time-invariant systems with arbitrary accuracy and simple block-oriented structure [19]. Also, Wiener model is extensively engaged in several engineering practices. A Wiener model constitutes of a linear time-invariant DPS followed by a static nonlinear element. The optimization task and control composition of the Wiener model is comparatively easier than the traditional nonlinear model (as the linear model is easily derivable from the block-oriented nonlinear model) [9,20].

The data acquired across WSN are usually restrained to a low dimensional subspace. Hence, a few principal components of covariance data represent the dimension of the data acquired [21]. The low dimensional signal subspace has been estimated in a distributed manner using distributed principal component analysis (D-PCA) or distributed Karhunen–Loeve (K–L) decomposition or subspace tracking [8,22–24]. The elemental idea behind K–L decomposition is to find out modes that are responsible for determining the dominant characteristics of the system. Some other techniques for model or dimension reduction are Green's function method, finite difference method (FDM), Galerkin method, eigenfunction method, finite element method (FEM) etc. However, K–L method is more suitable for dimension reduction as it is more accurate than all other methods [25]. The proposed work utilizes the approach involved in [8] and [22] to frame the cost function as constrained separable optimization tasks. Then, the unknown parameters are estimated in a distributed recursive manner. ADMM is employed along with coordinate descent method to handle the separable constrained optimization problem and then the unknown parameters of interest are estimated [8,22,26,27].

The rest of the proposed article is arranged in the following sequence. Problem formulation and preface are well thought out in Section 2. Distributed Wiener modeling based on consensus strategy is elaborated in Section 3. Section 4 of this article describes the convergence analysis of the proposed online distributed modeling methodology. Numerical test as an application of the deduced technique is described under Section 5. Finally, the conclusion of the proposed research work is drawn in Section 6.

The notations used here are as follows: Any alphanumeric having bar at its head is termed as vector quantity, e.g. $(\bar{\cdot})$. Any alphanumeric with bold case is noted as matrix, e.g. (***bold case***). Any alphanumeric with no bar and no bold-case is referred as a scalar quantity. $(\cdot)^T$ denotes the transposition, $\|\cdot\|^2$ denotes the Frobenius norm and $vec(\cdot)$ denotes the stacking of all columns of a matrix into a column vector. Some of the frequently used variables in this article are listed in Table 1. Other notations are defined wherever they are used.

Table 1
Frequently used variables.

$y(x_j,t)$	Measured data at location x_j at time instant t
$\bar{Y}_{x,t}$	Stacked sensor measurements at any time instant t
$\boldsymbol{\varphi}(x)$	Matrix of orthonormal spatial basis functions
$\boldsymbol{\Sigma}_x$	Covariance matrix of the measured data
$\bar{\gamma}(t)$	Temporal coefficients of the spatio-temporal output
$\hat{\bar{\gamma}}(t)$	Estimate of $\bar{\gamma}(t)$
$\bar{\gamma}_{t,j}$	Auxiliary variable for $\bar{\gamma}(t)$ at any node j
$\mathcal{N}_j$	Neighbors of sensing unit j
j'	Neighboring sensing unit of sensor node j

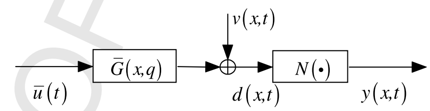

Fig. 1. Wiener distributed parameter system [9].

2. Problem formulation and preface

A Wiener distributed parameter system consists of a linear time-invariant (LTI) system in series with a static nonlinear element. A Wiener DPS can be presented as in Fig. 1.

$\bar{G}(x,q)$ $(1 \times m)$ represents LTI system transfer function, $N(\cdot) : \mathbb{R} \to \mathbb{R}$ is the nonlinear static element representation, t and x are the temporal variable and spatial variable respectively. q represents the forward shift operator. Referring to Fig. 1, intermediate variable $d(x,t)$ and input/output relationship of the system is expressed as

$$d(x,t) = \bar{G}(x,q)\bar{u}(t) + v(x,t), \tag{1}$$

$$y(x,t) = N\left(\bar{G}(x,q)\bar{u}(t) + v(x,t)\right), \tag{2}$$

where $\bar{u}(t) \in \mathbb{R}^m$ is the temporal input, $y(x,t) \in \mathbb{R}$ represents the spatio-temporal measured output at the sensor nodes or the output of the Wiener DPS and $v(x,t) \in \mathbb{R}$ refers to the process noise of the system. The transfer function $\bar{G}(x,q)$ representing LTI system of Wiener DPS can be explicitly written in the form of decomposable infinite orthogonal spatial basis functions $\left\{\boldsymbol{\varphi}_i(x)\right\}_{i=1}^{\infty}$ as

$$\bar{G}(x,q) = \sum_{i=1}^{\infty} \boldsymbol{\varphi}_i(x)\bar{G}_i(q), \tag{3}$$

where $\bar{G}_i(q)$ $(1 \times m)$ is denoting the traditional transfer function. From (3), Wiener DPS can be observed as the synthesizer of temporal and spatial variables.

With the concept of time-space separation framed in Fig. 2, Wiener DPS in Fig. 1 can be represented as Fig. 3. Since the function $F(\cdot)$ can be different from the function $N(\cdot)$, the considered assumption is very much suitable for a broad range of nonlinear systems [9]. DPS are widely represented through PDEs and are known to be of infinite dimension. In order to perfectly model and control any nonlinear infinite dimension system, an infinite number of sensors and actuators are required. This situation is not practically possible due to hardware and cost limitations. So, a limited number of sensors and actuators should be used in a trade-off with complexity and accuracy to analyze the system model.

Distributed Wiener modeling broadly includes three stages as depicted in Fig. 4. Firstly, a framework is designed to estimate the principal components of the data covariance matrix, which are used to reduce the dimension of the observed data. In the second stage of distributed modeling methodology, the parameters of the Wiener model are identified in a distributed manner. The third

Please cite this article in press as: S. Gupta et al., Nonlinear space–time varying parameter estimation using consensus-based in-network distributed strategy, Digit. Signal Process. (2018), https://doi.org/10.1016/j.dsp.2018.05.007