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Performance analysis and computational cost evaluation of high-resolution time-frequency distributions derived from compact support time-lag kernels

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ABSTRACT

This paper considers the objective performance evaluation of kernel-based time-frequency distributions (TFDs) using several concentration performance measures and resolution examination through a deep analysis of time slice plots. On the other hand, the numerical complexity of each TFD is evaluated; a parameter that is particularly critical when real-time implementation is intended. The performance of TFDs based on time-lag kernels with compact support (KCS) namely the Cheriet–Belouchrani (CB), the separable (CB) (SCB) and the polynomial CB (PCB) TFDs is compared to the well-known kernel-based TFDs using several tests on real-life and multicomponent signals with linear and nonlinear frequency modulation (FM) components including the noise effects and the influence of the kernel length. In all presented examples, the time-lag KCS TFDs, and particularly the PCB TFD, provide the best compromise between highest autoterm resolution and interference rejection while still requiring moderate computational costs thanks to the compact support nature of their kernels that reduces the number of points needing computation. On the other hand, the derived distributions do not require any smoothing window neither in time nor frequency in order to achieve the best time-frequency resolution. Furthermore, they have an extremely interesting practical advantage since their adjustment is performed by simply changing a single parameter which is integer for the PCB TFD.

1. Introduction

Real-life signals are generally classified as nonstationary, i.e. as signals with time-varying spectra. The study and analysis of these signals present a very important concept that finds its usage in various technical fields such as acoustics, seismic, radar, sonar, telecommunications and biomedical engineering. Among the available techniques, we are interested in kernel-based time-frequency distributions (TFDs) that perform a mapping of one-dimensional signal $x(t)$ into a two dimensional function of time and frequency $\mathrm{TFD}_x(t,f)$ expressed as [1–4]

$$\mathrm{TFD}_x(t,f) = \int\int\int_{-\infty}^{+\infty} e^{j2\pi\eta(s-t)}\phi(\eta,\tau)x\left(s+\frac{\tau}{2}\right) \times x^*\left(s-\frac{\tau}{2}\right)e^{-j2\pi f\tau}d\eta ds d\tau \quad (1)$$

where $\phi(\eta,\tau)$ is a two-dimensional kernel. Equivalently, one can use the time-lag kernel notation $G(t,\tau)$ expressed as the Fourier transform of the Doppler-lag kernel $\phi(\eta,\tau)$ with respect to η, i.e.

$$G(t,\tau) = \int_{-\infty}^{+\infty} \phi(\eta,\tau)e^{-j2\pi\eta t}d\eta; \quad (2)$$

so that the general class of quadratic TFDs can be defined in terms of the analytic signal $x_a(t)$ associated to the real signal $x(t)$ and the time-lag domain kernel as follows

$$\mathrm{TFD}_{x_a}(t,f) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} G(t-s,\tau)x_a\left(s+\frac{\tau}{2}\right)x_a^*\left(s-\frac{\tau}{2}\right) \times e^{-j2\pi f\tau}ds d\tau \quad (3)$$

Concerning the present paper, our motivation arises from four important problematics related to the field of time-frequency signal analysis (TFSA): First of all, the most efficient quadratic TFDs, that belong to the Cohen's class (Eqs. (1)–(3)), require the spec-

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ification of a kernel that defines the overall performance of the induced representation. Although many TFDs have been proposed in the literature, there is no specific *ideal* distribution that can be considered as the optimal choice for all possible cases and applications because every representation suffers from one or more drawbacks. This makes the building of new kernels for the analysis of time-varying spectra a promising and an open field of research.

Secondly, if time-frequency distributions of quadratic class constitute a powerful tool for nonstationary signals' analysis, their readability, however, is severely affected by the presence of interference terms that are automatically generated due to the bilinear form of these representations. The situation becomes more complex for noisy multicomponent signals while visual inspection is difficult and very subjective. This justifies the need for an objective informational performance measure for TFDs. This is extremely interesting for TFSA in order to allow automatic tuning of the TFD's parameters so that the performance of the generated distribution is optimized. Otherwise, important signal characteristics may be corrupted or become lost while the major challenge is to accurately estimate these characteristics whatever is the application: telecommunications, seismic, sound processing, blind source separation, interference rejection in spread spectrum communications systems, estimation of direction of arrival, multicomponent target detection, and watermarking in multimedia, just to name a few.

For some critical fields like biomedical engineering, time-frequency methods have proved a valuable tool based on their ability to highlight and describe time-varying characteristics [5]. For example, in the field of electroencephalogram (EEG) signal analysis, in particular for newborn babies, it is often intuitively expected that one needs to use TFDs which reduce the effects of crossterms while giving a good resolution [6]. Their ability to show how the energy of the signal is distributed over the 2D t–f domain helps to identify important features such as the number of signal components, rate of change, and regions of energy concentration [6]. The latter correspond, visually, to the choice of the most appealing t–f diagram.

In order to provide an objective assessment for quantifying the concentration and resolution performances, we introduce first a comparative study between the most used concentration-based performance measures applied to time-frequency signal analysis namely the Rényi entropy, the ratio of norms and the Stankovic measure. Then, the most accurate between them is selected to optimize a selection of time-frequency representations including the time-lag KCS-based TFDs. Supported by a deep analysis of time slices, resolution performance is inspected.

Thirdly, for most of the best-known quadratic TFDs, time-frequency resolution is commonly enhanced through the introduction of external windows that smooth the distribution in the time and frequency axes. This makes the TFD's setting harder, slower and more complicated because there are many parameters to adjust: the type of the windows, their respective lengths and the kernel's smoothing parameters. In fact, the most practical benefit of the proposed time-lag KCS-based distributions over the most commonly used TFDs in the literature is that external windowing is no longer needed in order to smooth the generated distribution in time and/or frequency. This is due to the fact that the window is the compact support kernel itself that conserves this property when moving from the time-lag domain to the Doppler-lag domain and becomes even thinner and more concentrated around the origin. Furthermore, the kernels' tuning is performed through a single parameter that is integer for the PCB. This constitutes a very specific feature of this kernel that is particularly interesting for automated optimization and real-time implementation. However, controlling the bandwidth extent is more flexible using the CB and SCB kernels because they use real smoothing parameters.

The fourth point concerns the perspective of real-time implementation of an embedded electronic time-frequency analyzer which has the role of acquiring real-life data, processing, and providing the most accurate information relative to nonstationary signals' energy for specific applications. However, in addition to the data length to be processed in real-time, the required computation load is the most constraining in the development of embedded electronic systems like the widely used FPGA and FPGA/DSP programmable chips. Computational requirements depend mainly on the overall number of arithmetic operations that are reduced to simple additions and multiplications. This is expressed by the resource utilization, especially in terms of the number of real adders and multipliers, that is also proportionally related to power consumption. Knowing that one complex multiplication requires four real multiplications and two additions, it is important to evaluate the computational cost in order to determine the hardware resources needed to implement a given distribution in terms of number of real embedded adders and multipliers. From Eq. (3), we see that implementing a kernel-based TFD involves common operations of Hilbert transform, autocorrelation function and FFT. Hence, the computational cost required to implement a given time-frequency representation is directly related to the kernel's order of complexity in the time-lag plane.

The remainder of this paper is organized as follows. In Section 2, we describe mathematically and graphically the compact support kernels in the time-lag domain. Section 3 introduces a comparative study between the most used theoretical measures that deal essentially with signal concentration and serve as objective performance measures for TFDs. The resolution property is described in Section 4 by analysis of time slices of TFDs. Section 5 develops in depth the computational cost of the investigated kernels by giving details about the numerical evaluation of mathematical operations and functions' approximations. Section 6 is devoted to presenting comparative experimental results obtained by applications involving energy estimation of a real-life signal, linear and nonlinear multicomponent frequency modulated signals including the influence of noise and kernel length. Finally, Section 7 concludes the paper.

2. Time-lag kernels with compact support

2.1. The Cheriet–Belouchrani (CB) time-lag kernel

Applied to TFSA, the compact support kernel [7,8] has the following expression in the time-lag plane

$$G_{KCS}(t,\tau)=\begin{cases} e^{\frac{1}{2}\left(\frac{\gamma}{t^2+\tau^2-1}+\gamma\right)} & \text{if } t^2+\tau^2<1 \\ 0 & \text{otherwise}\end{cases} \tag{4}$$

where γ is a parameter that controls the kernel's bandwidth as σ controls the width of the bell curve for the Gaussian function. The CB kernel [8], referred to as KCS, is defined in the time-lag domain as

$$G_{CB}(t,\tau)=\begin{cases} e^{e_{CB}} & \text{if } \frac{t^2+\tau^2}{D^2}<1 \\ 0 & \text{otherwise}\end{cases} \tag{5}$$

where

$$e_{CB}=C+\frac{CD^2}{(t^2+\tau^2)-D^2} \tag{6}$$

is the exponent of the kernel; D is a predetermined parameter and C is a tuning positive real number that is inversely proportional to the kernel's bandwidth. Fig. 1 shows the plots of the KCS kernel in the time-lag domain for different values of C and $D=2.5$ while

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