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Accurate and computationally efficient interpolation-based method for two-dimensional harmonic retrieval ☆

Jiajia Wang, Weize Sun *, Lei Huang, Jihong Zhang

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ABSTRACT

Determining the frequencies of multiple resolvable exponentials is an important problem due to its application in diverse areas in science and engineering. In this paper, frequency estimation of two-dimensional (2-D) sinusoids is addressed. With the use of the periodogram in frequency domain, the required harmonics are first located coarsely. The characteristics of the 2-D spectrum is then analyzed, and the accurate estimates of the parameters are retrieved using an interpolation method iteratively. It is proved that at sufficiently high signal-to-noise ratio conditions, the harmonic estimates are asymptotically unbiased, and their variances are also analyzed. Furthermore, when only part of the data is observed, the proposed algorithm is tailored to get fast and accurate estimation results. Computer simulations are also included to compare the proposed approach with conventional 2-D harmonic retrieval schemes in terms of root mean square error performance and computational complexity.

1. Introduction

Estimating the harmonic frequencies from 2-D noisy sinusoids is an important issue in various areas, such as the problem of direction-of-arrival (DOA) and direction-of-departure (DOD) estimation in multiple input multiple output (MIMO) radar and the 2-D DOA estimation in array antenna for radar, sonar and wireless communications [1–4].

Lots of approaches have been suggested to estimate the 2-D frequencies in noisy sinusoids. Generally speaking, these methods can be categorized into two types, namely, non-parametric based and parametric based approaches [5,6]. 2-D Fourier transform is the most widely used non-parametric approach to obtain 2-D spectral estimation. In spite of its computational attractiveness when using fast Fourier transform, this grid based approach suffers from poor resolution and resulting in a mismatch between the true frequency and the grids [7,8].

Instead of estimating the signal power spectrum, parametric based estimators, which assume that the signal satisfied a known functional form, try to determine the signal parameters accurately by solving closed-form equations. For example, the Maximum likelihood (ML) method [9], which exploits the Vandermonde structure of the signal model, can work well in 1-D situation but suffers from higher computational burden in the 2-D case. In order to reduce the computational complexity of the ML approach, subspace-based algorithms have been developed, which separate the received data into signal and noise subspaces by using eigenvalue decomposition (EVD) or singular value decomposition (SVD), such as estimation of signal parameters via rotational invariance technique (ESPRIT) [10–12], matrix enhancement and matrix pencil (MEMP) [13,14] and the principal-singular-vector utilization for modal analysis (PUMA) [15–18] approaches. The subspace-based methods are able to provide consistent estimates of frequencies when the signal-to-noise ratio (SNR) is sufficiently high. However, as long as SVD or EVD is used, the computational complexity is relatively high. Furthermore, for some methods, the step of 2-D frequency pairing is needed, leading to a higher complexity [13]. What's worse, when there are two or more frequencies in the same dimension with the same value, some of these algorithms fail to separate the parameters correctly, resulting in serious degraded performance [17].

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* Corresponding author.
E-mail address: proton198601@hotmail.com (W. Sun).

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Recently, an A&M estimator [19], which relies on interpolating on the Fourier coefficients of the received signal samples, is proposed. It is a powerful and efficient algorithm for estimating the frequencies of a 1-D single tone. Later on, Shanglin [20] proposes to combine the schemes of iterative frequency-domain interpolation and leakage subtraction to get accurate estimation results for multiple resolvable signals. However, for a more general 2-D case, a fast and accurate harmonic retrieval method in frequency domain is still an open problem.

Furthermore, in some applications, only part of the data is received [21], which is, the rest of the data are missed or corrupted by noises that exactly cancel the signal values. This usually happens when part of the transmit or receive antennas employ a sub-Nyquist sampling technique which is common in compressive sensing based systems. To overcome such problem, one commonly used technique is to use the matrix completion approaches to obtain the completed data and then use the regular frequency estimators to obtain the parameters from the completed matrix [22,23]. However, the completion methods are normally based on SVD, therefore a very high computational cost is required.

In this paper, we derive an accurate and efficient approach for 2-D multiple harmonics retrieval method based on the interpolation on Fourier coefficients. This approach is more computationally efficient than the state-of-the-art non-parametric estimators. Moreover, we investigate the behavior of the proposed method and show that its estimation performance approaches the Cramér–Rao lower bound (CRLB) [24] when SNR is sufficiently high. Furthermore, when only part of data is received, the method is more efficient than the state-of-the art algorithms, because it avoids the completion step using SVD or EVD.

The rest of the paper is organized as follows. The data model and related background are first presented in Section 2, and two efficient and accurate multi-components estimation algorithms are derived for 2-D fully and partly observed data, respectively, in Section 3. The performance analysis of the proposed estimator for fully observed data case is given in Section 4. In section 5 experimental results of the proposed and state-of-the-art methods are provided. Finally, conclusion is drawn in Section 6.

2. Data model

In this article, the problem of 2-D frequency estimation of multiple signals is tackled. The signal model for 2-D frequency estimation is given as

$$x(m_1,m_2)=\sum_{k=1}^{K}s_k(m_1,m_2)+w(m_1,m_2),\quad m_1=1,2,\dots,M_1, m_2=1,2,\dots,M_2 \tag{1}$$

where K is the number of signals, M_1 and M_2 are the lengths of the first and second dimension, and

$$s_k(m_1,m_2)=A_k e^{j2\pi f_{k1}m_1}e^{j2\pi f_{k2}m_2},\quad k=1,2,\dots,K \tag{2}$$

is the kth noise-free signal term. The A_k is the complex amplitude, while $f_{k1}\in(0,1)$ and $f_{k2}\in(0,1)$ are the frequencies in the first and second dimension of the kth cisoid. The terms $w(m_1,m_2)$ are assumed to be additive white Gaussian noise (AWGN) of which the real and imaginary parts are also AWGN with mean zero and variance $\sigma^2/2$. The SNR of the kth component is defined to be $\rho_k=|A_k|/\sigma^2$.

More generally, we consider the situation that only part of the data is observed. Denoting $\mathbf{M}_{\boldsymbol{\Omega}}$ as the data observed, $M_{\boldsymbol{\Omega}}(m_1,m_2)$ is the product of $x(m_1,m_2)$ and "observation mask" $G(m_1,m_2)$:

$$M_{\boldsymbol{\Omega}}(m_1,n_1)=x(m_1,n_1)\cdot G(m_1,n_1) \tag{3}$$

where

$$G(m_1,n_1)=\begin{cases}1, & (m_1,n_1)\subset\boldsymbol{\Omega}\\ 0, & (m_1,n_1)\not\subset\boldsymbol{\Omega}\end{cases} \tag{4}$$

and $\boldsymbol{\Omega}$ is the index set of observation matrix.

3. Proposed method

3.1. Frequency estimation of a single cisoid

Our main objective is to estimate the 2-D frequencies in (1). It is well known that the **ML** estimator of one single frequency is given by the argument of the periodogram maximizer [25]. Based on the idea of Periodogram, the 2-D frequency estimation of one component ($K=1$) can be achieved by [27]:

$$(\hat{u}_1,\hat{u}_2)=\arg\max_{u_1,u_2}|X(u_1,u_2)|^2=\arg\max_{u_1,u_2}\left|\sum_{m_1=0}^{M_1-1}\sum_{m_2=0}^{M_2-1}x(m_1,m_2)e^{-j\frac{2\pi}{M_1}u_1m_1}e^{-j\frac{2\pi}{M_2}u_2m_2}\right|^2 \tag{5}$$

where $(\hat{u}_1,\hat{u}_2)$ is the maximum amplitude bins of the 2-D spectrum. Furthermore, we have:

$$f_1=\frac{\hat{u}_1+\delta_1}{M_1},\ f_2=\frac{\hat{u}_2+\delta_2}{M_2} \tag{6}$$

where $\delta_1,\delta_2\in[-0.5,0.5]$ are the frequency residuals of two dimensions, respectively.

When the SNR and data length are sufficiently high, i.e., SNR $\to\infty$, $M_1\to\infty$ and $M_2\to\infty$, the $(\hat{u}_1,\hat{u}_2)$ can be correctly computed by taking the maximum amplitude bin of the 2-D spectrum, and the problem is reduced to retrieving the frequency residuals. Denoting $\hat{\delta}_1$, $\hat{\delta}_2$ as the estimates of δ_1,δ_2, the Fourier coefficients, $X_{(1,\pm0.5)}$ and $X_{(2,\pm0.5)}$, located in $(\hat{u}_1+\hat{\delta}_1\pm0.5,\hat{u}_2+\hat{\delta}_2)$ and $(\hat{u}_1+\hat{\delta}_1,\hat{u}_2+\hat{\delta}_2\pm0.5)$ can be written as: