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Signal enumeration in Gaussian and non-Gaussian noise using entropy estimation of eigenvalues

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ABSTRACT

In this paper, a novel method based on the entropy estimation of the observation space eigenvalues is proposed to estimate the number of independent sources impinging on a sensor array. In this method we do not need to know a priori information about the noise model and we can use it in any Gaussian or non-Gaussian model of observations and noise. Our analytical results show that the proposed algorithm is consistent and an approximation for probability of false alarm and an upper bound for probability of missed detection are derived analytically. The performance of the proposed algorithm is compared with the existing methods in the presence of Gaussian and non-Gaussian noise via the simulations. It is shown that this information theoretic method called EEE, has a better performance than those methods in the literature, especially in non-Gaussian noise environment.

1. Introduction

Signal enumeration has an important role in several fields such as brain imaging [1], neural networks [2], audio signals separation [3] and finance [4]. Also, in Direction Of Arrival (DOA) estimation, the number of sources must be known [5].

Some of the proposed solutions for signal enumeration problem are based on information theoretic criteria, for example Akaike-Information Criterion (AIC) [6], Bayesian Information Criterion (BIC) [7], Minimum Description Length (MDL) [8], and Predictive Description Length (PDL) [9]. AIC and MDL, minimize the Kullback–Leibler distance between the observations and the data model that is estimated by the maximum likelihood estimator. The performance of MDL and AIC, has been studied in [10], and [11]. In [12], the probability of missed detection in MDL is derived using the Tracy–Widom distribution for the largest eigenvalue of noise subspace and Gaussian distribution for signal subspace. In [13], new frameworks for analytically evaluating the statistical performance of eigen-decomposition based detectors are considered. Also, in there the exact and asymptotic bounds of the overestimation probability of AIC and MDL are discussed. In [14], the performance of information theoretic based-estimators have been analysed in the case of both Gaussian sources and Gaussian noise. They used the asymptotic distribution of the sample eigenvalues and it is shown that the performance of MDL estimator is not very sensitive to the actual distribution of source signals.

Perry and Wolfe, considered the problem of optimal rank estimation by developing a decision-theoretic rank estimation such as min–max algorithm [15]. In [16], the number of sources is estimated using hypothesis testing as a Neyman–Pearson approach. Also, the probability of overestimation and its bounds are computed. The signal strength required for a high probability detection has been analysed and the results are combined with the random matrix theory (RMT) concepts and then a new signal enumeration algorithm has been developed. In [17], Lu and Zoubir developed a two-step test for signal enumeration, where both of tests are based on the thresholding approach. The first step is similar to [16], except the thresholds used in [16] and [17] are based on Tracy–Widom distribution and Marčenco–Pastur distribution, respectively. In [18], signal enumeration problem is investigated using a criterion based on Generalized Bayesian Information Criterion (GBIC). In the GBIC algorithm, the density of sample eigenvalues has been incorporated with the statistics in BIC, and two algorithms, denoted by $GBIC_1$ and $GBIC_2$, have been developed which are suitable for small sample size and large sample size of observation time, respectively. In [19], by a correlation matrix decomposition method and using the directions of arrival of the signals, the number of independent sources is estimated. In [20], signal enu-

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meration problem is considered using the squared Euclidean norm of the product vector of the steering and Hankel matrices in low SNR regime.

In a different work done by Baik et al. [21], it is shown that only the signal with strength greater than a certain threshold can be detected. This phenomenon called phase transition and it says the signals with smaller strength than the threshold will be considered as the noise signal.

In the most published works, for example [6–20], the authors assumed that the noise is Gaussian and white both spatially and temporally. But in many cases, due to the impulsive nature of the noise, such as sonar [22,23] and impulsive man-made noise in urban and indoor wireless systems [24,25], the noise model is often non-Gaussian or impulsive.

Based on the best knowledge of the authors, the only work to estimate the number of signals in the presence of impulsive noise, is [26]. In there, signal enumeration is investigated via bootstrap method and robust statistics to estimate the source number in an environment with an impulsive noise model. They combined two robust estimators, i.e. the minimum covariance determinant (MCD) estimator and the MM-estimator, with the bootstrap and applied them in the presence of non-Gaussian noise. Also, in [17], since no Gaussian assumption for data model is used for the first step test, it is claimed that the proposed test can be used for the non-Gaussian noise cases, but no result has been presented in this regard.

In this paper, we use a novel approach based on the information earned from the estimated eigenvalues in a sensor array. We use N temporal samples to generate P (the number of array sensors) spatial samples to compute the number of signals. By implementing the kernel methods, the information (entropy) of the estimated eigenvalues will be computed to separate the observation space into signal and noise subspaces. In this method, called Entropy Estimation of Eigenvalues, *EEE*, we do not need to know a priori information about the observations. We only assume that the sources are independent. Then, the proposed algorithm has the ability to deal with the signal enumeration problem in the presence of Gaussian and non-Gaussian noise models. It will be shown that the proposed algorithm is consistent, i.e. EEE could detects true number of sources with probability approaching one when the number of observations tends to infinity. Also, an approximation for false alarm probability and an upper bound for missed detection probability will be derived analytically. Based on the simulation results, we have shown that the proposed method performs better than all the methods reported in the literature in Gaussian and non-Gaussian noise. Especially, in impulsive or non-Gaussian noise model, its performance is obviously better than the others.

This paper is organized as follow: in section 2, we present models and preliminaries. In section 3, we present our motivation to signal enumeration by entropy estimation of the eigenvalues. In section 4, we show that the proposed algorithm is consistent. Sections 5 and 6, consist of the performance analysis of the proposed method, and simulation results, respectively. Finally, section 7 concludes the paper.

Notations: In this paper, we present the matrices by uppercase boldface, i.e. $\mathbf{X}$, and vectors by lowercase boldface, i.e. $\mathbf{x}$. We use $\hat{a}$ as an estimation of a. Also $\mathcal{N}(\mathbf{0}, \mathbf{\Lambda})$ denotes the Gaussian probability of density function (pdf) with zero vector mean and positive definite covariance matrix $\mathbf{\Lambda}$. $\boldsymbol{\lambda}_i^j$ is a vector with samples $(\lambda_i, \lambda_{i+1}, ..., \lambda_j)$. For a Random variable, *Almost sure convergence* also known as convergence with probability one that is shown by (w.p.1).

2. System model and preliminaries

2.1. Problem formulation

Assume that K sources emit their signals, independently, to P ($P > K$) sensors. The received signal at the receiver is denoted by

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{x}(t) = [x_1(t), \cdots, x_P(t)]^T$ is the received vector at the P sensors, and $\mathbf{A} = [\boldsymbol{a}_1, \cdots, \boldsymbol{a}_K]$ is the steering matrix where $\boldsymbol{a}_i, i = 1, \cdots, K$ are linearly independent P-dimensional vectors. Also, the components of the vector $\mathbf{s}(t) = [s_1(t), ..., s_K(t)]^T$ are zero mean and complex random processes. $\mathbf{s}(t)$ has positive definite covariance matrix $\mathbf{R}_S = diag(p_1, \ldots, p_K)$ where $p_i, 1 \le i \le K$, is the received power of the ith source. $\mathbf{n}(t)$ is the noise that is assumed to be additive white zero mean with unknown noise power (σ^2) which is independent from $\mathbf{s}(t)$. Using (1), the population covariance matrix of the received signal from eigen-decomposition can be formulated as follows [8]

$$\begin{aligned}\mathbf{C} &= E\left[\mathbf{x}(t)\mathbf{x}^H(t)\right] \\ &= \mathbf{A}\mathbf{R}_S\mathbf{A}^H + \sigma^2\mathbf{I}_P \\ &= \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H, \quad \mathbf{\Lambda} = diag\left(\lambda_1, \lambda_2, \ldots, \lambda_P\right),\end{aligned} \tag{2}$$

where $\mathbf{I}_P$ denotes the P-dimensional identity matrix, $\mathbf{U}$ is the eigenvectors matrix with dimension $K \times P$, and $(\lambda_1, \lambda_2, \ldots, \lambda_P)$ are the eigenvalues of the population covariance matrix and they are ordered as $\lambda_1 \ge ... \ge \lambda_K > \lambda_{K+1} = ... = \lambda_P$ [8]. The first K eigenvalues, i.e. $(\lambda_1, \lambda_2, ..., \lambda_K)$, are called *signal subspace* eigenvalues, because they are contributed by the source signals and noise. The other smallest $(P - K)$ eigenvalues which belong to the noise subspace, equal to σ^2 [8]. In a limited observation time like as $[t_1, \cdots, t_N]$, an approximation of the sample covariance matrix, can be written as

$$\begin{aligned}\hat{\mathbf{C}} &= \frac{1}{N}\sum_{i=1}^{N}\left[\mathbf{x}(t_i)\mathbf{x}^H(t_i)\right] \\ &= \hat{\mathbf{U}}\hat{\mathbf{\Lambda}}\hat{\mathbf{U}}^H, \hat{\mathbf{\Lambda}} = diag\left(\hat{\lambda}_1, \hat{\lambda}_2, ..., \hat{\lambda}_P\right),\end{aligned} \tag{3}$$

where $\left(\hat{\lambda}_1, \hat{\lambda}_2, ..., \hat{\lambda}_P\right)$ are the sample eigenvalues and $\hat{\lambda}_1 \ge \hat{\lambda}_2 \ge ... \ge \hat{\lambda}_P$. Using the weak law of large numbers (WLLN), $\hat{\mathbf{C}}$ and $\hat{\mathbf{\Lambda}}$ in (3) tend to the population covariance matrix $\mathbf{C}$, and eigenvalue matrix $\mathbf{\Lambda}$, asymptotically as $N \to \infty$, respectively [16].

2.2. Preliminaries

In this subsection, we present some used concepts in our analysis to estimate the number of sources.

2.2.1. Kernel functions

In [27], Paninski showed that the estimation of the probability spaces, only from observation samples, is an important challenge and could be a much difficult problem, and a solution on how to combat with these type of challenges is: *nonparametric estimators*. Also, three algorithms were investigated by Paninski to estimate the entropy: maximum likelihood (ML) [28,29], Miller–Madow bias correction [30], and jackknifed-MLE [31]. All of the above investigated algorithms, are nonparametric estimators that it means they need to estimate the probability space from the observed samples. We will estimate the entropy directly from the observations without knowing about its probability distribution function. This method is based on the kernel functions which are developed by

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