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Computationally efficient direction of arrival estimation with unknown number of signals

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ABSTRACT

In this paper, we investigate the problem of direction of arrival (DOA) estimation with unknown number of signals in the framework of beamforming. We show that the real part of the array output covariance matrix (R-AOCM) can be reformulated as an entire AOCM of a virtual array with available signal model for fast DOA estimate. By introducing an optimization problem to minimize the variance of the weighted output of this virtual array, DOA can be found by a novel real-valued real part Capon (R-Capon) estimator accordingly. Moreover, we prove that the rank of the R-AOCM is always no less than that of the entire AOCM, which suggests that R-Capon outperforms the standard Capon in scenarios with small numbers of snapshots. We also prove that the inverse of the R-AOCM can be equivalently jointed by those of two sub-matrices of about half sizes, and hence R-Capon has a significantly reduced computational complexity. These advantages as well as the theoretical analysis are finally verified by numerical simulations over a wide range of scenarios.

1. Introduction

It is of great interest to find the direction-of-arrivals (DOAs) of multiple narrow-band signals in many applications such as radar, sonar, passive localization and wireless communication [1] [2]. Over the past several decades, a variety of super-resolution subspace-based algorithms including multiple signal classification (MUSIC) [3], estimation of signal parameters via rotational invariance techniques (ESPRIT) [4] and their numerous derivations have been proposed [5–10]. Among those techniques most are based on eigenstructure analysis, in which the determination of effective rank of the array output covariance matrix (AOCM) is generally required to distinguish the exact signal- and the noise- subspaces, which is hard to be established without a prior knowledge of the signal environment [11] [12]. If the number of sources is incorrectly estimated, the performance of those subspace-based algorithms will deteriorate significantly [13–16].

Akaike information criterion (AIC) and minimum description length (MDL) [20] are two of the most important methods for determining the number of sources. Unfortunately, numerical experimental evidence shows that they do indeed tend to estimate a wrong number of components for a small number of snapshots and a low signal-to-noise ratio (SNR), and hence their performance may be not satisfying in practice [21]. In order to estimate source DOAs without knowing the number of sources, many other approaches without source number detection have been proposed recently [22] [23]. Nevertheless, these techniques are inefficient and computationally intensive, which may be too expensive for real-time applications.

Beamforming techniques can be applied to solve the DOA estimation problem without knowing the number of signals [24] [25]. One of the most popular beamformers is the minimum variance distortionless response (MVDR) (also known as Capon) [26–28]. The primary advantage of the Capon algorithm is its easy application with no dependence on array configurations as well as close performance to MUSIC [31]. However, the conventional Capon involves a spectral search requiring to compute the product of the steering vector and the inverse of the entire AOCM. Because both the steering vector and the AOCM are generally modeled as complex-valued, this spectral search requires expensive complex-valued computations accordingly.

In this paper, we focus on the problem of DOA estimation with unknown number of sources, where efficient real-valued computations are taken into account to reduce the computational complexity. We propose a novel Capon-like DOA estimator named the real part Capon (R-Capon) because it exploits only the real part of the AOCM (R-AOCM) instead of the entire AOCM. The proposed method can be used for fast DOA estimation with significantly reduced complexity. Here, we briefly summarize the contributions of this paper.

1) Unlike most state-of-the-art algorithms which generally consider the problem of DOA estimation based on conventional

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signal models resulted from the entire AOCM, a new virtual signal model based on only the R-AOCM is established. It is shown that this virtual signal model can be further used to find signal DOAs because the R-AOCM can be reformulated as an entire AOCM of a virtual array equivalently.
2) A novel modification of the conventional Capon beamformer is proposed for DOA estimation without knowing the number of signals. Compared with the standard Capon, the proposed method exploits only the R-AOCM instead of the entire AOCM and it only requires to compute the inverse of the real matrix R-AOCM. Furthermore, two theorems are provided to show that the inverse of the R-AOCM can be equivalently jointed by those of two sub-matrices of about half sizes, and hence the proposed algorithm in fact has a significantly reduced computational burden as compared to Capon.
3) The rank of the R-AOCM is found to be always no less than that of the entire AOCM, and the proposed technique are found to outperform the standard Capon with small numbers of snapshots. As a result, the proposed estimator obtains reduced computational complexity with improved accuracy as compared to the conventional Capon in such scenarios.

The remainder of this paper is organized as follows. The signal model for DOA estimation with basic assumption as well as a brief review on the conventional Capon is given in section 2. In Section 3, the virtual signal model contained in the R-Capon is discussed, based on which the detailed derivations of the proposed R-Capon algorithm is described. Section 4 presents the theoretical analysis of the proposed method, in which three theorems providing insights into the rank of the R-AOCM and the inverse of the R-AOCM are proved. Numerical simulations are given in Section 5 to demonstrate the effectiveness of the proposed method and to verify the theoretical analysis, and the conclusions of this work are summarized in section 6.

Mathematical notations: Throughout the paper, matrices and vectors are denoted by upper- and lower- boldface letters, respectively. Complex- and real- vectors and matrices are denoted by single-bar- and double-bar- boldface letters, respectively. $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ stand for the real part- and the imaginary part- of the embraced element, respectively. In addition, $j=\sqrt{-1}$, $(\cdot)^*$ is conjugate, $(\cdot)^T$ is transpose, $(\cdot)^H$ is conjugate transpose, $E[\cdot]$ is mathematical expectation, $\|\cdot\|_F^2$ is Frobenius norm, $\mathrm{rank}(\cdot)$ is rank of the embraced matrix, $\mathbb{I}$ is the identity matrix, $\mathbf{0}$ is a matrix or vector with all zero elements, $\mathrm{diag}\{\cdot\}$ is a diagonal matrix composed of the embraced elements and $\mathbb{J}$ is the exchange matrix with ones on its anti-diagonal and zeros elsewhere.

2. Signal model and conventional capon

Assume that K uncorrelated narrow-band signals $s_k(t)$, $k\in[1,K]$ with unknown DOAs $\boldsymbol{\theta}\triangleq[\theta_1,\theta_2,\ldots,\theta_K]$ impinge from far-field on a uniform linear array (ULA) composed of M antenna elements simultaneously. In most subspace-based high-resolution DOA estimators, M is generally assumed to be larger than K [13] [29]. In the proposed technique, it is assumed $M>2K$, which is reasonable for large arrays [30]. Although this is more strict as compared to the conventional Capon, it is to be shown this assumption allows a significant reduction on complexity.

Let the first sensor be the reference point, the array output at snapshot $t, t\in[1,T]$ is given by [1–14]

$$\mathbf{x}(t)=\mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t)+\mathbf{n}(t), \tag{1}$$

where $\mathbf{A}(\boldsymbol{\theta})=\left[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\ldots,\mathbf{a}(\theta_K)\right]$ is the $M\times K$ array manifold with each column

$$\mathbf{a}(\theta)=\left[1,e^{j(2\pi/\lambda)d\sin\theta},\cdots,e^{j(2\pi/\lambda)d(M-1)\sin\theta}\right]^T \tag{2}$$

denoting the $M\times 1$ steering vector, where λ is the wavelength of the narrow-band signals. $\mathbf{n}(t)$ is the $M\times 1$ additive white Gaussian noise (AWGN) satisfying

$$\begin{cases} E\left[\mathbf{n}(t)\mathbf{n}^T(t)\right]=\mathbf{0} & \text{(a)}\\ E\left[\mathbf{n}(t)\mathbf{n}^H(t)\right]=\sigma_n^2\mathbb{I}, & \text{(b)}\end{cases} \tag{3}$$

where σ_n^2 is the noise power, $\mathbb{I}$ is the identity matrix. $\mathbf{s}(t)$ is the $K\times 1$ signal vector, which satisfies

$$\begin{cases} E\left[\mathbf{s}(t)\mathbf{s}^T(t)\right]=\mathbf{0} & \text{(a)}\\ E\left[\mathbf{s}(t)\mathbf{s}^H(t)\right]=\mathbb{S}, & \text{(b)}\end{cases} \tag{4}$$

where $\mathbf{0}$ is the zero matrix and $\mathbb{S}$ is an $M\times M$ real diagonal, denoting the source covariance matrix. Assumption (4) is made to guarantee that the incident signals are circular, which is commonly used as a standard hypothesis for narrow-band signal DOA estimate [17–19]. The $M\times M$ AOCM is given by

$$\mathbf{R}=\mathrm{E}\left[\mathbf{x}(t)\mathbf{x}^H(t)\right]=\mathbf{A}(\boldsymbol{\theta})\mathbb{S}\mathbf{A}^H(\boldsymbol{\theta})+\sigma_n^2\mathbb{I}. \tag{5}$$

In practice, the theoretical AOCM in (5) is unavailable, and we can use T snapshots of the observed data to obtain the estimated AOCM (EAOCM) as

$$\widehat{\mathbf{R}}=\frac{1}{T}\sum_{t=1}^{T}\mathbf{x}(t)\mathbf{x}^H(t). \tag{6}$$

If K is known or has been estimated by methods [20–23] in advance, subspace-based approaches such as MUSIC [3], ESPRIT [4] and their derivations [5–10] generally compute the eigenvalue decomposition (EVD) of the EAOCM to extract the exact signal- or noise- subspaces to derive different cost functions for further DOA estimate. It has been shown in [13] [14] that the performances of those techniques will deteriorate significantly if K is incorrectly estimated.

To avoid signal number detecting, the conventional Capon [26–31] defines a weighted array output $\mathbf{y}(t)=\mathbf{w}^H\mathbf{x}(t)$ and finds DOAs by minimizing the power of $\mathbf{y}(t)$ as

$$\min_{\mathbf{w}}\ \mathrm{E}\left[\mathbf{y}(t)\mathbf{y}^H(t)\right]=\mathbf{w}^H\mathbf{R}\mathbf{w},\quad \text{s.t. } \mathbf{w}^H\mathbf{a}(\theta)=1. \tag{7}$$

By solving (7), DOAs can be found by searching the peaks of

$$f_{\mathrm{Capon}}(\theta)=\frac{1}{\mathbf{a}^H(\theta)\widehat{\mathbf{R}}^{-1}\mathbf{a}(\theta)} \tag{8}$$

over $[-\pi/2,\pi/2]$. The peak number of $f_{\mathrm{Capon}}(\theta)$ gives signal amount while the peak locations indicate signal DOAs.

3. Proposed R-Capon algorithm

3.1. Virtual signal model

We are to show in this subsection that the R-AOCM contains a virtual signal model available for DOA estimate. Let $\mathbb{R}_{\mathrm{re}}\triangleq\mathrm{Re}(\mathbf{R})$. Using $\mathbb{R}_{\mathrm{re}}=\frac{1}{2}(\mathbf{R}+\mathbf{R}^*)$ and (5), we have

$$\begin{aligned}\mathbb{R}_{\mathrm{re}}&=\frac{1}{2}\left[\mathbf{A}(\boldsymbol{\theta})\mathbb{S}\mathbf{A}^H(\boldsymbol{\theta})+\mathbf{A}^*(\boldsymbol{\theta})\mathbb{S}\mathbf{A}^T(\boldsymbol{\theta})\right]+\sigma_n^2\mathbb{I}\\ &=\underbrace{\left[\mathbf{A}(\boldsymbol{\theta})\ \mathbf{A}^*(\boldsymbol{\theta})\right]}_{M\times 2K}\times\underbrace{\frac{1}{2}\begin{bmatrix}\mathbb{S} & \mathbf{0}\\ \mathbf{0} & \mathbb{S}\end{bmatrix}}_{2K\times 2K}\times\underbrace{\begin{bmatrix}\mathbf{A}^H(\boldsymbol{\theta})\\ \mathbf{A}^T(\boldsymbol{\theta})\end{bmatrix}}_{2K\times M}+\sigma_n^2\mathbb{I}\\ &=\widetilde{\mathbf{A}}(\boldsymbol{\theta})\widetilde{\mathbb{S}}\widetilde{\mathbf{A}}^H(\boldsymbol{\theta})+\sigma_n^2\mathbb{I},\end{aligned} \tag{9}$$