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Fourier–Bessel series expansion based empirical wavelet transform for analysis of non-stationary signals

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ABSTRACT

In this paper, a new method has been presented for the time–frequency (TF) representation of non-stationary signals. The existing empirical wavelet transform (EWT) has been enhanced using Fourier–Bessel series expansion (FBSE) in order to obtain improved TF representation of non-stationary signals. We have used the FBSE method for the spectral representation of the analyzed multi-component signals with good frequency resolution. The scale-space based boundary detection method has been applied for the accurate estimation of boundary frequencies in the FBSE based spectrum of the signal. After that, wavelet based filter banks have been generated in order to decompose non-stationary multi-component signals into narrow-band components. Finally, the normalized Hilbert transform has been applied for the estimation of amplitude envelope and instantaneous frequency functions from the narrow-band components and obtained the TF representation of the analyzed non-stationary signal. We have applied our proposed method for the TF representation of multi-component synthetic signals and real electroencephalogram (EEG) signals. The proposed method has provided better TF representation as compared to existing EWT method and Hilbert–Huang transform method, especially when analyzed signal possesses closed frequency components and of short time duration.

1. Introduction

Non-stationary signal analysis methods are focused to model the inherent time-varying characteristics of the analyzed signals recorded in several areas namely, communications, speech analysis and synthesis, radar, biomedical, and mechanical engineering [1]. The conventional Fourier transform based methods are not well suited for the spectral analysis of such signals. Moreover, the real life complicated biological signals [2–5], finance signals [6], civil structure vibration signals [7,8] are highly non-stationary in nature which require adequate analysis based on their information content.

In literature, several time–frequency (TF) domain based methods exist for analysing non-stationary signals namely short-time Fourier transform (STFT) [9], wavelet transform (WT) [10], Wigner–Ville distribution (WVD) [1], and Hilbert–Huang transform (HHT) [11]. The features can be extracted in TF domain for the classification of non-stationary signals [12]. The STFT provides TF representation of non-stationary signals based on moving window concept. But, the use of fixed moving window imposes a trade off between time and frequency resolutions in TF plane [13]. In order to overcome the limitations of STFT, the WT was proposed for the efficient TF representation of non-stationary signals. The WT uses pre-fixed basis functions and decomposes signals into different oscillatory levels with the transient characteristics of the analyzed signals retained [14]. However, due to predefined filter banks, WT fails to decompose signals according to the presence of information content. Therefore, it is difficult to determine instantaneous amplitude (IA) and instantaneous frequency (IF) functions from the decomposed components corresponding to actual mono-component signals. The wavelet packet transform [15,16] was proposed in order to enhance signal adaptability. However, this approach is still limited by the use of prefixed basis functions. In [17], authors proposed tunable-Q wavelet transform (TQWT) for the analysis of non-stationary signals. In TQWT method, the *Q* factor of the wavelet transform can be tuned in accordance to the oscillatory nature of the signal. In [18], authors proposed synchrosqueezed wavelet transform as the combination of wavelet transform and reallocation methods. This method achieved better TF resolution as compared to conventional wavelet transform. In [19], authors introduced variational mode decomposition (VMD) method which decomposes real valued signal into finite number of components. The VMD method has been found suitable for analysing non-stationary signals and applied in [20] for instantaneous voiced and

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non-voiced detection from speech signals and in [21] for speech enhancement.

Majority of the existing methods rely on pre-fixed basis functions to analyze the signals and hence are considered to be rigid or non-adaptive. However, the non-adaptive methods find difficulty in analyzing physical signals due to the existence of closely spaced frequency components. In the HHT based [11] TF representation, empirical mode decomposition (EMD) method adaptively decomposes analyzed signals into amplitude and frequency modulated components known as intrinsic mode functions (IMFs). Later, the Hilbert transform (HT) was applied in each of the IMFs for the estimation of IA and IF functions and TF representation was generated. In [22], authors proposed empirical decomposition method for separating amplitude modulation (AM) and frequency modulation (FM) parts from each IMF in order to estimate more meaningful IFs. In [23], authors cancelled the occurrences of riding waves from the empirically decomposed FM parts of the IMFs. However, the EMD method suffers from mode mixing problem and thus, IF functions cannot be effectively estimated [24]. In [25], authors proposed adaptive Fourier decomposition method in order to decompose non-stationary signals into a number of Fourier intrinsic band functions and provided time–frequency–energy distribution of the analyzed signals. In [26], authors introduced swarm decomposition based on fosters rules of biological swarms for the analysis of non-stationary signals. The method achieved good performance in extracting the components from the analyzed signals. In [27], TF representation was proposed using improved eigenvalue decomposition of the Hankel matrix together with Hilbert transform. The TF representation using WVD is highly affected by the presence of cross-terms between the signal components [28,29]. This puts a major limitation towards the efficient estimation of instantaneous frequencies of the analyzed signals in multi-component situation.

In [30], empirical wavelet transform (EWT) was proposed for the analysis of non-stationary signals. In [31], the authors explored the EWT method for multivariate signals and EWT based multivariate TF representation was proposed. It should be noted that, EWT is an adaptive decomposition method which extracts narrow-band frequency components from the analyzed signal based on the frequency information content in the signal spectrum. It decomposes signals with adaptive wavelet based filters after finding the boundary frequencies in the fast Fourier transform (FFT) based spectrum. The TF representation based on EWT, can be obtained after applying the Hilbert transform on the narrow band frequency components. However, EWT fails to represent closely spaced frequency components in the TF plane [32]. Moreover, it is very difficult to estimate the accurate frequency components for the short duration signals using EWT method due to the use of Fourier spectrum.

Thus, an improved method is desirable in order to encounter the aforementioned shortcomings of the existing EWT method. In this work, the conventional FFT based spectrum has been replaced with the spectrum obtained using Fourier–Bessel series expansion (FBSE). It should be noted that, FBSE coefficients are useful for the spectral analysis of non-stationary signals due to the non-stationary nature of Bessel functions bases in the FBSE [33–35]. We have employed the existing scale-space based boundary detection method for the spectrum segmentation purpose. Finally, the normalized Hilbert transform (NHT) [22] based TF representation has been generated for analysing multi-component non-stationary signals. The proposed method has been applied on synthetically generated multi-component AM and FM signals as well as on real electroencephalogram (EEG) signals. We have obtained better TF representation using proposed FBSE-EWT method as compared to existing EWT and HHT based TF representation for majority of the considered cases.

The reminder of the paper is organised as follows: Section 2 briefly discusses the existing methods, Section 3 demonstrates the proposed method. The experimental results have been presented in Section 4, while Section 5 discusses the effectiveness of the proposed method. Finally, Section 6 concludes the paper.

Table 1
Mathematical expressions of EWT scaling and wavelet functions.

Functions	Mathematical representation
Scaling [30]	$\Lambda_i(\omega) = \begin{cases} 1, & \text{if } \lvert\omega\rvert \le (1-\xi)\omega_i. \\ \cos\left(\frac{\pi\eta(\xi,\omega_i)}{2}\right), & \text{if } (1-\xi)\omega_i \le \lvert\omega\rvert \le (1+\xi)\omega_i. \\ 0, & \text{otherwise} \end{cases}$
Wavelet [30]	$\Theta_i(\omega) = \begin{cases} 1, \\ \quad \text{if } (1+\xi)\omega_i \le \lvert\omega\rvert \le (1-\xi)\omega_{i+1}. \\ \cos\left(\frac{\pi\eta(\xi,\omega_{i+1})}{2}\right), \\ \quad \text{if } (1-\xi)\omega_{i+1} \le \lvert\omega\rvert \le (1+\xi)\omega_{i+1}. \\ \sin\left(\frac{\pi\eta(\xi,\omega_i)}{2}\right), \\ \quad \text{if } (1-\xi)\omega_i \le \lvert\omega\rvert \le (1+\xi)\omega_i. \\ 0, \text{otherwise}. \end{cases}$

2. A brief explanation of EWT, boundary detection based on scale-space representation, FBSE, normalized Hilbert transform, and performance evaluation measure

Here, in the following sub-sections, we have described the EWT method, the scale-space representation, the FBSE based spectrum representation, the NHT based TF representation, and performance evaluation in terms of mean square error (MSE) measure.

2.1. Empirical wavelet transform

The EWT is an adaptive signal decomposition method which was proposed in [30] for the analysis of non-stationary signals. The inherent mechanism of EWT is based on the formation of adaptive wavelet based filters. These wavelet based filters possess support in the spectrum information location of the analyzed signal. The obtained sub-band signals after EWT decomposition have specific centre frequencies with compact frequency supports. The EWT method is summarised in the following steps [30]:

1. The FFT method is used to obtain frequency spectrum of the analyzed signal in the frequency range $[0, \pi]$.
2. The frequency spectrum is segmented into N number of contiguous segments using EWT boundary detection method. In this work, we have used scale-space based boundary detection method [36] in order to find optimal set of boundary frequencies denoted as ω_i. In the next subsection, the scale-space based boundary detection method is discussed in brief. It should be noted that, the first and last boundary frequencies are prefixed to 0 and π, respectively. Thus, EWT boundary detection method is used to find the rest of the $N-1$ intermediate boundary frequencies.
3. The empirical scaling and wavelet functions are defined in each segment as the set of band-pass filters. The idea of construction of Littlewood–Paley and Meyer's wavelets is used for the construction of wavelet based filters [10,30].

The mathematical expressions of empirical scaling function $\Lambda_i(\omega)$ and wavelet function $\Theta_i(\omega)$ have been presented in Table 1. In the table, the function $\eta(\xi,\omega_i)$ is expressed as [30],

$$\eta(\xi,\omega_i) = \psi\left(\frac{(\lvert\omega\rvert - (1-\xi)\omega_i)}{2\xi\omega_i}\right) \tag{1}$$

where $\psi(z)$ is an arbitrary function defined as [30],