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Brief paper

Deviation bounds in multi agent systems described by undirected graphs[☆]

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ABSTRACT

The theory of port-Hamiltonian systems is used to derive upper bounds for the state deviations in multi-agent systems described by undirected graphs pinned to a reference signal. The upper bounds for the deviations in networks of first or second order agents, respectively, depend on the minimal eigenvalue of the extended Laplacian of the system. In networks of first order agents, the deviations decay exponentially with a rate depending on the same minimal eigenvalue. In case networks of second order systems meet specific design properties, it can be shown that the deviations also decay exponentially with half the rate compared to first order systems.

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1. Introduction

A Multi-Agent System (MAS) describes a group of autonomous agents operating in a networked environment. Control engineers are interested in designing strategies for a MAS to achieve global control objectives through distributed sensing, communication, computing, and control. A common control objective is "consensus" where local algorithms ensure that all agents in the system converge to the same output or state value. A simple yet robust output-feedback controller to achieve consensus is designed in Münz, Papachristodoulou, and Allgöwer (2011). Consensus algorithms, which are robust to time delays, network size, and modelling errors, can be found in Das and Lewis (2010), Liu, Lu, and Chen (2010), Moreau (2004), Münz, Papachristodoulou, and Allgöwer (2010), Tian and Liu (2009) and Yang, Roy, Wan, and Saberi (2011).

In the area of "pinning control", a fraction of the nodes is connected (i.e., "pinned") to a reference signal. For pinning control of networks of first-order agents see Chen, Chen, Xiang, Liu, and Yuan (2009), Chen, Liu, and Lu (2007), Liu, Chen, and Lu (2009), Ren (2007) and Wang and Chen (2002). The results show that if the directed graph has a spanning tree, all agents approach a prescribed value if some are pinned. Consensus of double integrators was studied in Ren (2008). If a group reference velocity is available to each agent, then consensus is reached asymptotically if the directed interaction graph has a directed spanning tree and the gain for the velocity matching with the group reference velocity is above a certain bound. If the reference state is only available to a subset of the agents, then consensus is reached asymptotically if and only if the network is strongly connected. Lu, Ho, and Wang (2009) show that linearly coupled stochastic neural networks can be controlled by a minimal number of controllers.

One of the most difficult problems in the area of pinning control is to choose the best set of pinned nodes. For scale-free networks it is much more effective to pin some highly connected nodes compared to randomly selected nodes, Wang and Chen (2002). In random networks, there is no significant difference between pinning specific or random nodes, Li, Wang, and Chen (2004). Yu, Chen, and Lü (2009) revealed that a network can realise synchronisation under any linear feedback pinning scheme by adaptively adjusting the coupling strength. V-stability was used in Xiang and Chen (2007, 2009) to develop pinning schemes. The determinants of the principle minors are used in Xiong, Ho, and Huang (2010) to compute which nodes should be pinned. An approach to select strongly connected components was developed in Lu, Li, and Rong (2010). It was further shown in Song and Cao (2010) that nodes whose out-degrees are bigger than their in-degrees should be pinned. Further, the randomly pinning scheme may not guarantee the synchronisation of directed complex networks. Second-order nonlinear MASs were studied in Song, Cao, and Yu (2010).

In the area of "string stability", a group of vehicles drives in a platoon or string. In a unidirectional string, each vehicle

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follows its direct predecessor whereas in a bidirectional string, the distance towards the following vehicle is also used. The first vehicle follows a reference signal. This can be seen as a special case of a pinned network. Due to the simple network structure, it is trivial to ensure that all vehicles follow the trajectory. The main control objective is to design local controllers such that the distances between the vehicles remain bounded, independently of the string size, i.e. "string stability". It was shown in Barooah and Hespanha (2005) and Seiler, Pant, and Hedrick (2004) that both unidirectional and symmetric bidirectional linear strings with two integrators in the open loop and constant spacing are always string unstable. Approaches to guarantee string stability include using: (i) a time headway, Chien and Ioannou (1992); (ii) heterogeneous controllers, Khatir and Davison (2004); (iii) information of the lead vehicle, Darbha, Hedrick, Chien, and Ioannou (1994); or, (iv) the reference velocity, Barooah, Mehta, and Hespanha (2009). In Barooah et al. (2009), a linear bidirectional string is approximated as a PDE to derive stability bounds. This work was later extended in Hao and Barooah (2012) and Hao, Yin, and Kan (2012). Lately, it was shown in Knorn, Donaire, Agüero, and Middleton (2014) that symmetric bidirectional strings can be modelled as port-Hamiltonian systems, see van der Schaft and Jeltsema (2014) and van der Schaft and Maschke (2013).

This paper extends Knorn et al. (2014) to undirected networks of single- or double-integrators, showing that:

(i) The deviations between the states and the reference signal are bounded and the upper bound depends on the smallest eigenvalues of the extended Laplacian matrix describing the pinned network, i. e. $\lambda_{\min}(\bar{\mathcal{L}})$.
(ii) In some classes of systems the deviations can be guaranteed to decay exponentially with a rate that also depends on $\lambda_{\min}(\bar{\mathcal{L}})$.
(iii) Examples are presented to illustrate the results.

Work on (i) was inspired by the problem of string stability, which aims to design local controllers ensuring the existence of a uniform bound of the inter vehicle distances. In contrast, this paper derives bounds on the deviations in general undirected graphs. Note further that our results are an extension of the well-known problem of (leader-following) consensus and pinning control. But instead of investigating under which conditions consensus can be achieved or which nodes should be pinned, it is assumed that the pinned network will converge, and the behaviour of the *deviations* towards the desired equilibrium is studied. Note that some similar results studying homogeneous systems (i.e., MAS with identical agents) have been presented in the preliminary work Hao and Barooah (2011).

Section 2 clarifies mathematical preliminaries. Upper bounds and decay rates for the deviations are derived in Sections 3 and 4, respectively. Before concluding in Section 6, illustrative examples are presented in Section 5.

2. Notation and mathematical preliminaries

2.1. Notation

Consider the static vector $x \in \mathbb{R}^n$ and the time-varying vector $x(t) \in \mathbb{R}^n$. The L_2 vector norm is given by $|x|_2 = |x| = \sqrt{x^{\mathrm{T}}x}$ and the L_2 and L_∞ vector function norms by $\|x(\cdot)\|_2 = \sqrt{\int_0^\infty |x(t)|^2 \mathrm{d}t}$ and $\|x(\cdot)\|_\infty = \sup_{t\geq 0}|x(t)|$, respectively. For a scalar function $H(x)$ of a vector $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$ its gradient is $\nabla H(x) = [\frac{\partial H(x)}{\partial x_1}, \frac{\partial H(x)}{\partial x_2}, \ldots, \frac{\partial H(x)}{\partial x_n}]^{\mathrm{T}}$. The column vector of ones is $\underline{1}$ and $\vec{e}_i \in \mathbb{R}^n$ is the ith canonical vector of length n. We denote the diagonal matrix $A \in \mathbb{R}^{n\times n}$ with diagonal entries $a_1, \ldots, a_n$ as $A = \mathrm{diag}(a_1, \ldots, a_n)$. Given A is symmetric positive definite ($A > 0$), $x^{\mathrm{T}}Ax \leq \lambda_{\max}(A)|x|^2$ where $\lambda_{\max}(A)$ is the maximal eigenvalue of A, (Bernstein, 2009). $\lambda_{\min}(A)$ denotes the minimal eigenvalue of A. The identity matrix of dimension $n \times n$ is defined as I_n. Further, $\dot{x}(t) := \frac{\mathrm{d}x(t)}{\mathrm{d}t}$, $\ddot{x}(t) := \frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2}$ and "iff" = "if and only if".

2.2. Consensus networks

In its simplest case, a consensus network consists of a group of n_{a} agents, that are simple integrators

$$\dot{x}_i(t) = u_i(t) \quad \text{for } i \in \{1, 2, \ldots, n_{\mathrm{a}}\} \tag{1}$$

where $u_i(t)$ is the control input. It is the aim to reach consensus in the network, i.e., the states of each agent to converge to the weighted average of the states of its neighbours:

$$u_i(t) = \sum_{j=1, j\neq i}^{n_{\mathrm{a}}} a_{ij}(x_j(t) - x_i(t)) \tag{2}$$

where a_{ij} is the weight of the connection between agents i and j. There is no connection between i and j iff $a_{ij} = 0$.

Considering double integrator agents leads to

$$\ddot{x}_i(t) = u_i(t) \quad \text{with} \tag{3}$$

$$u_i(t) = \sum_{j=1, j\neq i}^{n_{\mathrm{a}}} a_{ij}(x_j(t) - x_i(t)) + \sum_{j=1, j\neq i}^{n_{\mathrm{a}}} r_{ij}(\dot{x}_j(t) - \dot{x}_i(t)) \tag{4}$$

where r_{ij} is the weight of the connection between the first derivatives of agents i and j. We assume $a_{ij} \neq 0$ iff $r_{ij} \neq 0$.

2.3. Graph theory

Consider the network (3)–(4). The agents can be regarded as "nodes" or "vertices" v of a graph. An "edge" e starts at node i and ends at node j iff $a_{ij} \neq 0$. In case the input equations are symmetric, i. e. $a_{ij} = a_{ji}$ and $r_{ij} = r_{ji}$, the graph is undirected. Then, it can be described by the Laplacian matrix, which is the product of the oriented incidence matrix $\mathcal{B} \in \mathbb{R}^{n_{\mathrm{a}}\times n_{\mathrm{e}}}$ with its transpose (where n_{a} or n_{e} are the number of agents or edges, respectively), such that $\mathcal{L} = \mathcal{B}\mathcal{B}^{\mathrm{T}}$. $\mathcal{B}$ is obtained by arbitrarily choosing a direction for all e and setting $(\mathcal{B})_{ve} = 1$ if e enters v, $(\mathcal{B})_{ve} = -1$ if e leaves v and $(\mathcal{B})_{ve} = 0$ otherwise. (For an example see Section 5.)

2.4. Pinning control and reference following

Some applications require the network to converge to a given reference. Since it is often impossible, undesirable, or unnecessary to connect all agents to the reference, only some nodes are pinned. We assume that the network is connected. Hence, pinning a single node is sufficient, Lu et al. (2009), but pinning more nodes will lead to a better performance, Patterson and Bamieh (2010). Consider (1)–(2). Pinning the first $n_{\mathrm{p}} < n_{\mathrm{a}}$ nodes to the scalar reference signal $x^*(t)$ yields

$$u_i(t) = \sum_{j=1, j\neq i}^{n_{\mathrm{a}}} a_{ij}(x_j(t) - x_i(t)) + \alpha_i(x^*(t) - x_i(t)), \tag{5}$$

for $i \leq n_{\mathrm{p}}$, where α_i is the weight of the connection between i and the reference. For (3)–(4), adding pinning control yields

$$u_i(t) = \sum_{j=1, j\neq i}^{n_{\mathrm{a}}} a_{ij}(x_j(t) - x_i(t)) + \sum_{j=1, j\neq i}^{n_{\mathrm{a}}} r_{ij}(\dot{x}_j(t) - \dot{x}_i(t)) + \alpha_i(x^*(t) - x_i(t)) + \rho_i(\dot{x}^*(t) - \dot{x}_i(t)), \tag{6}$$

for $i \leq n_{\mathrm{p}}$, where ρ_i is the weight of the connection between the first derivatives of i and the reference. Extending the graph theory above, define the extended oriented incidence matrix $\bar{\mathcal{B}} := (\vec{e}_1, \ldots, \vec{e}_{n_{\mathrm{p}}}, \mathcal{B})$. Following the relationship $\mathcal{L} = \mathcal{B}\mathcal{B}^{\mathrm{T}}$, we define the extended Laplacian as $\bar{\mathcal{L}} := \bar{\mathcal{B}}\bar{\mathcal{B}}^{\mathrm{T}}$.

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